# Chapter 18
# Inflation and Unemployment

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Inflation and unemployment are two of the most important macroeconomic problems. Indeed, the main goals of macroeconomic stabilization policy are to promote price stability and to prevent cyclical unemployment. In this chapter we shall study the relationship between inflation and unemployment. As we shall see in the next chapter, understanding the link between these two variables is crucial for understanding how the supply side of the economy works.

For many years after World War II, most economists and policy makers believed that there is a fundamental tradeoff between inflation and unemployment: if you want less inflation, you have to live with permanently higher unemployment, and vice versa. If this view were correct, inflation and unemployment should tend to move in opposite directions. A glance at Figure 1 reveals that this is often not the case. For example, in several years during the 1970s there was a simultaneous rise in inflation and unemployment, and in several years during the 1990s both of these problems simultaneously diminished. A satisfactory theory of inflation and unemployment must be able to account for these facts.

In the following sections we will try to construct such a theory. In doing so, we will build on the theory of wage and price formation underlying our theory of structural unemployment in chapters 13 and 14. As we will demonstrate below, this framework can explain the short-run link between inflation and cyclical unemployment as well as the factors determining the long-run equilibrium rate of unemployment (the 'natural' rate). By cyclical unemployment we mean unemployment *in excess of* the natural rate, that is, unemployment caused by the recurring cyclical downturns of the economy. Note that cyclical unemployment can also become *negative* during the expansion phase of the business cycle when excess aggregate demand drives actual unemployment below the natural rate. In that case we shall see that there will be a tendency for inflation to accelerate.

We will start by brushing up our theory of price and wage formation introduced in chapters 13 and 14. We will then show how this theory may explain the short-run relation between inflation and unemployment, and we will confront our theory with the data. In the final part of the chapter we will study how productivity growth affects the relationship between inflation and unemployment and discuss whether the process of wage and price formation has changed in recent years, as some observers have argued.

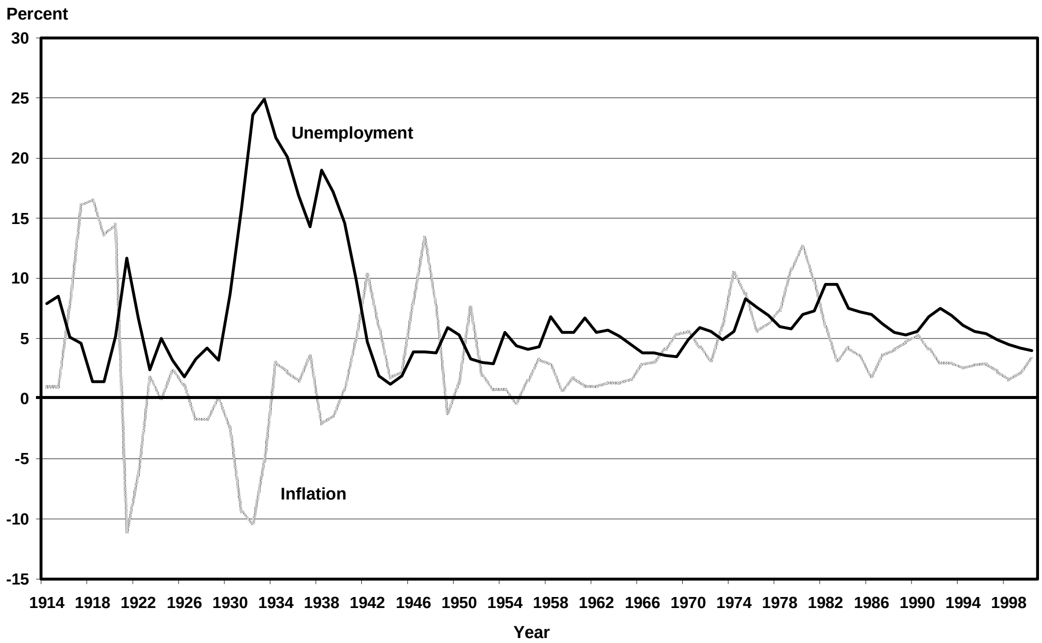


**Figure 18.1.a: Inflation and unemployment in the United States**

Note: The rate of inflation is measured by the percentage increase in the consumer price index.

Source: "International Historical Statistics", R.B. Mitchell, Macmillian 1998; and Bureau of Labor Statistics

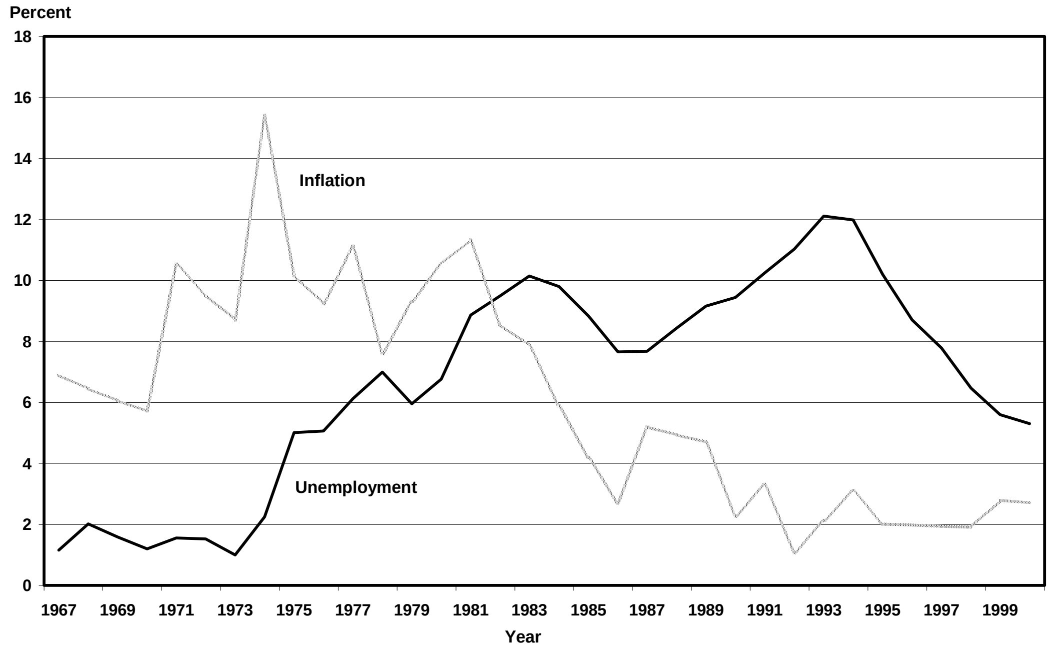


**Figure 18.1.b: Inflation and unemployment in Denmark**

Note: The rate of inflation is measured by the percentage increase in the consumer price index.

Source: ADAM database

# 1 The Formation of Prices and Wages

### Price Setting under Monopolistic Competition

Inflation is defined as a continuous rise in the general level of nominal prices and wages. Obviously, a theory of inflation therefore requires a theory of price and wage formation. In chapters 13 and 14 we have already presented such a theory, which we briefly restate here for convenience, starting with the theory of price setting.

We consider an economy which is divided into $n$ production sectors. In each sector a representative firm is the sole producer of a particular differentiated product. Consumer demand $D$ for the output of firm (sector) $i$ is given by the demand curve

$$D\left(P_i\right) = \left(\frac{P_i}{P}\right)^{-\sigma} \frac{Y}{n}, \qquad \sigma > 1 \tag{1}$$

where $P_i$ is the price of good $i$, $P$ is the general price level, $Y$ is total GDP, and $\sigma$ is the numerical price elasticity of demand reflecting the degree to which consumers are willing to substitute good $i$ for other goods.[1] As long as $\sigma$ is finite, the demand curve is downward-sloping, and the firm then enjoys some monopoly power.

The output $Y_i$ of firm $i$ is a simple linear function of its labour input $L_i$:

$$Y_i = aL_i. \tag{2}$$

Equation (2) implies that the marginal and average productivity of labour is constant and equal to the parameter $a$. This implicitly assumes that the firm's existing capital stock is not fully utilized. As long as there is some unused capacity, output may be increased without driving down the marginal productivity of labour, as postulated in equation (2).

The nominal wage rate paid by firm $i$ is $W_i$, and the marginal productivity of labour is $a$. The firm's marginal cost is therefore equal to $W_i/a$. From microeconomic theory we know that a monopolist wishing to maximize profits will expand output and sales until marginal revenue equals marginal cost. Since $\sigma$ is the numerical price elasticity of demand, we also know that firm $i$'s marginal revenue equals $P_i\frac{\sigma-1}{\sigma}$.[2] Hence the firm's optimal price can be found from the condition $P_i\frac{\sigma-1}{\sigma} = W_i/a$, implying

$$P_i = m\frac{W_i}{a}, \qquad m \equiv \frac{\sigma}{\sigma-1} > 1 \tag{3}$$

Thus the monopolistically competitive firm sets its price as a mark-up over its unit labour cost $W_i/a$. The mark-up factor $m$ is seen to be larger, the smaller the price elasticity of demand $\sigma$. This is intuitive, since a low price elasticity means that the firm has considerable market power.

[1] As mentioned in Chapter 13, the demand curve (1) may be derived from the solution to the consumers' problem of utility maximization if utility functions are of the CES form. In that case the parameter $\sigma$ is the representative consumer's elasticity of substitution between good $i$ and any other good. See Exercise 13.3.

[2] The total revenue of firm $i$ is $TR_i \equiv P_iY_i$, and its marginal revenue is $MR_i \equiv dTR_i/dY_i = P_i + \left(\frac{dP_i}{dY_i}\right)Y_i = P_i\left(1+\frac{dP_i}{dY_i}\frac{Y_i}{P_i}\right)$, where $\frac{dP_i}{dY_i}\frac{Y_i}{P_i}$ is the *inverse* of the price elasticity of demand. Using the fact that $Y_i = D(P_i)$ and inverting the firm's demand curve (1), we obtain $P_i = P\left(\frac{Y_i}{Y/n}\right)^{-1/\sigma}$ from which you may verify that $\frac{dP_i}{dY_i}\frac{Y_i}{P_i} = -\frac{1}{\sigma}$. It then follows that $MR_i = P_i\left(1+\frac{dP_i}{dY_i}\frac{Y_i}{P_i}\right) = P_i\left(1-\frac{1}{\sigma}\right)$.

When the firm has set its optimal price as given by (3) it follows from (1) that its output will be $Y_i = \left(\frac{mW_i}{aP}\right)^{-\sigma} \frac{Y}{n}$, and hence its labour demand as a function of the wage rate will be:

$$L_i\left(W_i\right) = \frac{1}{a}\left(\frac{mW_i}{aP}\right)^{-\sigma} \frac{Y}{n}, \tag{4}$$

implying

$$\frac{dL_i}{dW_i} = -\frac{\sigma}{a}\left(\frac{mW_i}{aP}\right)^{-\sigma-1} \frac{mY}{aPn} = -\frac{\sigma L_i}{W_i} < 0. \tag{5}$$

We see that labour demand goes down as the wage rate goes up, because a rise in the wage rate drives up the relative price of good $i$ which in turn reduces demand for the firm's product. Note that the price elasticity of output demand $\sigma$ is also the wage elasticity of labour demand.

The relationships (4) and (5) will be useful for our study of wage formation to which we turn next.

### Union Wage Setting

Suppose that workers in each sector are organized in an industry trade union which monopolizes the supply of labour to firm $i$.[3] Suppose further that, by appropriate choice of units, the exogenous number of working hours per worker is equal to 1. The number of workers employed in sector $i$ will then be equal to firm $i$'s labour input $L_i$. The total number of workers educated and trained to work in sector $i$ is denoted by $N$, which we take to be exogenous. Hence the number of workers who are initially attached to sector $i$, but who fail to find a job in that sector will be equal to $N - L_i$. Each of these workers is expected to be able to earn the alternative real income $\overline{V}/P$, where $\overline{V}$ is the nominal 'outside option', defined as the expected nominal income obtainable outside sector $i$.

[3] As we explained in Chapter 14, collective bargaining agreements negotiated by trade unions are often automatically extended to cover the non-unionized parts of the labour market. Hence the formation of wages can be governed by union behaviour even if only a fraction of workers are actually union members.

Because of its monopoly position, the trade union may dictate the wage rate $W_i$ to firm $i$. The union sets the wage rate so as to maximize the expected real income $\Omega(W_i)$ of its members, given by

$$\Omega(W_i) = \frac{L_i(W_i)}{N}\frac{W_i}{P} + \frac{N - L_i(W_i)}{N}\frac{\overline{V}}{P} = \frac{1}{NP}\left[L_i(W_i)\left(W_i - \overline{V}\right) + N\overline{V}\right]. \tag{6}$$

The fraction $L_i(W_i)/N$ is the probability that the representative union member will find a job in sector $i$ at the real wage $W_i/P$, and $(N - L_i(W_i))/N$ is the probability that he will have to look for a job outside the sector in which case he expects to earn the real income $\overline{V}/P$.

The union takes $N$, $\overline{V}$ and $P$ as given, since it is too small to be able to affect the economy-wide variables $\overline{V}$ and $P$, and since it has inherited a given membership $N$ from the past. Hence maximization of the expected real income $\Omega(W_i)$ in (6) is equivalent to maximization of the magnitude $L_i(W_i)\left(W_i - \overline{V}\right)$ which may be interpreted as the nominal 'net profit' earned by the union. The union can be seen as a monopoly seller of the commodity called 'labour'. For each unit of labour sold the union obtains a nominal 'price' equal to the wage rate $W_i$, but it also incurs an opportunity cost equal to the alternative income $\overline{V}$ which a union member could have earned outside the sector. For an ordinary monopoly firm maximization of profit requires marginal revenue to be equal to marginal cost. We also recall that if the price is equal to $P_i$ and the numerical price elasticity of demand for the monopolist's product is $\sigma$, marginal revenue is given by $P_i\frac{\sigma-1}{\sigma}$ so that profit maximization implies $P_i\frac{\sigma-1}{\sigma} = MC_i$, where $MC_i$ is the marginal cost of production. In the present context of our monopoly trade union where the 'product' being sold is 'labour', the product price $P_i$ must be replaced by the wage rate $W_i$, but the wage elasticity of labour demand is still equal to the price elasticity of product demand, $\sigma$, as we have seen earlier. Furthermore, the analogue of the firm's marginal cost $MC$ is the union's opportunity cost $\overline{V}$ of 'selling' an extra unit of labour. Hence the union's optimal wage claim $W_i$ must fulfill $W_i \frac{\sigma-1}{\sigma} = \bar{V}$, or

$$W_i = m\overline{V}, \qquad m \equiv \frac{\sigma}{\sigma - 1}. \tag{7}$$

Thus the union sets the wage rate as a mark-up over the 'outside option', that is, the income which its members expect to be able to earn outside the sector. The mark-up $m$ will be higher the lower the wage elasticity of labour demand, $\sigma$.

Note that a unionized labour market is not the only economic environment which generates a wage equation like (7). As we saw in chapter 13, the theory of *efficiency wages* also implies a wage setting schedule of the same general form as (7). Thus our theory of wage formation does not rely crucially on the existence of strong trade unions.

Let us now use our theory of wage and price setting to analyze the relationship between inflation and unemployment.

## 2 The Phillips curve

### Deriving the Link between Inflation and Unemployment

If a worker fails to find a job in the sector to which he is initially attached, he will look for work elsewhere. We assume that his probability of finding a job outside sector $i$ is simply equal to the general rate of employment in the labour market which by definition equals $1 - u$, where $u$ is the rate of unemployment. Hence the probability that he will remain unemployed and have to rely on unemployment benefits will be equal to $u$. The nominal income which a worker expects to be able to earn outside his initial sector may therefore be specified as

$$\overline{V} = (1 - u)\, W^e + uB^e. \tag{8}$$

where $W^e$ is the expected average nominal wage level, and $B^e$ is the nominal rate of unemployment benefit which the worker expects to collect if he ends up being unemployed.

In chapters 13 and 14 where we focused on the long run, we assumed that expected wages and benefits were equal to their respective actual levels ($W^e = W$ and $B^e = B$), as must be the case in a long run equilibrium. In the present short-run context we allow for the fact that expectations may not always be correct.[4]

By definition, the expected nominal wage level equals the expected level of real wages, $W^*$, multiplied by the expected general price level, $P^e$:

$$W^e \equiv P^e W^*. \tag{9}$$

The rate of unemployment benefit is assumed to be indexed to the general wage level, with a constant replacement ratio $c$ which is known to all workers. The expected benefit rate is then given by

$$B^e = cW^e, \qquad 0 < c < 1. \tag{10}$$

It follows from (8) through (10) that the nominal outside option $\overline{V}$ will be the same for all workers regardless of the particular sector $i$ in which they start out. Hence (7) implies that unions in all sectors will set the same wage rate so that we may write $W = W_i$ for all $i$, where $W$ is the general level of nominal wages. Using this observation and inserting (8) through (10) into (7), it then follows that the general wage level is given by

$$W = m\left[1 - (1-c)\,u\right] P^e W^*. \tag{11}$$

It will be convenient to transform (11) into logarithms, since we will thereby obtain a linear relationship. Taking natural logs on both sides of (11) and using the approximation $\ln[1 - (1-c)\,u] \approx -(1-c)\,u$ (which is a good approximation given that $(1-c)\,u$ is fairly

[4]For simplicity, equation (8) assumes that wage setters know the current unemployment rate $u$ even though they do not know the current general wage level and the level of benefits (which is tied to the general wage level, see equation (10)). This assumption may not be a bad approximation to reality, since unemployment statistics tend to be available with a very short time lag, whereas wage statistics are available with a somewhat longer time lag. Thus most OECD countries publish monthly unemployment figures with a short time lag, whereas wage statistics are typically published on a quarterly basis and with a longer time lag.

close to zero), we get

$$w - p^e = \ln m + \ln W^* - (1 - c)\,u, \qquad w \equiv \ln W, \qquad p^e \equiv \ln P^e. \tag{12}$$

Equation (12) is the economy's *wage curve* stating that the real wage claimed by trade unions is a declining function of the rate of unemployment, since a higher unemployment rate reduces the value of the representative worker's outside option. The higher the replacement rate in the system of unemployment compensation, the weaker is the wage moderating effect of unemployment, since a higher replacement rate increases the value of the outside option by reducing the income loss associated with unemployment.

We may also take natural logs on both sides of (3) to obtain a logarithmic version of the economy's *price setting curve*:

$$p - w = \ln m - \ln a, \qquad p \equiv \ln P. \tag{13}$$

Adding equations (12) and (13) we find

$$p - p^e = 2 \ln m + \ln W^* - \ln a - (1 - c)\,u. \tag{14}$$

It seems reasonable to assume that the expected level of real wages is positively related to the average labour productivity $a$. Suppose therefore that $W^* = \omega a$, implying

$$\ln W^* = \ln \omega + \ln a, \tag{15}$$

where $\omega$ is a positive constant. Using (15), we may now rewrite (14) as: $p - p^e = 2 \ln m + \ln \omega - (1 - c)\,u$, or $p - p_{-1} - (p^e - p_{-1}) = 2 \ln m + \ln \omega - (1 - c)\,u$ or:

$$\pi = \pi^e + 2 \ln m + \ln \omega - (1 - c)\,u, \qquad \pi \equiv p - p_{-1}, \qquad \pi^e \equiv p^e - p_{-1}. \tag{16}$$

The subscript '-1' indicates that the variable in question refers to the previous time period, whereas variables without a subscript refer to the current period. Recalling that the change in the log of some variable $x$ roughly equals the relative change in $x$, it follows that $\pi$ is the *actual* rate of inflation whereas $\pi^e$ is the *expected* rate of inflation. Thus equation (16) says that the actual rate of inflation varies positively and one-to-one with the expected rate of inflation, and negatively with the rate of unemployment (since the replacement rate $c$ is less than one).

Equation (16) is our first main result, linking inflation to unemployment and expected inflation. It is useful to relate this result to the concept of the natural rate of unemployment introduced in earlier chapters.

**The Natural Rate of Unemployment**

Equation (16) may be rewritten as

$$\pi = \pi^e - (1-c)(u - \overline{u}), \tag{17}$$

where

$$\overline{u} \equiv \frac{2\ln m + \ln \omega}{1-c}. \tag{18}$$

The variable $\overline{u}$ is the *natural rate of unemployment*, defined as the rate of unemployment prevailing in a long-run equilibrium where expected inflation coincides with actual inflation ($\pi^e = \pi$). We see from (18) that the natural or 'structural' rate of unemployment varies positively with 1) the mark-up factor $m$, 2) the benefit replacement rate $c$, and 3) the expected 'normal' real wage $\omega$.[5] In other words structural unemployment will be higher the lower the degree of competition in product and labour markets, the more generous the benefit system, and the more ambitious the real wage aspirations of workers.

The magnitude $(u - \overline{u})$ is the amount of *cyclical unemployment.* According to (17) our theory therefore implies that actual inflation will be higher the higher the expected rate of

[5] Strictly speaking, $\omega$ is that part of output per worker which is expected to be paid out as wages to the average worker, but for a given level of productivity the expected real wage is proportional to $\omega$.

inflation and the lower the amount of cyclical unemployment. Note that inflation depends on the *difference* between the actual and the natural rate of unemployment.

Equation (16), from which we derived the natural unemployment rate, is a version of the so-called expectations-augmented Phillips curve. In the next section we shall trace the historical development of this theory of inflation and unemployment.

**The Simple Phillips Curve versus the Expectations-Augmented Phillips Curve**

If the general price level is expected to be stable, we have $\pi^e = 0$, and equations (16) and (17) then reduce to

$$\pi = (1 - c)(\overline{u} - u) = 2 \ln m + \ln \omega - (1 - c) u. \quad (19)$$

Equation (19) is a version of the *simple Phillips curve* according to which a lower rate of unemployment is always associated with a higher rate of inflation, and vice versa, as long as the structural characteristics of the labour and product markets embodied in our parameters $m$, $\omega$ and $c$ are unchanged.

The Phillips curve owes its name to the New Zealand-born economist A.W. Phillips. In 1958 he published a seminal article documenting a systematic negative relationship between unemployment and wage inflation in Britain in the period 1861-1913.[6] In addition, Phillips showed that his curve estimated from the 1861-1913 data fitted the observations of wage inflation and unemployment for the period 1948-1957 almost perfectly! The apparent stability of the relation between unemployment and wage inflation over a period of almost

[6] See A.W. Phillips: 'The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861-1957', *Economica*, New Series, vol. 25, November 1958. The curve fitted by Phillips was not linear, like our equation (19). Instead, he found that the data were best explained by an equation of the form

$$\Delta w = 9.638 u^{-1.394} - 0.9,$$

where $\Delta w$ is the percentage rate of change of the average money wage rate. However, the crucial point is that Phillips found a significant negative relationship between unemployment and wage inflation.

100 years made a great impression on the economics profession. Hence the Phillips curve was rapidly incorporated into macroeconomics where it provided the 'missing link' in the standard Keynesian macro model, replacing the unsatisfactory assumption of a fixed money wage rate.

The simple Phillips curve quickly came to be seen as a menu from which macroeconomic policy makers could choose. A policy maker with strong aversion to inflation could keep inflation low if he was willing to live with permanently high unemployment. On the other hand, a policy maker whose main concern was unemployment could maintain a (very) low rate of unemployment for ever by accepting a high (but stable) rate of inflation. The Phillips curve was thus regarded as a *stable tradeoff* which policy makers could *exploit* at their will.

For a while after the publication of Phillips' article there did indeed seem to be a stable tradeoff between inflation and unemployment. This is shown in Figure 18.2 which illustrates a simple Phillips curve relationship between these two variables in the United States up until the late 1960s.

However, even in the heyday of the Phillips curve during the 1960s there were sceptics who argued that this curve did not represent a stable tradeoff which could be freely exploited by policy makers. One such sceptic was the later Nobel Prize winner Milton Friedman. In December 1967 Friedman delivered a famous presidential address to the American Economic Association.[7] In this speech he predicted that the simple Phillips curve would break down if monetary policy makers tried to push unemployment below a critical level which he termed the 'natural' rate of unemployment. Friedman pointed out that workers care about their real wages rather than their money wages. He went on to argue that observations of a higher actual rate of inflation would lead to a higher expected rate of

[7]Milton Friedman: 'The Role of Monetary Policy', American Economic Review, vol. 58, March 1968, pp. 1-17.

inflation. This in turn would lead to higher required rates of increase of money wages, as workers tried to protect their real wages. The Phillips curve had been estimated using data for a historical period where the price level had been fairly stable. In such circumstances the expected rate of inflation was probably close to zero, as implicitly assumed in our simple Phillips curve (19). However, if policy makers drove unemployment down to low levels, thereby generating positive rates of inflation for an extended period, the resulting increase in *expected* inflation would cause a *shift* in the simple Phillips curve. According to Friedman, the shift in the Phillips curve would continue, causing a steady acceleration of inflation, as long as the unemployment rate were kept below its long run equilibrium level (the 'natural' rate).

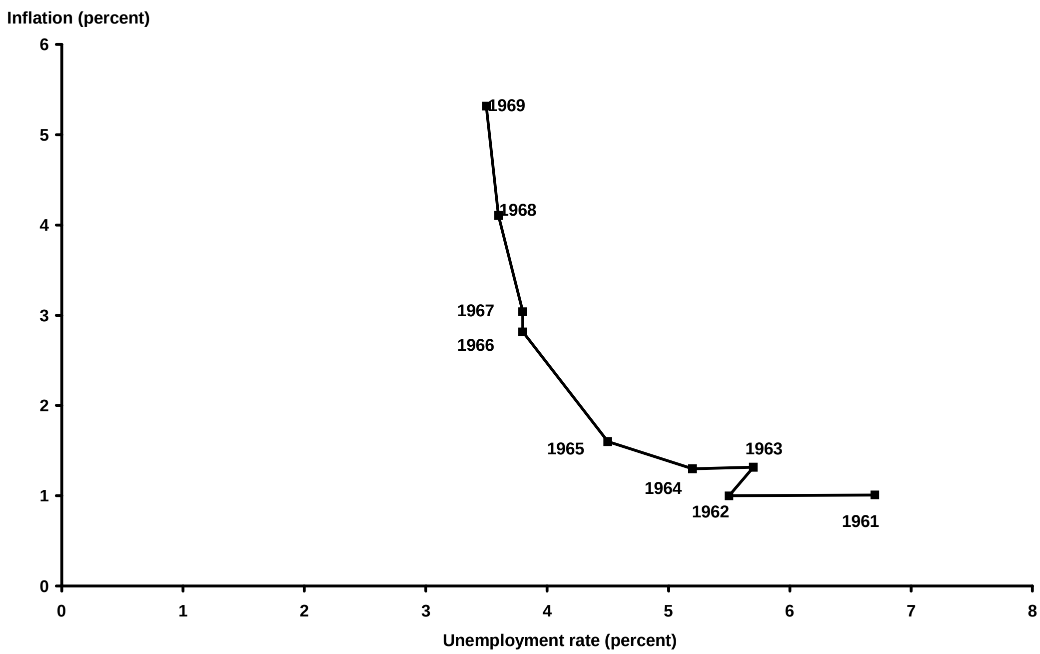


**Figure 18.2: The simple U.S. Phillips curve of the 1960s**

Source: "Int. Historical Statistics", R.B. Mitchell, Macmillian 1998; and Bureau of Labor Statistics

Let us offer an interpretation of Friedman in terms of our own theory of wage and price formation.[8] We start by restating the link between unemployment and the real wage

[8]Friedman did not present a formal model to illustrate his theory, so it is not easy to identify the 'correct' interpretation of his paper. The verbal discussion in Friedman's 1968 article seems to assume perfect

claims of workers. From equations (12) and (15) we get

$$w - p^e = \eta - (1 - c)\, u, \qquad \eta \equiv \ln (m\omega a)\,. \tag{20}$$

The expression on the right-hand side of (20) is the *target real wage* of workers, that is, the real wage rate which they claim at the prevailing level of unemployment. Given their expectations $p^e$ of the level of prices, unions set the money wage rate $w$ with the purpose of attaining this real wage target. Equation (20) may be rewritten as $w - p = \eta - (1 - c)\, u - (p - p^e) = \eta - (1 - c)\, u - [(p - p_{-1}) - (p^e - p_{-1})]$, or

$$w - p = \eta - (1 - c)\, u - (\pi - \pi^e)\,. \tag{21}$$

Since (17) implies that $\pi - \pi^e = (1 - c)\,(\overline{u} - u)$, we may also write (21) as

$$w - p = \overbrace{\eta - (1 - c)\, u}^{\text{target real wage}} - (1 - c)\,(\overline{u} - u)\,. \tag{22}$$

Equation (22) shows that when unemployment is below the natural rate so that $\overline{u} > u$, the *actual* real wage $w - p$ attained by workers will be lower than the *target* real wage. As long as $\overline{u} > u$, workers will repeatedly find that their actual real wage is lower than the target real wage they expected to obtain when they decided on their money wage claims. The reason is that since $\pi - \pi^e = (1 - c)\,(\overline{u} - u)$ according to (17), workers *underestimate* the rate of inflation when unemployment is below the natural rate. In that situation Friedman argued that workers will gradually raise their expected rate of inflation and hence their required rate of increase of money wages, and this will lead to accelerating inflation. To illustrate, suppose for simplicity that workers have so-called *static expectations*, expecting that this period's inflation rate will correspond to the rate of inflation observed during the previous period:

$$\pi^e = \pi_{-1}. \tag{23}$$

competition in labour and product markets. By contrast, our analysis assumes imperfect competition which is more realistic. However, the interpretation presented here is consistent with Friedman's main conclusion that the long-run Phillips curve is vertical at the natural rate of unemployment.

Inserting (23) into (17), we get

$$\Delta\pi \equiv \pi - \pi_{-1} = (1 - c)(\overline{u} - u), \tag{24}$$

which shows that inflation will *accelerate* when unemployment is *below* its natural rate, and *decelerate* when unemployment is *above* the natural level. If policy makers push unemployment below the natural rate, say, through an expansionary monetary policy, they will not only have to live with a permanently higher *level* of inflation, as the simple Phillips curve suggests. Instead, they will in fact have to accept a permanently *accelerating* rate of inflation, as workers and firms gradually revise their expected rate of inflation upwards. In graphical terms, the simple Phillips curve will *shift upwards* whenever $\pi^e$ increases, as illustrated in Figure 18.3.

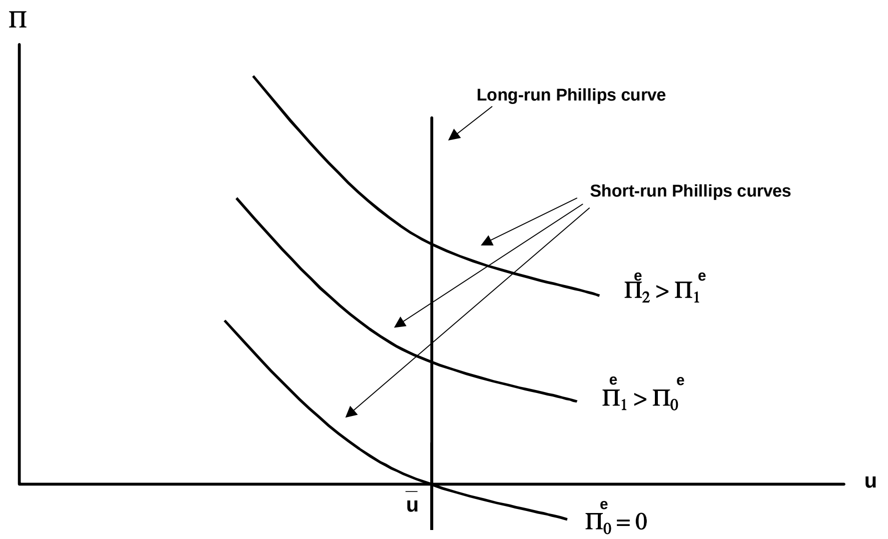


**Figure 18.3:The expectations-augmented Phillips curve**

Since permanently rising or falling inflation is incompatible with a long run equilibrium, Friedman's theory implied that there is no *permanent* trade-off between inflation and unemployment. In the long run only one rate of unemployment - the natural rate - is consistent with a stable inflation rate. In that sense the long run Phillips curve is *vertical*.

The simple negative Phillips curve relationship between inflation and unemployment is just a short run tradeoff which will hold only as long as the expected rate of inflation stays constant. For this reason the simple downward-sloping Phillips curve (defined for a given expected rate of inflation) may also be called the *short-run* Phillips curve.

Note that the acceleration of inflation occurring when unemployment is below the natural rate can be seen as the result of the *incompatible real income claims* of workers and employers. The actual real wage appearing on the left-hand side of (22) is determined by the firm's mark-up pricing rule (13) and cannot be influenced by workers. When $u < \overline{u}$, the real wage claimed by workers exceeds the actual real wage implied by the price setting of firms. Hence the total real income claims of workers and employers exceed the available real national income. The consequence of this incompatibility of income claims is rising inflation, as workers accelerate their nominal wage increases in their frustrated efforts to capture a larger share of the income pie.

Because our equation (17) 'augments' the simple Phillips curve by the expected rate of inflation, it is usually referred to as the *expectations-augmented Phillips curve*. The theory of the expectations-augmented Phillips curve was developed simultaneously by Milton Friedman and his U.S. colleague Edmund Phelps.[9] For this reason the theory is also referred to as the 'Friedman-Phelps natural rate hypothesis'. Friedman's version of the theory became more famous, because it contained the most explicit prediction that the simple Phillips curve relationship would break down after a period like the 1960s in which policy makers had driven unemployment down to historically low levels. This prediction turned out to be prophetic, as illustrated in Figure 18.4. The figure shows how the U.S. Phillips curve relationship tended to shift outwards in several years between 1969 and 1980. The fact that Friedman anticipated this development made his theory of the natural rate

[9] See Edmund S. Phelps: 'Money Wage Dynamics and Labor Market Equilibrium', *Journal of Political Economy*, vol. 76 (July/August, Part 2), 1968, pp. 678-711.

of unemployment enormously influential. During the 1970s the expectations-augmented Phillips curve was therefore readily absorbed into mainstream macroeconomic theory. The expectations-augmented Phillips curve (24) implies that when unemployment is at its natural rate, inflation will neither decelerate nor accelerate. For this reason the natural unemployment rate is also called the Non-Accelerating-Inflation-Rate-of-Unemployment, or just the NAIRU, for short.

Another implication of the expectations-augmented Phillips curve (24) is that there is *nominal inertia*: if unemployment is at its natural rate, the inflation prevailing today will automatically continue tomorrow because it is built into expectations. To bring down inflation, it is necessary to push the actual unemployment rate above the NAIRU for a while.

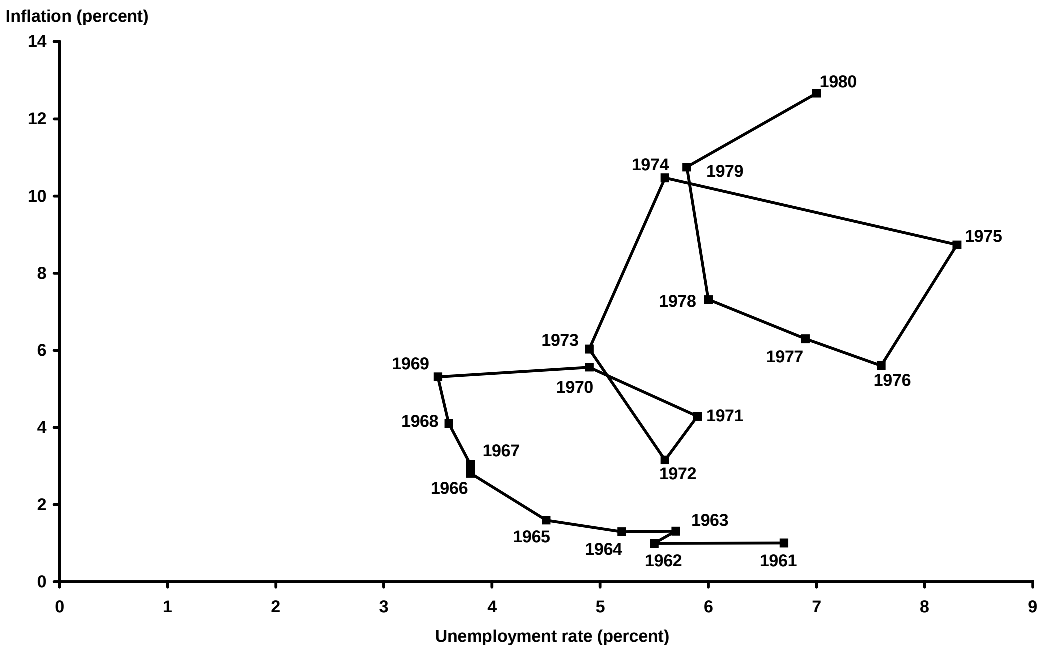


**Figure 18.4: The breakdown of the simple Phillips curve in the United States**

Source: "Int. Historical Statistics", R.B. Mitchell, Macmillian 1998; and Bureau of Labor Statistics

**Does the Phillips Curve Theory Fit the Data?**

Our expectations-augmented Phillips curve with static inflation expectations (24) is an equation of the form

$$\Delta\pi = \alpha - \beta u, \tag{25}$$

where $\alpha$ and $\beta$ are positive constants. Thus our theory implies that the *change* in the inflation rate should be negatively related to the rate of unemployment. If we have data for inflation and unemployment, we can use econometric techniques (regression analysis) to estimate the magnitude of the parameters $\alpha$ and $\beta$. In this way we can check whether the estimated parameter values have the 'correct' positive sign expected from theory, and we can test whether they are significantly different from zero in a statistical sense.

Figure 18.5.a shows observations of the unemployment rate and the annual change in the rate of consumer price inflation in the United States in the period 1962-1995.

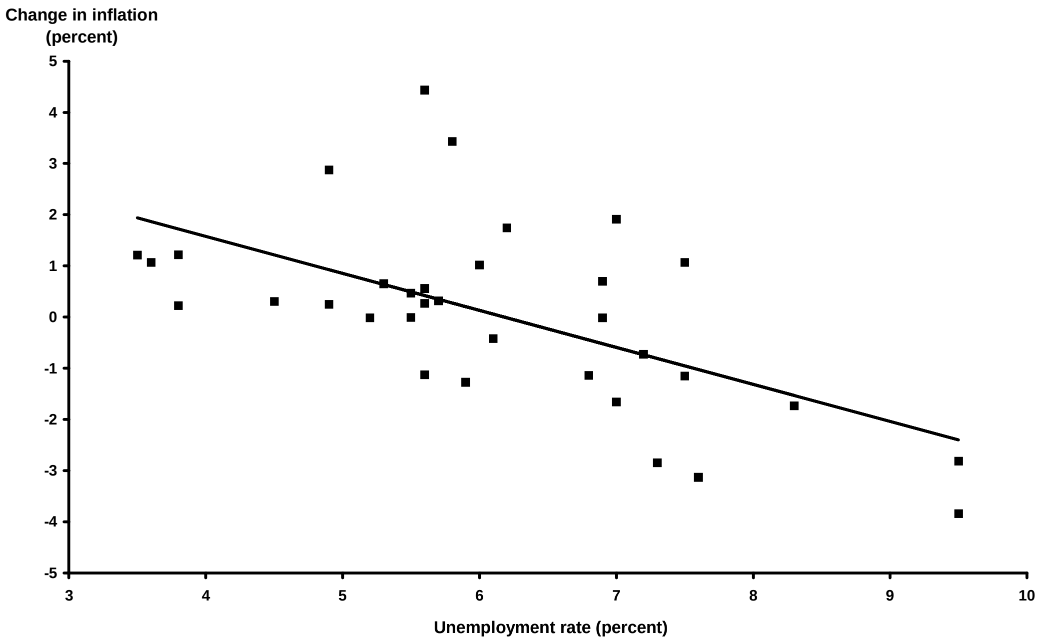


**Figure 18.5.a: Relation between unemployment and the change in inflation in the United States, 1962-1995**

Source: "International Historical Statistics", R.B. Mitchell, Macmillian 1998; and Bureau of Labor Statistics

The downward-sloping straight line in the figure is a regression line indicating the 'average' relationship between unemployment and the change of inflation. The regression line has the following quantitative properties, where the figures in brackets indicate the standard deviations of the estimated coefficients, and where $R^2$ is the so-called coefficient of determination measuring the share of the variation in $\Delta\pi$ which is explained by our estimated regression line:

$$\Delta\pi = \underset{(1.081)}{4.467} - \underset{(0.172)}{0.723}\, u, \qquad R^2 = 0.36. \tag{26}$$

The coefficients in (26) do indeed have the signs we would expect from theory. They are also significantly different from zero in a statistical sense.[10] Figure 18.5.a also shows a fairly clear tendency for inflation to fall as unemployment goes up.

Note that the estimated coefficients in (26) enable us to offer an estimate of the natural rate of unemployment in the United States. According to (24) and (25), we have $\Delta\pi = (1-c)(\overline{u}-u) = \alpha - \beta u$. Since $\Delta\pi = 0$ when $\overline{u} = u$, it follows that $\overline{u} = \alpha/\beta$. Inserting the estimated parameter values from (26), we find that $\overline{u} = 4.467/0.723 \approx 6.2$. This implies that the natural unemployment rate in the U.S. averaged around 6.2 percent in the estimation period 1962-1995.

In summary, the theory of the expectations-augmented Phillips curve seems roughly consistent with the U.S. data. At the same time we also see that the observed change in inflation has often deviated quite a lot from the estimated regression line. Indeed, the value of $R^2$ suggests that our theory can only explain a little over one third of the variation of inflation. This is not really surprising, given the strong simplifying assumptions we have made. Our assumption in (23) that expectations are static is rather primitive and mechanical. For example, in periods where the fiscal or monetary authorities announce a significant change in economic policies, the private sector may have good reasons to

[10] As a rule of thumb, the estimated coefficient should be numerically at least twice as big as its standard deviation to be statistically significant. This condition is easily met in our estimated equation (26).

For the benefit of readers who are familiar with regression analysis, the value of the Durbin-Watson statistic (1.515) indicates that there are no serious problems of autocorrelation in our regression.

believe that tomorrow's inflation rate will not simply equal the current rate of inflation.[11] As another example, our simple theory of production in equation (2) abstracts from the fact that production requires inputs of raw materials as well as labour input. Hence our Phillips curve does not capture changes in inflation which are driven by changes in the international prices of important raw materials such as oil.

Figure 18.5.b shows the relationship between unemployment and the change in inflation in Denmark.

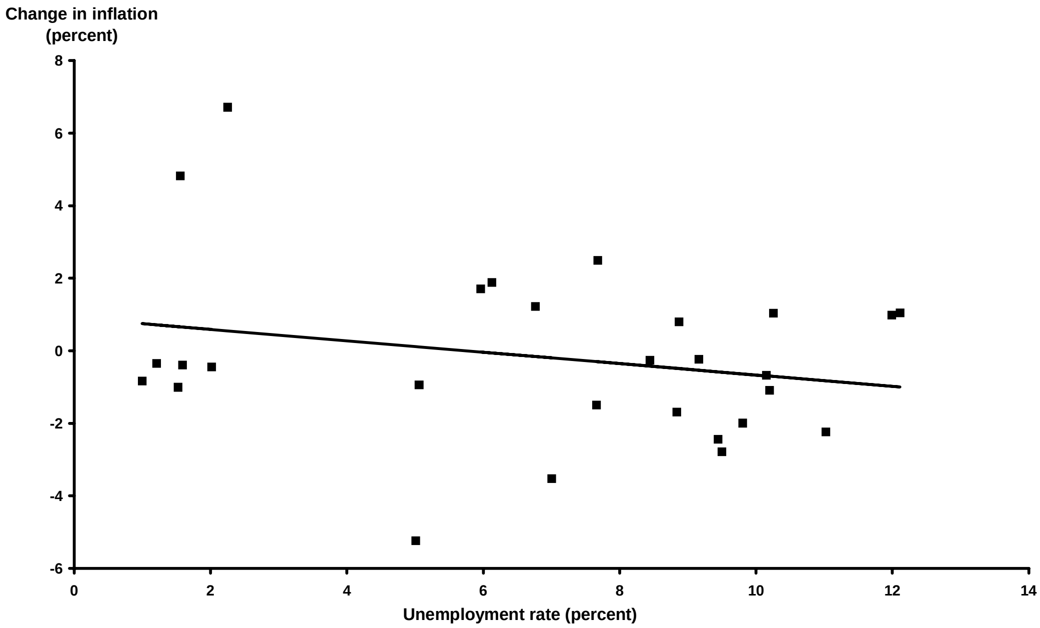


**Figure 18.5.b: Relation between unemployment and the change in inflation in Denmark, 1968-1995**

Source: ADAM data base

The estimated regression line included in the figure indicates a weakly negative relation between unemployment and the annual change in inflation, in accordance with our theory. However, a statistical analysis reveals that the slope of the regression line is not significantly different from zero. Figure 18.5.b also suggests to the naked eye that there has been no simple relationship between unemployment and inflation in Denmark. The reason is that,

[11]In Chapter 22 we shall discuss the formation of private sector expectations in more detail.

in the highly open Danish economy, a large part of private consumption and of inputs into production consists of goods imported from abroad. The domestic inflation rate is therefore heavily influenced by changes in import prices which are not related to the Danish unemployment rate. A satisfactory theory of inflation in a small open economy must therefore allow for the influence of import prices. In chapters 24 and 25 we shall have more to say about the relation between import prices and domestic inflation.

# 3 Productivity Growth, the Phillips Curve, and the 'New Economy'

In the second half of the 1990s many observers were struck by the remarkable performance of the U.S. economy. As you can see from Figure 18.6, after having been located to the far northeast of the unemployment-inflation scatter diagram, during the 1990s the short-run Phillips curve seemed to shift all the way back to the favourable position it had in the 1960s.

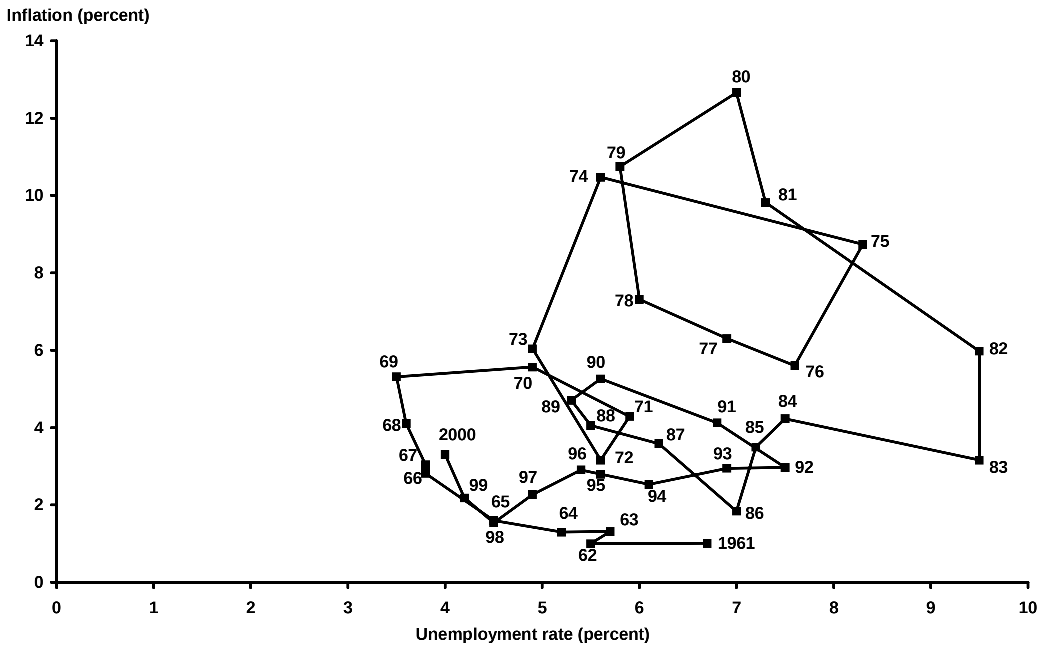


**Figure 18.6: The shifting short-run Phillips curve in the United States**

Source: "International Historical Statistics", R.B. Mitchell, Macmillian 1998; and Bureau of Labor Statistics

Apparently this remarkable shift was not a simple consequence of a fall in expected inflation generated by the observed fall in actual inflation since the early 1980s. This point is illustrated in Figure 18.7. The figure compares the actual inflation rate to the rate of inflation predicted from our Phillips curve (28) which was estimated from data for 1962-1995. We see that, from 1996 and onwards, the rate of inflation predicted from the historical link between unemployment and inflation systematically overshot the actual inflation rate. For example, based on the behaviour observed during the 1962-1995 period, one would have expected to see a U.S. inflation rate of 8.5 percent in 2000, but the actual inflation rate remained subdued to a level of 3.3 percent, despite the low rate of unemployment. In other words, it seemed that a *structural shift* took place in the U.S. economy around the mid-1990s, causing a breakdown of the expectations-augmented Phillips curve which had fitted the data reasonably well up until 1995.


**Figure 18.7: Actual and predicted inflation in the United States**

Note: The predicted inflation rate was found from equation (26).

Source: "Int. Historical Statistics", R.B. Mitchell, Macmillian 1998; and Bureau of Labor Statistics

Impressed by this development, many commentators argued that a 'New Economy' had arrived which did not obey the 'old rules of the game'. Some participants in the economic policy debate even claimed that it was time to scrap the established macroeconomic theory which apparantly could no longer explain what was going on. In particular, these proponents of the 'New Economy' paradigm argued that it is now possible to maintain a much lower unemployment rate in the U.S. without provoking high and accelerating inflation.

However, we shall now present a slight extension of our model of inflation to show that the recent development in the U.S. economy does not necessarily invalidate the theory of the expectations-augmented Phillips curve. This extension of our model will also enable us to illustrate how productivity growth may affect the short-run link between inflation and unemployment.

Our revised model builds on a modified specification of the real wage aspirations of workers. As an alternative to equation (15), we may specify the expected current real wage level $W^*$ (the real wage aspiration) as

$$W^* \equiv (1 + g^*) \, (W/P)_{-1} \, , \tag{27}$$

where $g^*$ is the expected rate of real wage growth, and $(W/P)_{-1}$ is the actual real wage rate observed during the previous period. As it stands, equation (27) is just an identity which must hold by definition, assuming that agents know the actual real wage earned during the previous period. From the price setting equation (3) it follows that last period's actual real wage was

$$(W/P)_{-1} = a_{-1}/m, \tag{28}$$

where $a_{-1}$ is average labour productivity during the previous period, and where we attach no time subscript to the mark-up $m$ which is assumed to stay constant over time. Inserting (28) into (27) gives $W^* = (1 + g^*) \, (a_{-1}/m)$, and taking logs, we get

$$\ln W^* \approx g^* + \ln a_{-1} - \ln m. \tag{29}$$

In deriving (29) we have used the approximation $\ln(1+g^*) \approx g^*$. This is reasonable, given that annual productivity growth is rarely far above zero. In a similar way, the *actual* rate of productivity growth $g$ is given by

$$a \equiv (1+g)\, a_{-1}, \tag{30}$$

so approximately we have

$$\ln a \approx g + \ln a_{-1}. \tag{31}$$

To derive our revised Phillips curve, we go back to our earlier equation (14) which was found by combining the firm's price setting curve and the union's wage setting curve:

$$p - p^e = 2\ln m + \ln W^* - \ln a - (1-c)\, u. \tag{14}$$

Substituting (29) and (31) into (14), we obtain our new Philips curve as follows: $p - p^e = \ln m + g^* - g - (1-c)\, u$, equivalent to $p - p_{-1} - (p^e - p_{-1}) = \ln m - (g - g^*) - (1-c)\, u$, from which:

$$\pi = \pi^e + \ln m - (g - g^*) - (1-c)\, u. \tag{32}$$

If we specify the natural rate of unemployment as

$$\overline{u} = \frac{\ln m - (g - g^*)}{1-c} \tag{33}$$

we may write (32) in the same form as our earlier equation (17):

$$\pi = \pi^e - (1-c)\,(u - \overline{u}) \tag{34}$$

Equation (33) says that the natural unemployment rate will ceteris paribus be lower, the greater the difference between actual productivity growth $g$ and expected real wage growth $g^*$. According to (28) and (30) the *actual* rate of real wage growth is always equal to the actual rate of productivity growth since

$$\frac{(W/P) - (W/P)_{-1}}{(W/P)_{-1}} = \frac{(a/m) - (a_{-1}/m)}{(a_{-1}/m)} = \frac{(1+g)\, a_{-1} - a_{-1}}{a_{-1}} = g. \tag{35}$$

However, in periods where actual productivity growth $g$ tends to *accelerate*, it seems plausible that expected real wage growth $g^*$ will lag somewhat behind $g$, since $g^*$ is likely to be affected by the experience of slower real wage growth during the previous years. In other words, if technological progress and the rate of productivity growth start to accelerate, it may take a while for workers' expected rate of real wage growth to catch up. During this adjustment period where real wage aspirations lag behind, the term $(g - g^*)$ in the numerator of (33) will be positive and will hence pull down the natural rate of unemployment. According to (34) the short run Phillips curve will then shift downwards so that inflation and unemployment may fall at the same time.

Perhaps this is what happened in the United States in the second half of the 1990s. As you can see from Figure 18.8, the growth rate of U.S. labour productivity has tended to be higher in recent years than during the previous decades. As indicated by the horizontal lines in Figure 18.8, the average growth rate of labour productivity during the period of the prolonged productivity slowdown from 1974 to 1995 was only 1.35 percent per year, whereas the average productivity growth rate rose to 2.42 percent per year during 1996-2001. During the long period of the productivity slowdown U.S. real wage growth was quite low, and workers probably got used to expecting only moderate annual growth rates of their real earnings. It seems likely that it has taken a while for the higher productivity growth and the associated higher real wage growth in the most recent years to feed into workers' expectations. In that case inflationary pressures were temporarily reduced by a fall in workers' real wage claims relative to the economy's productive potential.

To illustrate, take the simple case where workers have static expectations regarding the rate of real wage growth. This means that

$$g^* = g_{-1}. \tag{36}$$

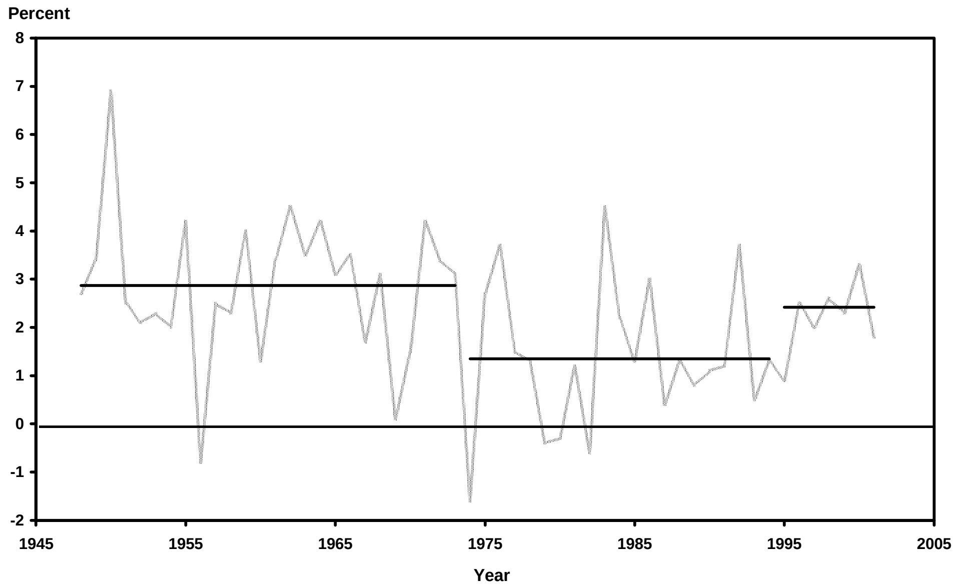


**Figure 18.8: Annual growth rate of labour productivity in the United States**

Note: Growth rate of output per hour in non-farm business sector. The horizontal lines are average growth rates over the subperiods indicated. Source: Bureau of Labor Statistics

Inserting (36) and (33) into (34), we then get

$$\pi = \pi^e - (g - g_{-1}) - (1 - c)\, u. \tag{37}$$

During a period where productivity growth is accelerating so that $g > g_{-1}$, we see from (37) that this can help to keep inflation down even if unemployment is falling.

The idea that accelerating productivity growth may explain the favourable shift in the U.S. Philips curve in the second half of the 1990s has received empirical support in a recent study by American economists Laurence Ball and Robert Moffitt (2001)[12]. They show that a Phillips curve which assumes that the real wage growth demanded by workers is a geometrically weighted average of realized past rates of real wage growth (which in turn depend on past productivity growth) can fully account for the favourable U.S. unemployment-inflation tradeoff in the late 1990s.

[12] Laurence Ball and Robert Moffitt: 'Productivity Growth and the Phillips Curve', National Bureau of Economic Research, NBER Working Paper 8421, August 2001.

Many recent prophets of the 'New Economy' have claimed that we have entered a new era in which rapid improvements in information and communication technology enable us to maintain low unemployment and higher growth without having to fear high inflation. The story we have told here acknowledges that an *acceleration* of productivity growth can temporarily improve the short run unemployment-inflation tradeoff by lowering the natural rate of unemployment during the period in which productivity growth picks up. However, our theory also implies that once workers' real wage aspirations catch up with the higher rate of productivity growth, the temporary drop in the NAIRU will be reversed, in contrast to the claims made by the most optimistic proponents of the New Economy.

Time will tell who is right. But as we have seen in the present section, it only takes a modest extension of the basic theory of the expectations-augmented Phillips curve to make the theory consistent with the most recent data on unemployment and inflation.

# 4 Summarising the Theory of Inflation and Unemployment

The basic messages of this chapter are captured by Figure 18.9 which illustrates how our theory of inflation and cyclical unemployment is related to our theory of wage and price formation and structural unemployment presented in chapters 13 and 14[13]. Along the horizontal axes in Figure 18.9 we measure the rate of employment $E$, that is, the total number of workers employed relative to the total labour force. By definition, the rate of employment is equal to one minus the unemployment rate ($E = 1 - u$). As we move to the left from point 1 on the horizontal axis, we gradually increase the rate of unemployment.

The upper part of Figure 18.9 restates our theory of the unionized labour market. On the vertical axis we measure the (logarithm of the) real wage. The *upward-sloping curve*

[13]Figure 18.9 is inspired by Richard Layard, Stephen Nickell and Richard Jackman (1991), *Unemployment, Macroeconomic Performance and the Labour Market*, Oxford: Oxford University Press. This book is recommended reading if you want to dig deeper into the problems of unemployment and inflation.

in the upper part of Figure 18.9 is given by the following equation which is obtained from (12) by inserting the definition $u \equiv 1 - E$:

$$\text{Wage curve:} \qquad w - p^e = \ln m + \overbrace{\ln W^*}^{\ln \omega + \ln a} - (1-c) + (1-c)\, E. \tag{38}$$

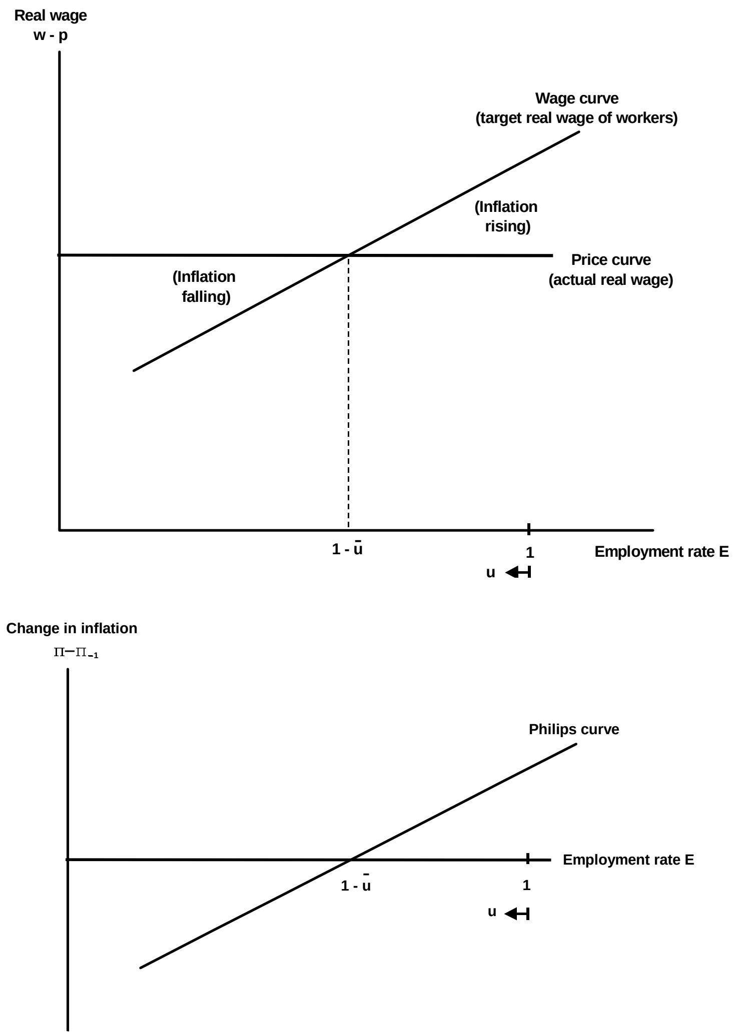


**Figure 18.9. The relation between real wages, inflation and unemployment**

Equation (38) is the *wage curve* indicating how the representative industry union's *target* real wage increases with the rate of employment. As the employment rate goes up (as unemployment falls), the value of workers' outside option increases, leading to more aggressive union real wage claims. The *position* of the wage curve (the intercept with the vertical axis) depends on the real wage aspiration of workers, captured by our variable $W^*$, on the replacement rate $c$ which influences the value of the outside option, and on the union's mark-up ($m$) of wages over the outside option which in turn depends on the wage elasticity of labour demand. The *slope* of the wage curve $(1-c)$ is steeper the lower the replacement rate: a lower value of unemployment benefits relative to the wage level implies a greater income loss from unemployment and therefore means that falling employment has a stronger moderating impact on wage claims[14].

The *horizontal line* in the upper part of Figure 18.9 shows the *actual* real wage implied by the price setting behaviour of firms. The equation for this curve is found by a simple rearrangement of equation (13) in the text, yielding

$$\text{Price curve:} \qquad w - p = \ln a - \ln m. \tag{39}$$

Because of our simplifying assumptions that labour productivity $a$ and the firm's mark-up factor $m$ are both independent of the rate of employment, the price curve is completely flat[15].

The natural rate of employment $\overline{E} \equiv 1 - \overline{u}$ is given by the intersection of the wage curve and the price curve. At the natural employment rate the target real wage of workers is consistent with the actual real wage implied by the price setting behaviour of firms, and the expected price level $p^e$ in (38) equals the actual price level $p$ in (39) so that workers obtain the real wage they are striving for. As labour productivity $a$ increases over time, we see from (39) that the price curve will shift upwards. However, since our equation (15) implies that $\ln W^* = \ln \omega + \ln a$, productivity growth will give a similar boost to the real wage aspiration of workers. In fact, it follows from (38) and (39) that productivity growth

[14]You might think that since a rise in $c$ reduces the slope of the wage curve at the same time as it raises the intercept with the vertical axis, the net effect on the natural unemployment rate is ambiguous. However, from equation (18) you can see that a higher replacement rate does in fact raise the natural unemployment rate, in accordance with our theory in chapters 13 and 14.

[15]In Exercise 18.1 we shall ask you to analyse the more complicated case where the mark-up varies with the state of the business cycle.

causes the wage curve and the price curve to shift upward by exactly the same distance, leaving the natural rates of employment and unemployment unaffected. In practice, as we have seen in the previous section, it may take a while for the real wage aspiration $W^*$ to catch up with higher productivity, and in the intervening period the natural employment rate may increase, as the wage curve shifts up less rapidly than the price curve. But in the longer run when real wage aspirations catch up, productivity growth cannot boost the natural employment rate. This is consistent with our observations in chapter 12 where we saw that over the long run there is no discernible trend in the unemployment rate, despite the secular growth in productivity.

On the vertical axis in the bottom part of Figure 18.9 we measure the change in the rate of inflation, $\pi - \pi_{-1}$. The upward-sloping curve in that part of the figure is found from equation (24), which we have rewritten in terms of the employment rate, using the definitions $E \equiv 1 - u$ and $\overline{E} \equiv 1 - \overline{u}$, to get

$$\text{Phillips curve:} \qquad \pi - \pi_{-1} = (1 - c)\left(E - \overline{E}\right). \tag{40}$$

In accordance with the usual terminology in modern economics textbooks, equation (40) is just termed the 'Phillips curve', although we remember that it is in fact a version of the *expectations-augmented* Phillips curve rather than the simple Phillips curve introduced by Phillips himself. According to (40) inflation will rise when the actual employment rate exceeds the natural rate, whereas inflation will fall when employment falls short of its natural level. When $E > \overline{E}$, the 'function' of accelerating inflation is to create *unanticipated inflation* so that workers end up with the actual real wages implied by the mark-up pricing of firms even though their *required* real wages are higher. By contrast, when $E < \overline{E}$ the workers' required real wage is more modest than the actual real wage implied by price setting behaviour. In this situation falling inflation takes workers by surprise, as the moderate required rates of money wage increase are passed on into lower than expected rates of price increase, and workers end up with higher real wages than they bargained for.

In the next chapter we shall see how the theory of inflation and unemployment summarized in Figure 18.9 can be turned into a theory of the economy's aggregate supply of goods, and how aggregate supply interacts with aggregate demand to determine total GDP as well as the rates of inflation and unemployment.

# 5 Summary

1. The crucial supply side mechanism in the macro economy is the relationship between inflation and unemployment. This relation can be derived from the theory of wage and price formation underlying our theory of structural unemployment, by allowing for the fact that the expected and actual levels of wages and prices need not coincide in the short run.

2. Under monopolistic competition profit-maximizing firms set their prices as a mark-up over their unit wage cost, assuming that labour is the only variable factor of production in the short run. In a similar way, decentralized monopoly trade unions will set their nominal wage claims as a mark-up over the representative union member's 'outside option'. The outside option is the union member's expected income obtainable outside the sector covered by the union. The size of the mark-up of firms and unions will be larger the lower the numerical price elasticity of demand for the firm' product, that is, the lower the degree of competition in output markets.

3. A higher rate of unemployment reduces the nominal and real wage rate claimed by unions by reducing the representative union member's expected income obtainable elsewhere in the labour market. A higher replacement rate in the system of unemployment compensation will weaken this effect of unemployment on wage claims by reducing the

income loss associated with unemployment. A higher expected inflation rate will increase the nominal wage claims of unions by increasing the nominal value of the representative union member's outside option.

4. The wage and price setting behaviour of unions and firms implies that the actual inflation rate is a decreasing function of the rate of unemployment and that it varies one-to-one with the expected inflation rate. If the expected inflation rate is zero (or constant), one obtains the simple Phillips curve which implies a permanent trade-off between inflation and unemployment. Empirically, the simple Phillips curve broke down in the 1960s, giving way to the theory of the expectations-augmented Phillips curve which allows for a positive expected rate of inflation adjusting endogenously over time.

5. The natural rate of unemployment is that rate of unemployment where the real wage claimed by workers coincides with the real wage implied by the firms' mark-up price setting. Whenever actual unemployment is below (above) the natural rate, the realized real wage will be lower (higher) than the real wage claimed by workers. Workers will therefore demand higher (lower) rates of increase in their money wages, and consequently inflation will accelerate (decelerate). An alternative but equivalent way of putting it is that workers will underestimate (overestimate) the rate of inflation when unemployment is below (above) the natural rate, and hence they will gradually revise their expected inflation rate upwards (downwards), causing accelerating (decelerating) rates of increase in money wages and prices.

6. The natural rate of unemployment is the only level of unemployment where the real income claims of firms and workers are compatible, and where inflation expectations are correct. Hence the natural rate of unemployment is the only unemployment rate consistent with a stable long run rate of inflation. The long run Phillips curve is therefore vertical at the natural unemployment rate.

7. The natural rate of unemployment (also called the 'structural' rate of unemployment)

varies positively with the degree of monopoly in goods and labour markets, with the unemployment benefit replacement rate, and with workers' expected 'normal' real wage (relative to the level of productivity).

8. The theory of the expectations-augmented Phillips curve is reasonably consistent with U.S. data on inflation and unemployment until the mid-1990s. After 1995 the observed inflation rate was unusually low, given the low level of unemployment. This may be explained by a modified version of the expectations-augmented Phillips curce where the real wage aspirations of workers adjust only gradually to a rise in the trend growth rate of labour productivity.

# 6 Exercises

**Exercise 1. The Phillips curve with endogenous mark-ups**

In the main text of this chapter we assumed for simplicity that firms' mark-up $m$ of price over marginal cost was an exogenous constant. However, empirical research for the United States suggests that the mark-up factor in that country actually tends to move in a countercyclical fashion. In other words, the mark-up tends to fall during business cycle expansions and to rise during recessions. The reason for this countercyclical behaviour of markups is not clear, but one possible explanation is that firms may face various types of costs of changing their prices. These costs may include the costs of printing new price catalogs, the costs of communicating price changes to the market, and perceived costs of annoying customers by too frequent price changes. For these reasons it may take a while for firms to raise their prices during expansions where wage costs go up due to falling unemployment, and it may take a while to reduce the customary rate of price increase during recessions when wage inflation goes down as a result of rising unemployment. In that case the mark-up will tend to move opposite to the unemployment rate. For concreteness, suppose this relationship is linear so that

$$m = \overline{m} \cdot e^{\gamma u}, \qquad \overline{m} > 1, \qquad \gamma \geq 0 \tag{1}$$

where $e$ is the exponential function, and $\gamma$ is a parameter measuring the sensitivity of the mark-up to changes in the unemployment rate $u$. In the main text of the chapter we focused on the case of $\gamma = 0$ where the mark-up in (1) becomes equal to the constant $\overline{m}$. In this exercise we ask you to study the implications of assuming $\gamma > 0$ which is more in line with the empirical evidence for the U.S.

In accordance with equation (3) in the main text, we assume that firms set their prices as a mark up over their unit labour cost $W/a$:

$$P = m \cdot \frac{W}{a} \tag{2}$$

We also maintain the theory that trade unions set the wage rate as a mark-up over their members' outside option, but we now allow for the possibility that the union mark-up $m^u$ may differ from the mark-up of firms. As a slight modification of equation (11) in the main text, the nominal wage rate claimed by unions is then given by

$$W = m^u \cdot \overbrace{\left[1 - (1-c)\,u\right] P^e W^*}^{\text{outside option } \overline{V}}, \qquad 0 < c < 1 \tag{3}$$

where we remember that $W^*$ is the expected level of real wages, and $c$ is the replacement rate in the system of unemployment compensation. Furthermore, we maintain our assumption in equation (15) that the expected real wage is proportional to average labour productivity:

$$W^* = \omega a, \qquad 0 < \omega < 1 \tag{4}$$

**1.** Derive an expectations-augmented Phillips curve on the basis of (1) through (4), using the same procedure as the one described on pp. 8-10 of the main text. State the restriction on parameter values which is necessary to ensure that higher unemployment will ceteris paribus reduce the rate of inflation. How does the parameter $\gamma$ affect the sensitivity of inflation to unemployment? Give an intuitive explanation for your answer.

**2.** Derive an expression for the natural rate of unemployment $\overline{u}$. Comment on the various factors determining $\overline{u}$. How does the parameter $\gamma$ affect the natural rate of unemployment? Try to give an intuitive motivation for your answer.

**3.** Draw a figure analogous to Figure 18.9 in the main text to illustrate the formation of real wages and the determination of the natural employment rate and the change in the rate of inflation, assuming static inflation expectations. When drawing your figure, you may assume that the restriction on parameter values which you derived in Question 1 is satisfied. Explain the factors determining the position and slope of the various curves.

**Exercise .2: Inflation, 'effective' unemployment, and active labour market policy**

The model of inflation and unemployment presented in the main text assumes that all workers are competing on equal terms and with equal intensity for the available jobs. In that case it seems reasonable that any individual worker's probability of finding a job is simply equal to the overall rate of employment, $1 - u$, as we assumed when specifying a worker's 'outside option'.

In the present exercise we assume instead that some of the workers recorded in the unemployment statistics are not fully 'effective' in competing for jobs, perhaps because their skills do not fully match the qualifications demanded by employers, or perhaps because they are not actively searching for a job all the time. We may model this in a simple way by assuming that only a fraction $s$ of the registered unemployed workers contribute fully to the available labour supply in the sense of being immediately ready and able to accept the jobs available. Thus we may specify the 'effective' labour supply $\widehat{u}$ from the pool of unemployed workers as

$$\widehat{u} = su, \qquad 0 < s < 1 \tag{1}$$

In the following, we will refer to $s$ as the Job-Search-and-Matching-Efficiency parameter (the JSME parameter).

If we normalize the total labour force to equal 1, the 'effective' unemployment rate $\widehat{u}$ measures the degree to which the registered unemployment rate $u$ reduces the effective supply of labour. For a worker who is fully qualified and ready to accept the available jobs, the probability of actually finding a job is therefore given by the 'effective' rate of employment $1 - \widehat{u}$, and his probability of ending up in the unemployment pool is equal to $\widehat{u}$. As a modification to equation (8) in the main text, we may therefore specify such a worker's outside option as

$$\overline{V} = (1 - \widehat{u})\, W^e + \widehat{u} B^e \tag{2}$$

As we assumed in the text, the expected general level of wages ($W^e$) and the expected rate of unemployment benefit ($B^e$) are still given by

$$W^e \equiv P^e W^* \tag{3}$$

$$B^e = cW^e, \qquad 0 < c < 1 \tag{4}$$

where $W^*$ is the expected level of real wages, $P^e$ is the expected price level, and $c$ is the replacement rate in the system of unemployment compensation. The expected real wage is proportional to average labour productivity $a$, that is

$$W^* = \omega a \tag{5}$$

and trade unions mark up wages over the outside option:

$$W = m\overline{V}, \qquad m > 1 \tag{6}$$

In a similar way, firms mark up the product price $P$ over unit labour cost $W/a$:

$$P = m\frac{W}{a} \tag{7}$$

**1.** Derive a wage curve from equations (1) through (6), that is, derive an expression for the log of the expected real wage. How does the JSME parameter $s$ affect the real wages claimed by unions (the target real wage)? Explain your finding.

**2.** Derive an expectations-augmented Phillips curve, using (7) and the wage curve you derived in Question 1. How does the JSME parameter $s$ affect the slope of the short-run Phillips curve? Explain.

**3.** Derive an expression for the natural rate of unemployment, using your result in Question 2. How does the JSME parameter $s$ affect the natural unemployment rate? Explain.

Let us now analyze the effects of active labour market policy. Suppose that a fraction $\ell$ of the unemployed workers is enrolled in public education and training programs aimed at improving their qualifications for the available jobs. Such programs may increase the JSME parameter $s$ partly by improving the match between the qualifications demanded by employers and the skills possessed by the unemployed, and partly by increasing workers' motivation to look for jobs (say, because it makes more attractive jobs available to them). Hence we assume that $s$ is an increasing function of $\ell$:

$$s = \overline{s} \cdot \ell^{\eta}, \qquad 0 < \overline{s} < 1, \qquad \eta \geq 0 \tag{8}$$

The elasticity $\eta$ is a parameter measuring the degree to which the labour market programs succeed in actually upgrading the skills and motivation of the unemployed. However, equation (8) does not capture all of the effect of active labour market programs. When a person is enrolled in such a program, he will often not be immediately available for a job in case he gets a job offer, or he may not have time to look for a job. For simplicity, let us assume that only that fraction $1-\ell$ of the unemployed which is not currently engaged in education and training is able to take a job. Moreover, let us assume that these job seekers have benefited from previous training so that their Job-Search-and-Matching-Efficiency corresponds to the value of $s$ specified in (8). Instead of equation (1), the 'effective' unemployment rate will then be given by

$$\widehat{u} = s\,(1-\ell)\,u \tag{9}$$

where $s$ is determined by (8). In the questions below we will assume that $\ell$ is a policy instrument controlled by the makers of labour market policy.

**4.** Derive the expectations-augmented Phillips curve and the natural rate of unemployment when effective unemployment is given by (9) and the JSME parameter is given by (8). Comment on your expressions.

**5.** How does the natural unemployment rate react to an increase in the proportion of the unemployed enrolled in active labour market programs? (hint: derive $\partial\overline{u}/\partial\ell$). Explain the offsetting effects of an increase in $\ell$.

**6.** Suppose that the government wishes to minimize the natural rate of unemployment via its active labour market programs. Derive the value of $\ell$ which will achieve this goal (hint: remember that a necessary condition for minimization of $\overline{u}$ is $\partial\overline{u}/\partial\ell = 0$). Give an intuitive interpretation of your result. Discuss briefly whether the government should necessarily push active labour market policy to the point implied by your formula (hint: are there any costs and benefits of active labour market policy which we have not included in our analysis).

**Exercise 3. Wage formation, price formation and the NAIRU**

Empirical estimates of the natural unemployment rate (the NAIRU) typically find that the evolution of the NAIRU tends to track the evolution of the actual rate of unemployment fairly closely, at least in the short and medium run. This exercise extends our theory of the Phillips curve in order to explain why the NAIRU tends to move in the same direction as the actual unemployment rate in the shorter run.

We consider the following modified version of our previous theory of price and wage formation:

$$\text{Price formation:} \qquad P = m \cdot \frac{W}{a}, \qquad m > 1 \tag{1}$$

$$\text{Wage claim of the representative union:} \qquad W = m\overline{V} \tag{2}$$

$$\text{The 'outside option' of union members:} \qquad \overline{V} = (1 - p^u)\, W^e + p^u B^e \tag{3}$$

$$\text{Expected general wage level:} \qquad W^e = P^e \omega a \tag{4}$$

$$\text{Expected rate of unemployment benefit:} \qquad B^e = cW^e, \qquad 0 < c < 1 \tag{5}$$

Probability of remaining unemployed in case of job loss:

$$p^u = u + \theta \left( u - u_{-1} \right), \qquad \theta \geq 0, \qquad p^u \leq 1 \tag{6}$$

We use the usual notation, that is, $P$ is the price level, $W$ is the nominal wage rate claimed by the representative trade union, $a$ is average labour productivity (exogenous), $m$ is the representative firm's mark-up factor (exogenous), $\overline{V}$ is the expected nominal income obtainable outside the sector considered, $W^e$ is the expected general level of money wage rates, $P^e$ is the expected price level, $\omega$ is the expected ratio between the real wage and average labour productivity (exogenous), $B^e$ is the expected nominal rate of unemployment benefit, $c$ is the replacement rate (exogenous), $p^u$ is the representative worker's expected probability of remaining out of work if he fails to find a job in his original sector, and $u$ and $u_{-1}$ is the unemployment rate in the current and in the previous period, respectively.

The new feature of the model above is equation (6) which says that, ceteris paribus, an unemployed worker has a smaller chance of finding a job if unemployment is rising than if unemployment is falling.

**1.** Explain briefly the assumptions and the theory behind equations (1) and (2).

**2.** How does equation (6) deviate from the theory of the main text of this chapter? Discuss briefly whether the specification in (6) is plausible?

**3.** Derive the wage curve showing how the log of the expected real wage ($w - p^e$) claimed by trade unions depends (among other things) on the level of unemployment and on the change in the rate of unemployment (use our usual notation $w \equiv \ln W$ and $p^e \equiv \ln P^e$). Comment on the expression for the wage curve.

**4.** Use the model (1) through (6) to derive an expectations-augmented Phillips curve of the form

$$\pi = \pi^e + 2\ln m + \ln \omega - \gamma u - \gamma\theta\left(u - u_{-1}\right), \qquad \gamma \equiv 1 - c > 0 \tag{7}$$

where $\pi \equiv p - p_{-1}$, $\pi^e \equiv p^e - p^e_{-1}$, and $p \equiv \ln P$. Comment on the expression in (7) and compare with the expectations-augmented Phillips curve derived in the main text of the chapter.

In the following we assume that inflation expectations are static so that $\pi^e = \pi_{-1}$.

**5.** Define the *long run NAIRU* as the rate of unemployment $\overline{u}$ which will be realized when the rate of inflation as well as the rate of unemployment are constant over time, that is when $\pi = \pi_{-1}$ and $u = u_{-1}$. Derive an equation for the long run NAIRU and use this expression to explain the factors which determine the equilibrium rate of unemployment in the long run.

**6.** Define the *short run NAIRU* as the rate of unemployment $\overline{u}_s$ which will be compatible with a constant inflation rate in the *short* run where we do not necessarily have $u = u_{-1}$ (at the short run NAIRU we thus have $\pi = \pi_{-1}$ but not necessarily $u = u_{-1}$). Derive an expression for the short run NAIRU and show that $\overline{u}_s$ may be written as a weighted average of the long run NAIRU and last period's actual rate of unemployment $u_{-1}$. Which parameter determines how much the short run NAIRU is affected by last period's actual unemployment rate?

**7.** Assume that in period 0 unemployment increases from the long run NAIRU ($\overline{u}$) to the level $\overline{u} + \Delta u_o$. Will it be possible to return to the unemployment rate $\overline{u}$ in period 1 without creating higher inflation? Motivate your answer.

**8.** The recent years have witnessed a large number of mergers and acquisitions in the private business sectors of most OECD economies. Discuss whether and how this trend may affect the long run NAIRU. Do we observe other trends in the private business sector which tend to pull the long run NAIRU in the opposite direction?