Chapter 19
Output and Inflation in the Short Run: Aggregate Supply and Aggregate Demand

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5. september 2003

What factors determine the rates of output, employment and inflation in the short run? This is a basic issue in macroeconomics, and it is the question we will now address, building on the three previous chapters. We will explain the short run levels of output and inflation as the result of the interaction of the aggregate supply of and the aggregate demand for goods and services. We will also explain how the gradual adjustment of prices and wages over time - reflected in changes in the rate of inflation - tends to pull economic activity towards its 'natural' rate.

One important feature of our analysis is that long run macroeconomic equilibrium involves a constant level of *inflation*, that is, a constant percentage rate of price increase, but not necessarily a constant *level* of prices. As we saw in the previous chapter, when unemployment is at its natural rate, inflation will tend to continue forever, simply because the public *expects* it to continue. Another insight from this chapter is that the conduct of monetary policy has important implications for the way the economy works.

In Chapter 15 we saw that economic activity follows a smooth underlying growth trend over the long run, but that it typically deviates from trend in the short run. There are two reasons why output may deviate from trend. First of all, even if unemployment stays at its natural rate, the corresponding 'natural' level of output may fluctuate due to short-run fluctuations in labour productivity. Second, output will tend to fluctuate around its trend to the extent that unemployment fluctuates around its natural rate.

In the previous chapter we found that the rate of employment will deviate from the natural rate when people underestimate or overestimate the rate of inflation. This might seem to suggest that economic activity can only deviate from its long run trend for extended periods of time if expectations are 'sticky' in the sense that economic agents keep on overestimating or underestimating the rate of inflation for quite a while. In practice the deviations of output from trend may also reflect that it takes time for the rate of inflation to adjust even after people have adjusted their inflation expectations. The fact is that most nominal prices and wages only adjust sluggishly over time. For example, a survey of pricing behaviour in the United States found that 39 percent of firms only change their prices once a year, and another 10 percent change their prices less than once a year.[1] One popular explanation for short-run price rigidity is that firms face various costs of changing their prices, including the costs of communicating new prices to customers. Hence it is inoptimal for firms to adjust prices too frequently. When you interpret the macro model set up in this chapter, you should therefore keep in mind that the sluggish adjustment of inflation implied by the model - and hence the sluggish adjustment of output to its trend level - does not necessarily reflect that people cling stubbornly to unrealistic inflation expectations for long periods of time; it may also reflect that it takes time for changes in expectations to feed into the actual price level because it is costly for firms and workers to change prices and wages too frequently.

We start this chapter by showing how our theory of inflation and unemployment implies a positive short-run link between inflation and total output. This link will be called 'the

[1] See Alan S. Blinder, 'On Sticky Prices: Academic Theories Meet the Real World', in *Monetary Policy*, N.G. Mankiw, ed., (Chicago: University of Chicago Press, 1994): pp. 117-154.

short-run aggregate supply curve'. We then turn to the determination of aggregate demand for goods and services, establishing a link between the real rate of interest and aggregate demand. This is followed by a discussion of monetary policy which shows that because of the way in which nominal interest rates typically react to a change in inflation, we get a negative relationship between inflation and the aggregate demand for output. This relationship is called 'the aggregate demand curve'. In the final section we show how the interaction of the aggregate supply and demand curves determines output and inflation in the short run, and how these variables adjust over time to their long-run equilibrium levels.

1 Aggregate supply

In this part of the chapter we will show that our theory of inflation and unemployment presented in Chapter 18 implies a positive short-run relationship between total output and inflation. This relationship, called the short-run aggregate supply curve, will be one of the two crucial building blocks in our theory of the determination of output and inflation.

Deriving the Aggregate Supply Curve

In Chapter 18 we derived the so-called expectations-augmented Phillips curve. As you recall, this 'curve' had the form

$$\pi = \pi^e + \gamma\left(\overline{u} - u\right), \qquad \gamma > 0, \tag{1}$$

where π and π^e are the actual and expected rates of inflation, respectively, and u and $\overline{u}$ are the actual and natural rates of unemployment. Our specific theory in Chapter 18 implied that $\gamma = 1 - c$, where c is the ratio of unemployment benefits to wages, realistically assumed to be less than one. However, for the purpose of this chapter the only important assumption is that γ is positive, reflecting that cyclical unemployment $u - \overline{u}$ reduces inflation by moderating the wage claims of workers.

In Chapter 18 we also assumed that total output Y is proportional to the number of employed workers L, with a proportionality factor a indicating the average productivity of labour[2]. By definition, total employment L equals the rate of employment $1-u$ multiplied by the total labour force N. Hence we have

$$Y = aL = a(1-u)N. \tag{2}$$

By analogy, we may define the 'natural' level of output $\overline{Y}$ as the volume of output produced when employment is at its natural rate $1-\overline{u}$. In other words,

$$\overline{Y} = a(1-\overline{u})N. \tag{3}$$

It is convenient to rewrite these equations in natural logarithms, since changes in logs can be interpreted as percentage changes and are therefore independent of our units of measurement. Taking logs on both sides of (2) and using the approximation $\ln(1-u) \approx -u$, we get $y \equiv \ln Y = \ln a + \ln(1-u) + \ln N$, implying

$$y \approx \ln a + \ln N - u. \tag{4}$$

In a similar way we find from (5) that

$$\overline{y} \equiv \ln \overline{Y} \approx \ln a + \ln N - \overline{u}. \tag{5}$$

Subtracting (5) from (4), we get

$$y - \overline{y} = \overline{u} - u. \tag{6}$$

We see from (6) that the percentage deviation of actual from natural output $(y-\overline{y})$ equals the difference between the natural and the actual unemployment rate. This simple one-to-one relationship between $y-\overline{y}$ and $\overline{u}-u$ is due to our simplifying assumption in (2)

[2]In Chapter 18 we explained that the constancy of average and marginal labour productivity a requires that firms have some idle plant and equipment so that they can increase employment without running into diminishing returns.

that the production function is linear. The only thing that matters for our analysis below is that $y - \overline{y}$ is roughly *proportional* to $\overline{u} - u$.

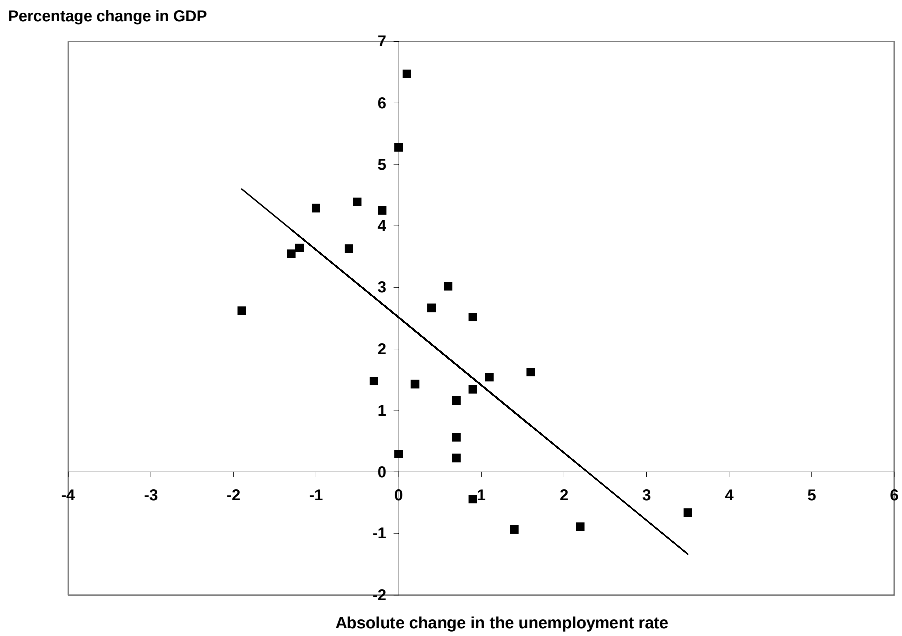

Figure 19.1: Okun's Law for Denmark, 1971-1995
The scatterplot shows the annual percentage change in GDP against the absolute change from the previous year in the rate of unemployment. Each dot corresponds to a single year. For the statistically proficient, the regression line for Denmark has R^2=0.41 and t=-3.98.
Source: OECD and DSTB (Statistics Denmark)

Let us use the notation Δx to indicate the change in variable x. If $x \equiv \ln X$, it follows that Δx is the percentage change in X. Suppose now that the natural unemployment rate is constant over time so that $\Delta \overline{u} = 0$. According to (6) we then have

$$\Delta y = \Delta \overline{y} - \Delta u, \tag{6.a}$$

where Δy is the percentage growth rate of actual GDP, and $\Delta \overline{y}$ is the growth rate of natural output. Equation (6.a) implies a negative link between the absolute change in unemployment and the percentage change in actual GDP. Such a link between unemployment and real GDP is well established empirically and is known as *Okun's Law*, named after the American economist Arthur Okun who first documented this 'law'.[3] We already came across Okun's Law in Chapter 12. Figure 19.1 illustrates it once again for the case of Denmark. It is interesting to note from the estimated regression line in Figure 19.1 that in Denmark, a one-percentage point reduction in unemployment does on average seem to be associated with a roughly one percentage point increase in GDP, just as we postulate in equation (6.a).

Inserting (6) into our expectations-augmented Phillips curve (1), we end up with the Short-Run Aggregate Supply (SRAS) curve:

$$\pi = \gamma \left(y - \overline{y} \right) + \pi^{e}. \tag{7}$$

The short-run aggregate supply curve in (7) summarizes the supply side of the economy. It implies that, for a given expected rate of inflation, the actual inflation rate varies positively with the percentage deviation between actual and natural output. Moreover, the impact of $y - \overline{y}$ on inflation is given by the slope of the Phillips curve, captured in our parameter γ: a steeper Phillips curve yields a steeper aggregate supply curve.

Because the expected inflation rate is here taken as given, the curve is a *short-run* relationship. Over time, the expected inflation rate will gradually adjust in reaction to previous inflation forecast errors. When π^e changes, it follows from (7) that the short-run aggregate supply (SRAS) curve will shift upwards or downwards. This is illustrated in Figure 19.2 which shows three different SRAS curves corresponding to three different levels of the expected inflation rate. In long-run equilibrium, when expected inflation equals actual inflation, we see from (7) that output must be equal to its 'natural' level $\overline{y}$. The natural rate of output is independent of the rate of inflation, since the natural unemployment rate $\overline{u}$ is independent of π. The Long-Run Aggregate Supply (LRAS) curve is therefore vertical,

[3] Arthur M. Okun, 'Potential GNP: Its Measurement and Significance', in *Proceedings of the Business and Economics Statistics Section, American Statistical Association* (Washington, D.C.: American Statistical Association, 1962).

as shown in Figure 19.2.

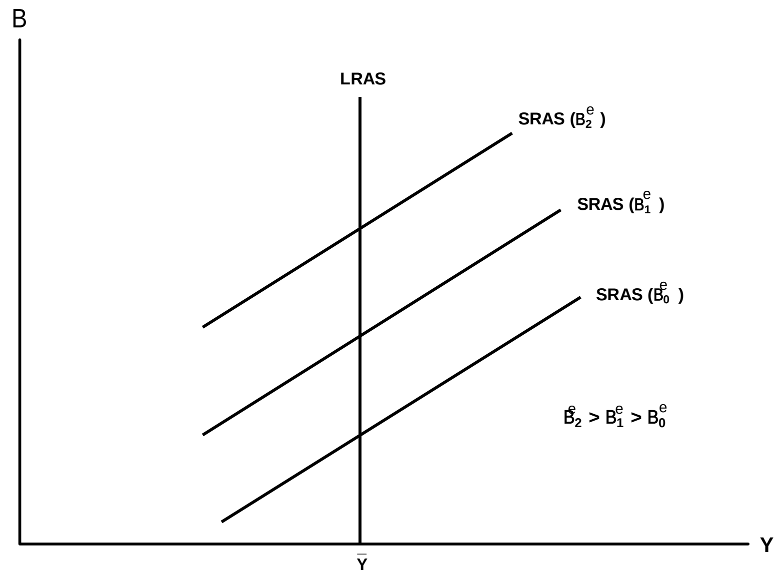

Figure 19.2: Aggregate supply in the short run (SRAS) and in the long run (LRAS)

Allowing for Supply Shocks

A change in the expected inflation rate is not the only source of shifts in the short-run aggregate supply curve. To see this, we go back to equation (18) in Chapter 18 which showed that the natural rate of unemployment is given by

$$\overline{u} = \frac{2\ln m + \ln \omega}{1 - c}, \tag{8}$$

where m is the mark-up factor in the price setting curve of firms, and ω is an indicator of the real wage aspirations of workers (the ratio between the expected real wage and average labour productivity). Recalling from (5) that $\overline{y} \approx \ln a + \ln N - \overline{u}$, it follows from (7) and (8) that the short-run aggregate supply curve will shift *upwards* in case of a rise in the natural rate of unemployment. A rise in $\overline{u}$ will occur if firms raise their profit margin m (say, due to merger activity which increases the market power of firms), if the real wage claims of workers become more aggressive, implying a rise in ω, or if the system of unemployment compensation becomes more generous so that c goes up.[4]

Further, the short-run fluctuations in productivity growth which were illustrated in Figure 18.8 will also cause shifts in the SRAS curve. Let us denote the trend level of labour productivity by a^*. The percentage deviation of actual productivity a from its underlying trend level may then be approximated by

$$s = \ln a - \ln a^*. \tag{10}$$

When s is positive, we say that the economy experiences a *positive productivity shock*, whereas a negative value of s reflects a so-called *negative productivity shock*, meaning that actual labour productivity is below its long-run growth trend. We now define the (log of the) 'normal' level of output $\overline{y}_o$ as that volume of output which is produced when unemployment is at its natural rate *and* labour productivity is at its 'normal' level a^*:

$$\overline{y}_o \equiv \ln a^* + \ln N - \overline{u}. \tag{11}$$

You may think of normal output $\overline{y}_o$ as the output level corresponding to the smooth underlying growth trend which we estimated in Chapter 15 to separate the cyclical component of GDP from its long-run growth component. By comparison, the 'natural' level of output

[4] The inflationary effect of a rise in unemployment benefits may be demonstrated as follows: inserting (5) and (8) and using the fact that $\gamma = 1 - c$, we may rewrite the SRAS curve (7) as

$$\pi = \pi^e + (1-c)\overbrace{\left(y - y^f\right)}^{<0} + 2\ln m + \omega, \qquad y^f \equiv \ln(aN), \tag{9}$$

where y^f is the log of 'full-employment output', that is, the level of output which would prevail if each and every worker in the labour force N were fully employed. Since the economy never operates at a zero unemployment rate, we have $y < y^f$. We then see from (9) that an increase in the replacement rate c will indeed shift the short-run aggregate supply curve upwards. We also see that a rise in c will make the SRAS curve *flatter*. The reason is that a higher replacement rate reduces a worker's income loss in case he is fired. This makes real wage claims less sensitive to unemployment so that cost pressures on firms are less affected by changes in economic activity.

$\overline{y} = \ln a + \ln N - \overline{u}$ is the volume of output corresponding to the natural unemployment rate and the *actual* productivity level a. Thus the natural rate of output will fluctuate around normal output to the extent that actual productivity fluctuates around normal productivity, since it follows from (5), (10) and (11) that

$$\overline{y} = \ln a + \ln N - \overline{u} = \ln a^{*} + \ln N - \overline{u} + s = \overline{y}_{o} + s. \tag{12}$$

Equation (12) may be inserted into (7) to give the modified short-run aggregate supply curve

$$\pi = \pi^{e} + \gamma\left(y - \overline{y}_{o}\right) - \gamma s. \tag{13}$$

From (13) we see that a positive productivity shock will shift the short-run aggregate supply curve downwards. The reason is that higher productivity enables output to grow without requiring a higher employment rate and hence without creating stronger wage cost pressure on firms. By analogy, a negative productivity shock (a negative value of s) will shift the SRAS curve upwards, because lower productivity means that employment will have to increase to maintain the same level of output, and higher employment means stronger wage pressure which feeds into a higher rate of inflation.

The magnitude $y - \overline{y}_{o}$ is the percentage deviation between actual output and 'normal' output (trend output). This is usually referred to as the 'output gap'. Thus we may summarize our theory of aggregate supply in equation (13) by saying that the actual inflation rate will be higher 1) the higher the expected rate of inflation, 2) the larger the output gap, and 3) the lower the level of actual labour productivity relative to trend productivity (the lower the value of s).

This completes our analysis of the short-run aggregate supply curve. As we have seen, the curve shows the short-run relation between inflation and output implied by the behaviour of wage setters and price setters. To find the point on the SRAS curve where the economy will actually be located, we must introduce the aggregate demand side.

2 Aggregate Demand

The Goods Market

We start our analysis of the demand side of the economy by recalling that equilibrium in the goods market requires the aggregate demand for goods to be equal to total output. In this chapter we will focus on a closed economy (we will consider the open economy in later chapters). Aggregate goods demand then consists of the sum of real private consumption C, real private investment I, and government demand for goods and services, G. Hence goods market equilibrium requires

$$Y = C + I + G. \tag{14}$$

In Chapter 16 we saw that private investment behaviour can be summarized in an investment function of the form $I = I(Y, r, K, \varepsilon)$, where r is the real interest rate, K is the predetermined capital stock existing at the beginning of the current period, and ε is a parameter capturing the 'state of confidence', reflecting the expected growth of income and demand. For the purpose of short-run analysis, we may treat the predetermined capital stock as a constant and leave it out of our behavioural equations.[5] We may then write private investment demand as

$$I = I(Y, r, \varepsilon), \qquad I_Y \equiv \frac{\partial I}{\partial Y} > 0, \qquad I_r \equiv \frac{\partial I}{\partial r} < 0, \qquad I_\varepsilon \equiv \frac{\partial I}{\partial \varepsilon} > 0, \tag{15}$$

where the signs of the partial derivatives of the investment function follow from the theory we developed in Chapter 16. Thus, investment increases with current output and with growth expectations ε, whereas it decreases with the real interest rate.

[5] Of course, if we are also interested in the long run we should ideally include the dynamics of capital accumulation. However, since this will complicate the formal analysis considerably, given that we also want to study the dynamics of output as well as inflation, we shall have to leave the inclusion of capital stock adjustment for a more advanced macro course.

Our theory of private consumption presented in Chapter 17 implies a consumption function of the form $C = \widetilde{C}(Y - T, r, V, \varepsilon)$, where T denotes total tax payments so that $Y - T$ is current disposable income, and V is non-human wealth. We assume that the future income growth expected by consumers equals the growth expectations ε of business firms, since firms are owned by consumers. Our analysis in Chapter 16 showed that the market value of non-human wealth is a decreasing function of r, since a rise in the real interest rate will ceteris paribus drive down stock prices as well the value of the housing stock. In other words, $V = V(r)$, and $dV/dr < 0$. To simplify exposition, we will use this relationship to eliminate V from the consumption function and simply write

$$C = C(Y - T, r, \varepsilon), \qquad 0 < C_Y \equiv \frac{\partial C}{\partial (Y - T)} < 1, \qquad C_r \equiv \frac{\partial C}{\partial r} \lesseqgtr 0, \qquad C_\varepsilon \equiv \frac{\partial C}{\partial \varepsilon} > 0. \tag{16}$$

The signs of the partial derivatives were motivated in Chapter 17. From that chapter we recall that the real interest rate has an ambiguous effect on consumption, due to offsetting income and substitution effects, although the negative impact of a higher interest rate on private wealth suggests that the net effect on consumption is likely to be negative. The analysis in Chapter 17 also implied that the marginal propensity to consume current income is generally less than one, as we assume above.

Let us denote total private demand by $D \equiv C + I$. To avoid complications arising from the dynamics of government debt accumulation, we will assume that the government balances its budget so that $T = G$. It then follows from (15) and (16) that the goods market equlibrium condition (14) may be stated in the form

$$Y = D(Y, G, r, \varepsilon) + G. \tag{17}$$

We will now consider the signs and magnitudes of the partial derivatives of the private demand function $D(Y, G, r, \varepsilon)$. Since $D \equiv C + I$, it follows from (15) and (16) that $D_Y \equiv \partial D/\partial Y = C_Y + I_Y > 0$. The derivative D_Y is the marginal private propensity to spend, defined as the increase in total private demand induced by a unit increase in income. We will assume that the marginal spending propensity is less than one so that

$$0 < D_Y \equiv \frac{\partial D}{\partial Y} \equiv C_Y + I_Y < 1. \tag{18.a}$$

The assumption that $D_Y < 1$ guarantees that the *Keynesian multiplier* $\widetilde{m} \equiv 1/(1 - D_Y)$ is positive. Recall from your basic macro course that the Keynesian multiplier measures the total increase in aggregate goods demand generated by a unit increase in some exogenous demand component, provided that intererest rates and prices stay constant. The Keynesian multiplier captures the phenomenon that once economic activity goes up, the resulting rise in output and income induces a further increase in private consumption and investment, which generates an additional rise in output and income that in turn causes a new round of private spending increase, and so on. Below we shall return to the role played by the Keynesian multiplier in our theory of aggregate demand.

Since $T = G$, we see from (16) that

$$D_G \equiv \frac{\partial D}{\partial G} \equiv -\frac{\partial C}{\partial (Y - T)} = -C_Y < 0. \tag{18.b}$$

Given that $C_Y < 1$, it follows that the net effect of a unit increase in government demand on aggregate (private plus public) demand will be $1 + D_G = 1 - C_Y > 0$. In other words, a fully tax-financed increase in public consumption will only be partially offset by a fall in private consumption, so the net effect on aggregate demand will be positive. This assumes that at least part of the increase in taxes is expected to be temporary, for as we saw in Chapter 17, a permanent tax increase will tend to generate a equivalent fall in private consumption.

The effect of a rise in the real interest rate on private demand is given by $D_r \equiv \partial D/\partial r = C_r + I_r$. The (negative of the) derivative D_r measures the effect of a rise in the real interest rate on the private sector savings surplus. The private sector savings surplus is defined as $SS \equiv S - I$, where private saving is given by $S \equiv Y - T - C$. Hence we have $\partial SS / \partial r = -C_r - I_r = -(C_r + I_r) \equiv -D_r$. There is strong empirical evidence that a higher real interest rate raises the private sector savings surplus. For example, Figure 19.3 illustrates a clear positive correlation between SS and a measure of the real interest rate in Denmark. Even though economic theory does not unambigously determine the sign of the derivative C_r, we may therefore safely assume that

$$D_r \equiv \frac{\partial D}{\partial r} \equiv C_r + I_r < 0. \tag{18.c}$$

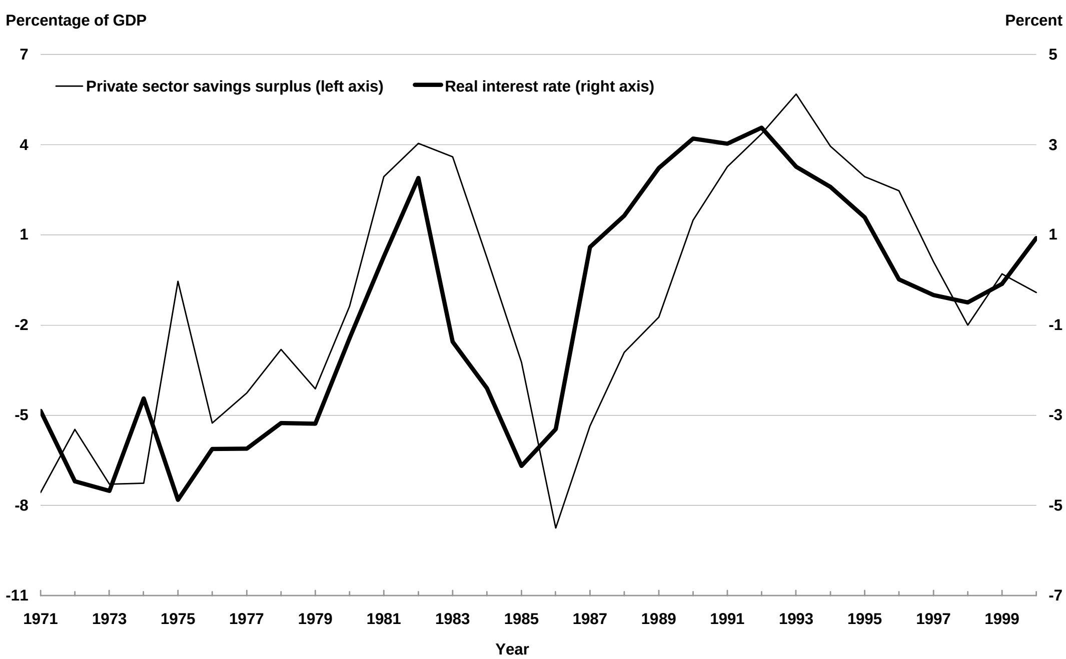

Figure 19.3: The real interest rate and the private sector savings surplus in Denmark, 1971-2000

Note: The real interest rate is measured as the after-tax nominal interest rate on 10-year government bonds minus an estimated trend rate of inflation which includes the rate of increase of housing prices.

Source: Erik Haller Pedersen, 'Udvikling i og måling af realrenten', Danmarks Nationalbank, Kvartalsoversigt, 3. kvartal, 2001, Figur 6.

Finally we see from (15) and (16) that the effect on private demand of more optimistic growth expectations is

$$D_\varepsilon \equiv \frac{\partial D}{\partial \varepsilon} \equiv C_\varepsilon + I_\varepsilon > 0. \tag{18.d}$$

It will be convenient to rewrite the goods market equilibrium condition (17) such that output, government spending and the confidence variable ε appear as percentage deviations from their trend values. In the appendix to this chapter, we show that (17) implies an approximate relationship of the form

$$y - \overline{y}_o = \alpha_1 (g - \overline{g}) - \alpha_2 (r - \overline{r}) + v, \tag{19}$$

$$g \equiv \ln G, \qquad \overline{g} \equiv \ln \overline{G}, \qquad v \equiv \widetilde{m} \left(\overline{\varepsilon} D_\varepsilon / \overline{Y}_o \right) (\ln \varepsilon - \ln \overline{\varepsilon}), \tag{20.a}$$

$$\alpha_1 \equiv \widetilde{m} (1 - C_Y) \left(\overline{G} / \overline{Y}_o \right) > 0, \qquad \alpha_2 \equiv -\widetilde{m} \left(D_r / \overline{Y}_o \right) > 0. \tag{20.b}$$

The magnitudes $\overline{G}$, $\overline{r}$, and $\overline{\varepsilon}$ are the values of G, r and ε prevailing in a long-run equilibrium where (the log of) output y is at its trend value $\overline{y}_o$. Thus equation (19) says that the percentage deviation of output from trend can be approximated by a linear function of the deviations of r, g and $\ln \varepsilon$ from their trend values. Of course, (19) is just a particular way of stating that the aggregate demand for goods varies negatively with the real interest rate and positively with government spending and with expected income growth. Note that the long-run equilibrium real interest rate $\overline{r}$ can be found from the following condition for long-run goods market equilibrium:

$$\overline{Y}_o = D(\overline{Y}_o, \overline{G}, \overline{r}, \overline{\varepsilon}) + \overline{G}. \tag{21}$$

Notice also the role played by the Keynesian multiplier $\widetilde{m} \equiv 1/(1 - D_Y)$ in the definitions of the coefficients α_1 and α_2 given in (20.b). For example, if taxes are raised by one unit to finance a unit increase in government consumption, the immediate impact is a net increase in aggregate demand equal to $1 - C_Y$. But when the Keynesian multiplier effect is accounted for, the total increase in demand adds up to $\widetilde{m}(1 - C_Y)$. Therefore, if public consumption increases by one percent, the resulting percentage increase in total demand will be $\widetilde{m}\left(1-C_Y\right)\left(\overline{G}/\overline{Y}_o\right)$, given that the initial ratio of public consumption to total output is $\overline{G}/\overline{Y}_o$. This explains the coefficient α_1 on the percentage increase in government consumption $g-\overline{g}$ in equation (19). Similarly, if the real interest rate goes up by one percentage point, the resulting *percentage* drop in total demand is $D_r/\overline{Y}_o$. When this initial fall in demand is magnified by the Keynesian multiplier, the total percentage fall in demand adds up to $-\widetilde{m}\left(D_r/\overline{Y}_o\right)$, as shown by the expression for α_2 in (20.b). Thus the familiar Keynesian multiplier theory is built into our theory of aggregate demand.

Equation (19) is our preliminary version of the economy's aggregate demand curve. To confront this curve with our aggregate supply curve, we must turn (19) into a relationship between output and inflation. For this purpose we must study the relationship between inflation and the real interest rate, and that requires us to take a closer look at the money market and the conduct of monetary policy.

The Money Market

From chapters 2 and 3 and your basic macro course you may recall that equilibrium in the money market is obtained when

$$\frac{M}{P}=m\left(Y,i\right),\qquad m_Y\equiv\frac{\partial m}{\partial Y}>0,\qquad m_i\equiv\frac{\partial m}{\partial i}<0,\tag{22}$$

where $m\left(Y,i\right)$ is the real demand for money, i is the nominal interest rate, M is the nominal money supply, and P is the price level. The left-hand side of (22) is the supply of real money balances which must be equal to real money demand in equilibrium. Real money demand varies positively with income, reflecting that a rise in income leads to more transactions. At the same time money demand varies negatively with the nominal interest rate, since a higher interest rate raises the opportunity cost of holding money rather than interest-bearing assets. For concreteness, we will assume that the demand for real money balances can be approximated by a function of the form

$$m(Y,i) = kY^{\eta}e^{-\beta i}, \qquad k>0, \qquad \eta>0, \qquad \beta>0, \tag{23}$$

where e is the exponential function, and η is the income elasticity of money demand. Notice that the interest rate i appearing in the money demand function should be interpreted as a *short-term* interest rate, since the closest substitutes for money are the most liquid interest-bearing assets with a short term to maturity.

Monetary Policy Rules

To find the link between output and inflation on the economy's demand side, we need to know how the real interest rate r appearing in (19) is related to these two variables. This depends on the way monetary policy is conducted. Monetary policy regimes vary across time and space. Here we shall focus on two benchmark monetary policy rules which have received widespread attention in the literature. A monetary policy rule is a rule or principle prescribing how the monetary policy *instrument* of the central bank should be chosen. In practice, the main monetary policy instrument of the central bank is its short-term interest rate charged or offered vis á vis the commercial banking sector. Via their control of the central bank interest rate, monetary policy makers can roughly control the level of short term interest rates prevailing in the interbank market. The interbank market is the market for short-term credit where commercial banks with a temporary surplus of liquidity meet other commercial banks with a temporary liquidity shortage. The interbank interest rate in turn heavily influences the level of market interest rates on all types of short-term credit.

Under the *constant money growth rule* for the conduct of monetary policy the central bank adjusts its short-term interest rate to ensure that the forthcoming money demand results in a *constant growth rate of the nominal monetary base*. Assuming a constant money multiplier (that is, a constant ratio between the broader money supply and the monetary base), this will also ensure a constant growth rate of the broader money supply which includes bank deposits as well as base money. In an influential book published in 1960,[6] Milton Friedman argued that a constant money supply growth rate would in practice ensure the highest degree of macroeconomic stability which could realistically be achieved, since it would imply a stable increase in aggregate nominal income. This argument was based on Friedman's belief in a stable money demand function with a low interest rate elasticity. To see this point most clearly, suppose for simplicity that our parameter β in (23) is close to zero, and that the income elasticity of money demand η is equal to one. Money market equilibrium then roughly requires $M = kPY$, where k is a constant. Hence aggregate nominal income PY must grow roughly in proportion to the nominal money supply M. Securing a stable growth rate of M will then secure a stable growth rate of nominal income.

Friedman pointed out that we only have limited knowledge of the way the economy works. His studies of American monetary history also suggested that monetary policy tends to affect the real economy with long and variable lags.[7] Friedman therefore argued that the central bank may often end up *de*stabilizing the economy if it attempts to manage aggregate demand through activist monetary policy by constantly varying the growth rate of money supply in response to changing economic conditions. Moreover, according to Friedman the self-regulating market forces are sufficiently strong to ensure that real output and employment will be pulled fairly quickly towards their 'natural' rates, following an economic disturbance. Given that activist monetary policy may fail to stabilize the economy, and that the need for stabilization is limited anyway, Friedman concluded that his constant money supply growth rule would be the best way to conduct monetary policy.

[6] See Milton Friedman, *A Program for Monetary Stability*. New York: Fordham University Press, 1960.

[7] Milton Friedman and Anna Schwartz, '*A Monetary History of the United States, 1867-1960'*. Princeton, N.J., Princeton University Press, 1963. In chapter 21 we shall discuss the lags in monetary policy in more detail.

Friedman's arguments did not go unchallenged, but they had a substantial impact on many central bankers. In particular, the German Bundesbank adopted stable target growth rates for the money supply from the 1970s, and after the formation of Monetary Union the European Central Bank has maintained a target for the evolution of the money supply to support its target for (low) inflation.

What does the constant money growth rule imply for the formation of interest rates? To investigate this, suppose that the central bank knows the structure of the money market sufficiently well to be able to implement its desired constant growth rate μ of the nominal money supply. Using (22) and (23), we may then write the condition for money market equilibrium as

$$\frac{(1+\mu)\,M_{-1}}{(1+\pi)\,P_{-1}} = kY^{\eta}e^{-\beta i}, \tag{24}$$

where M_{-1} and P_{-1} are the nominal money supply and the price level prevailing in the previous period, respectively. We want to study how the economy behaves when it is not 'too' far off its long run trend. We therefore assume that the economy was in long-run equilibrium in the previous period. Ignoring growth for simplicity,[8] a long-run equilibrium requires that the real money supply be constant. This in turn means that the inflation rate π must equal the monetary growth rate μ. Moreover, in a long-run equilibrium with no supply shocks, output and the real interest rate must be at their trend levels $\overline{Y}_o$ and $\overline{r}$, and the nominal interest rate i must equal $\overline{r}+\mu$, given that $\pi=\mu$. If we denote the long-run value of the real money stock by m^*, our assumption that the money market was in long run equilibrium in the previous period then implies that

$$\frac{M_{-1}}{P_{-1}} = m^* = k\overline{Y}_o^{\eta}e^{-\beta(\overline{r}+\mu)}. \tag{25}$$

[8] If we assume that trend output $\overline{Y}_o$ grows at the constant rate x, the real money supply would have to grow at the rate ηx in a long run equilibrium with a constant interest rate (you may want to demonstrate this for yourself). This in turn would imply an equilibrium rate of inflation π^* equal to $\pi^*=\mu-\eta x$. Nevertheless, for constant values of η and x, you can easily show that the nominal interest rate would still react to changes in inflation and output in accordance with our equation (28) derived below.

Taking natural logs of (24), remembering that $M_{-1}/P_{-1} = m^*$, and using the approximations $\ln(1+\mu) \approx \mu$ and $\ln(1+\pi) \approx \pi$, we get

$$\mu - \pi + \ln m^* = \ln k + \eta y - \beta i, \tag{26}$$

where (25) implies

$$\ln m^* = \ln k + \eta \overline{y}_o - \beta\left(\overline{r} + \mu\right). \tag{27}$$

By inserting (27) into (26) and rearranging, you may verify that

$$i = \overline{r} + \pi + \left(\frac{1-\beta}{\beta}\right)(\pi - \mu) + \left(\frac{\eta}{\beta}\right)(y - \overline{y}_o). \tag{28}$$

Equation (28) shows how the short-term nominal interest rate i will react to changes in inflation and output if monetary policy aims at securing a constant growth rate μ of the nominal money supply. Since η and β are both positive, we see that the interest rate goes up whenever output y increases. If the numerical semielasticity β of money demand with respect to the interest rate is not too high ($\beta < 1$), we also see that the nominal interest rate will increase more than one-to-one with the rate of inflation, implying an increase in the *real* interest rate. Note that since the long-term equilibrium inflation rate equals the monetary growth rate, our parameter μ may be interpreted as the central bank's *target inflation rate.*[9]

[9] In a provocative essay Milton Friedman argued that the target inflation rate μ ought to be *negative* and numerically equal to the equilibrium real interest rate so that the nominal interest $i = \overline{r} + \mu$ becomes zero. Friedman's argument was that the marginal social cost of supplying money to the public is roughly zero, since printing money is virtually costless. To induce people to hold the socially optimal amount of money balances, the marginal private opportunity cost of money-holding - given by the nominal interest rate - should therefore also be zero. If the nominal interest rate is positive, people will economize on their money balances to hold more of their wealth in the form of interest-bearing assets. The resulting inconvenience of having to exchange interest-bearing assets for money more often to handle the daily transactions will yield a utility loss. According to Friedman this welfare loss can be avoided at zero social cost by driving the nominal interest rate to zero so that people are no longer induced to economize on their money balances. This recommendation of a steady rate of deflation to ensure a zero nominal interest rate is sometimes referred to as the 'Friedman Rule'. See Milton Friedman, "The Optimum Quantity of

As we have mentioned, some central banks have occasionally defined targets for the growth rate of the nominal money supply, in accordance with Friedman's recommendation. However, in an influential article, American economist John Taylor argued that rather than worrying too much about the evolution of the money supply as such, the central bank might as well simply adjust the short term interest rate in reaction to observed deviations of inflation and output from their targets.[10] Assuming that policy makers wish to stabilize output around its trend level $\overline{y}_o$, and denoting the inflation target by π^*, we may then specify the monetary policy rule proposed by John Taylor as

$$i = \overline{r} + \pi + h(\pi - \pi^*) + b(y - \overline{y}_o), \qquad h > 0, \qquad b > 0. \tag{29}$$

Equation (29) is the famous *Taylor Rule*. Recalling that the monetary growth rate μ may be interpreted as an inflation target, we see from (28) and (29) that the nominal interest rate follows an equation of the same form under the constant money growth rule and under the Taylor rule. However, there is an important difference. Under the constant money growth rule the coefficients in the equation for the interest rate depend on the parameters η and β in the money demand function. In contrast, under the Taylor Rule the parameters h and b in (29) are chosen directly by policy makers, depending on their aversion to inflation and output instability. According to Taylor it is important that the value of h is positive so that the real interest rate goes up when inflation increases. If $1 + h$ is less than one, a rise in inflation will drive down the real interest rate $i - \pi$, and this in turn will further feed inflation by stimulating aggregate goods demand, leading to economic instability. In

Money". In *The Optimum Quantity of Money and Other Essays*, pp. 1-50. Chicago: Aldine Publishing, 1969.

Many economists consider Friedman's recommendation to be theoretically interesting, but potentially dangerous in practice. They argue that a policy of deflation can trigger a destabilizing wave of bankruptcies if debtors have not fully anticipated the future fall in prices and the resulting increase in their real debt burdens. In chapter 21 we shall consider some further reasons why a negative inflation target may be undesirable.

[10] See John B. Taylor, 'Discretion versus Policy Rules in Practice', in *Carnegie-Rochester Conference Series on Public Policy*, vol. 39, 1993, pp. 195-214.

fact Taylor suggested that the parameter values $h = 0.5$ and $b = 0.5$ would lead to a good economic performance, given the structure of the U.S. economy.

Empirical studies have found that although central bankers never mechanically follow a simple policy rule, central bank interest rates do in fact tend to be set in accordance with equations of the general form given in (29). As we have seen, such interest rate behaviour is consistent with the constant money growth rule as well as the Taylor Rule. However, one problem with the former rule is that a constant monetary growth rate may not succeed in stabilizing the evolution of nominal aggregate demand if the parameters of the money demand function are changing over time. Such unanticipated shifts in the money demand function may occur when new financial instruments and methods of payment emerge as a result of financial innovation.

In part because of this problem with the constant money growth rule, monetary policy has increasingly come to be discussed in terms of the Taylor Rule in recent years.

	Estimate of	
	1+h	b
German Bundesbank[1]	1.31 (0.09)	0.25 (0.04)
Bank of Japan[2]	2.04 (0.19)	0.08 (0.03)
U.S. Federal Reserve[3]	1.83 (0.45)	0.56 (0.16)

Table 19.1: Estimated interest rate reaction functions of three central banks

Notes: Estimates based on monthly data. Standard errors in brackets.
[1] Estimation period 1979:3-1993:12. [2] Estimation period 1979:4-1994:12.
[3] Estimation period 1982:10-1994:12.
Source: Richard Clarida, Jordi Gali, Mark Gertler, 'Monetary Policy Rules in Practice - Some International Evidence', European Economic Review, vol. 42, 1998, pp. 1033-1067.

Table 19.1 shows econometric estimates of the 'Taylor' coefficients $1 + h$ and b in the three largest OECD economies where interest rate policies have not been significantly constrained by a target for the foreign exchange rate. The table is based on monthly data for the period from around 1980 to around 1994. The estimates relate to a target for the central bank's short term interest rate; the estimation method assumes that the actual interest rate adjusts gradually to this target. The figures in brackets are standard errors. All coefficients are statistically significant and are estimated with considerable accuracy, as seen from the fact that the coefficients are typically many times larger than their corresponding standard errors.

Table 19.1 shows that in recent years all of the three central banks have followed Taylor's recommendation that h should be considerably above zero to ensure a rise in the real interest rate in response to a rise in inflation.

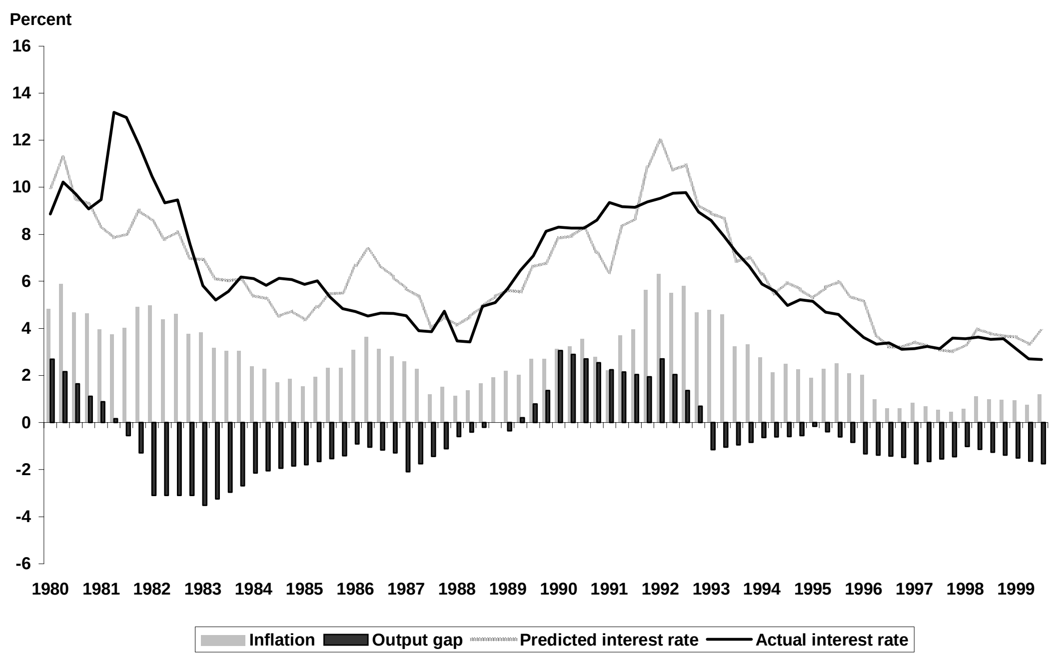

Figure 19.4: The Taylor Rule in Germany

Source: Danish Ministry of Finance

The Bank of Japan seems to react strongly to inflation but weakly to the output gap. By comparison, U.S. monetary policy makers seem to have been much more concerned about deviations of output from trend.

Figure 19.4 shows an interest rate reaction function of the form (29) for Germany, estimated from quarterly data for the period 1980:1 until 1999. The estimated coefficients $1 + h = 1.34$ and $b = 0.28$ are almost the same as those reported in Table 19.1, indicating considerable historical stability in Bundesbank policy. Figure 19.4 compares the actual short-term interest rate to the rate predicted by the estimated Taylor Rule. We see that, in general, the Taylor Rule gives a fairly good description of actual monetary policy. In chapter 21 we will discuss whether the Taylor Rule is in fact also an *optimal* monetary policy.

Monetary Policy and Long Term Interest Rates: the Yield Curve

The central bank can control the short-term interest rate via the choice of its own borrowing and lending rate. But the aggregate demand for goods and services depends to a large extent on the *long-term* interest rate, because a lot of business investment and household investment in durable goods such as housing relies on long term finance, reflecting the long-lived character of these capital goods. Hence the ability of the central bank to influence aggregate demand depends to a large extent on its ability to influence the long term interest rate via its control over the short term interest rate. In this section we study how changes in short-term interest rates engineered by monetary policy are transmitted to long-term interest rates.

For a start we assume that there are only two interest rates: the short-term interest rate i on loans maturing after a single period, and the long-term interest rate i^l on debt maturing after n periods.[11] Thus, an investor who buys the long-term debt instrument at the start of period t and holds it until it matures in period $t + n$ will earn the effective

[11] In practice, the interest rate on 10-year government bonds is often used as an indicator of the long-term interest rate, although governments and private mortgage credit institutions may also issue bonds with 20 or even 30 years to maturity.

nominal interest rate i_t^l in each period from time t until time $t+n$, since the effective yield depends on the price at which he purchased the instrument at time t. Suppose that financial investors consider the two types of debt instrument to be *perfect substitutes* for each other. In that case the effective interest rate on the long-term instrument must adjust to ensure that *the expected returns on short-term and long-terms instruments are equalized.* Specifically, in a financial market equilibrium an investor must expect to end up with the same stock of wealth at time $t+n$ whether he buys a long-term instrument and holds it until maturity, or whether he makes a series of n short-term investments, reinvesting in short-term instruments every time the instrument bought in the previous period matures. At the beginning of period t we therefore have the financial arbitrage condition

$$\left(1+i_t^l\right)^n = (1+i_t) \times \left(1+i_{t+1}^e\right) \times \left(1+i_{t+2}^e\right) \times \ldots\ldots \times \left(1+i_{t+n-1}^e\right), \tag{30}$$

where i_{t+j}^e is the short-term interest rate *expected* to prevail in future period $t+j$. The term on the left-hand side of (30) is the investor's wealth at time $t+n$ if he invests in the long-term instrument at time t and holds on to his investment. The right-hand side of (30) measures the wealth he expects to accumulate if he makes a series of short-term investments, reinvesting his principal plus interest in each period until time $t+n$. In equilibrium the two investment strategies must be equally attractive, given the perfect substitutability of short-term and long-term financial instruments.

According to equation (30) *the current long-term interest rate depends on expected future short-term interest rates.* This is referred to as the *expectations hypothesis.* If the length of our period is, say, a year, a quarter, or a month, the interest rates appearing in (30) will not be far above zero, and our usual approximation $\ln(1+i) \approx i$ will be fairly accurate. Taking logs on both sides of (30) and dividing through by n, we then get

$$i_t^l \approx \frac{1}{n}\left(i_t + i_{t+1}^e + i_{t+2}^e + \ldots\ldots + i_{t+n-1}^e\right). \tag{31}$$

Equation (31) says that *the current long-term interest rate is a simple average of the current and the expected future short-term interest rates.*

We have so far considered only two different debt instruments. In reality a large number of securities with many different terms to maturity are traded in financial markets. But the reasoning which led to equation (31) is valid for any $n \geq 2$, so (31) determines the entire *term structure of interest rates*, that is, the relationship between the interest rates on securities with different terms to maturity (different values of n).

From the term structure equation (31) one can derive the so-called *yield curve* which shows the effective interest rates on instruments of different maturities at a given point in time. According to (31) we have

$$i_t^l = i_t \qquad \text{iff} \qquad i_{t+j}^e = i_t \qquad \text{for all } j = 1, 2, ..., n-1. \tag{32}$$

In other words, if financial investors happen expect no changes in future short-term interest rates, the interest rates on long-term and short-term instruments will coincide, and the yield curve will be quite flat. Figure 19.5 shows that the yield curve in Denmark did in fact look this way on the 2nd of January 2001.

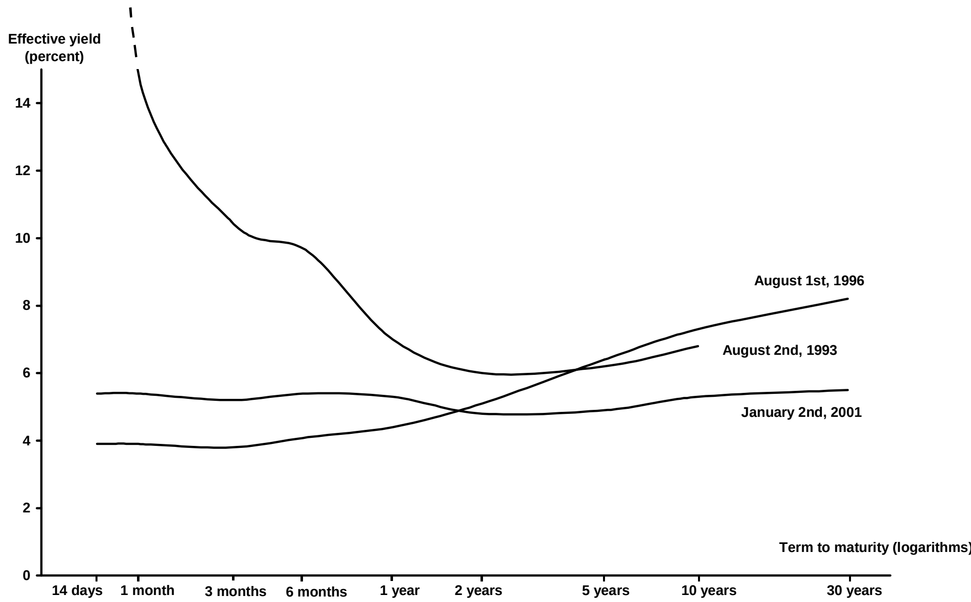

Figure 19.5: The term structure of interest rates in Denmark

Source: Danmarks Nationalbank

As we move from left to right on the horizontal axis, we consider instruments with increasing terms to maturity. The first point on the yield curve shows the market interest rate on interbank credit with 14 days until maturity. This interest rate is almost perfectly controlled by the interest rate policy of the Danish central bank (Danmarks Nationalbank). The last point on the yield curve plots the effective market interest rate on 30-year Danish government bonds. The flatness of the yield curve suggests that investors in Denmark roughly expected constant short-term interest rates at the beginning of 2001.

A rather flat yield curve is often considered to represent a 'normal' situation where investors have no particular reason to believe that tomorrow will be much different from today. But sometimes the situation is not normal. Figure 19.5 shows that short-term interest rates were way above long-term rates on the 2nd of August 1993. Around that date Denmark and many other European countries suffered from the speculative attack on the European Monetary System, the fixed exchange rate system existing before the formation of European Monetary Union. To stem the capital outflow generated by fears of a devaluation of the Danish krone, Danmarks Nationalbank drove up the 14-days interbank interest rate to the exorbitant height of 45 percent p.a.! The fact that long-term interest rates remained much lower indicates that investors did not expect the extreme situation in the short end of the market to last long.

In contrast, the yield curve had an unusually steep upward slope on the 1st of August 1996, as illustrated in Figure 19.5. At that time it was generally expected that the pace of growth in the European economy was about to increase significantly. Market participants therefore expected future monetary policy to be tightened to counteract inflationary pressures, and the expectation of higher future short-term interest rates drove current long-term rates significantly above the current short rate.

What does all this imply for monetary policy? The crucial implication is that *monetary policy can affect long-term interest rates significantly only by affecting expectations about future short-term interest rates.* For example, if the central bank engineers a unit increase in the current short rate i_t which the market considers to be purely temporary, the expected future interest rates appearing on the right-hand side of (31) will be unaffected, and the interest rate on long-term debt with n periods to maturity will only increase by $1/n$. If the short-term rate applies to an instrument with a term of one month, and the long-term rate relates to a 30-year bond, n will be equal to $12 \times 30 = 360$. In that case a one-percentage point increase in the short-term interest rate will only raise the long-term bond rate by a negligible 0.0028 percentage points, i.e., less than 0.3 basis points! At the other end of the spectrum is the situation where a change in the current short-term interest rate is expected to be permanent. According to (30) the long-term interest rate will then rise by the full amount of the increase in the short rate. This corresponds to the assumption of constant expectations in (32).

The difficulties of controlling long-term interest rates through central bank interest rate policy are illustrated in Figure 19.6. Despite the many successive cuts in the target short-term interest rate of the U.S. Federal Reserve Bank (the Federal funds target rate) undertaken during 2001 in reaction to economic recession, the long-term interest rate refused to come down significantly. This suggests that market participants expected a quick economic recovery which would induce the Fed to raise its interest rate again.

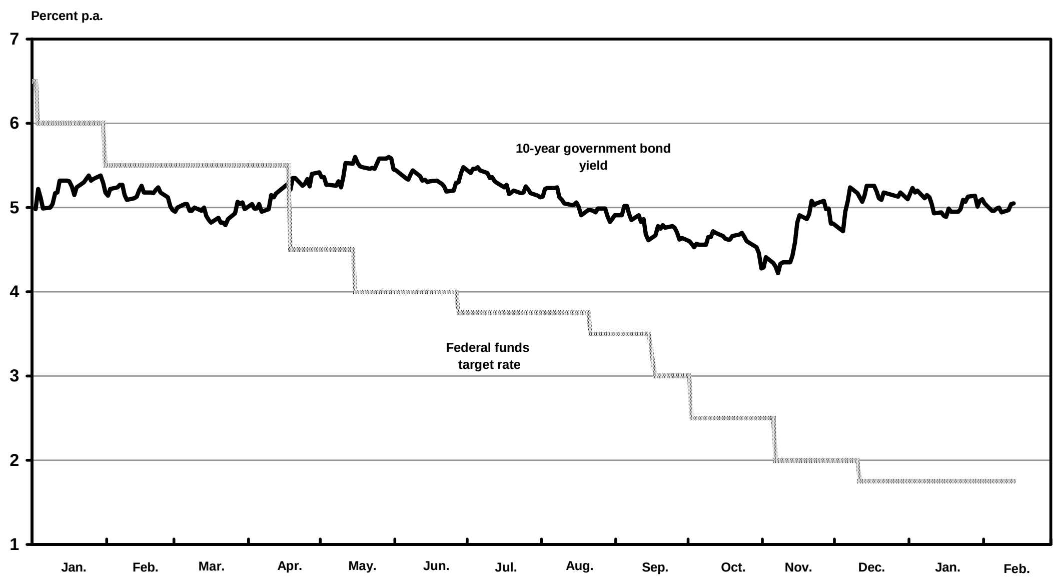

Figure 19.6: The decoupling of short-term and long-term interest rates in the United States, 2001-2002

Source: Danmarks Nationalbank

The fact that monetary policy works to a large extent through its impact on market expectations explains why central banks care so much about their communication strategies, and why market analysts scrutinize every statement by central bankers to find hints about future monetary policy. In any given situation, the transmission from a change in the central bank interest rate to the change in long-term market interest rates will depend on market expectations. These in turn will depend on context and historical circumstances.

In the analysis below we will ignore the complication that long-term interest rates do not always move in line with the short-term rates controlled by monetary policy. In fact we will assume that financial investors have static expectations so that (32) is satisfied. As the preceding analysis makes clear, this is a strong simplification. Yet we should not exaggerate the loss of generality implied by the assumption of static interest rate expectations. Figure 19.7 shows that the long-term market rate and the central bank interest rate do tend to move in tandem over the longer run, even though they may be out of line in the short run.

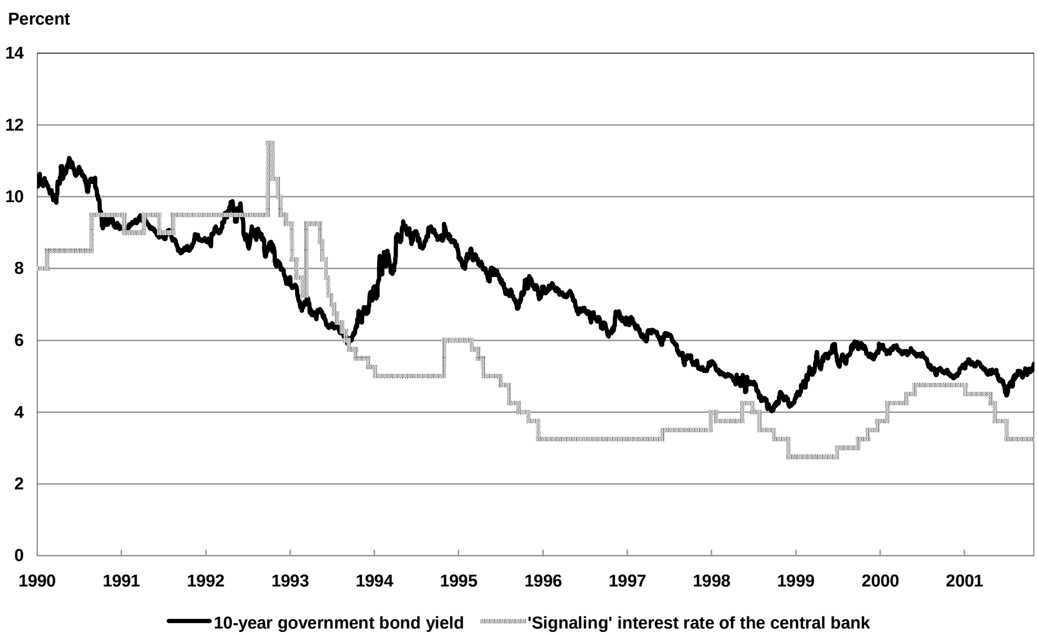

Figure 19.7. The 'signaling' interest rate of the central bank and the 10-year government bond yield in Denmark

Source: Danmarks Nationalbank

Moreover, certain components of aggregate demand may be directly affected by the short-term interest rate. For example, in countries like the United Kingdom and Sweden - and increasingly in Denmark - the interest rate paid by homeowners on their mortgage debt tends to follow the short-term market interest rates. In such an institutional setting the interest rate policy of the central bank may have a considerable impact on housing prices, housing investment and private consumption by directly affecting the user cost of housing.

The discussion in this section just serves to remind you that the conduct of monetary policy is a difficult matter because long-term interest rates often adjust to changes in the central bank interest rate with long and variable lags.

Deriving the Aaggregate Demand Curve

We are now ready to derive the relationship between the inflation rate and the aggregate demand for goods and services. This relationship, called the aggregate demand (AD) curve, is the second cornerstone of our model of the macro economy.

The first step in our derivation of the AD curve is the specification of the relationship between the nominal interest rate, the real interest rate, and inflation. We have not previously paid much attention to timing issues, but now we need to be precise regarding the dating of the rates of inflation appearing in our equations. For an investor incurring debt at the beginning of the current period, the actual real interest rate r^a paid between the current period and the next one is given by

$$1 + r^a \equiv \frac{(1+i)\,P}{P_{+1}} \equiv \frac{1+i}{1+\pi_{+1}} \qquad \Longrightarrow$$

$$r^a \approx i - \pi_{+1}. \tag{33}$$

The variables P and P_{+1} are the price levels prevailing at the start of the current and the next period, respectively, so π_{+1} is the percentage rate of price increase between those two points in time. The variable r^a is called the *ex post real interest rate*, because it measures the real interest rate implied by the *actual* rate of inflation, measured *after* the relevant time period has passed ('ex post'). However, since saving and investment decisions must be made 'ex ante', *before* the future rate of inflation is known with certainty, the real interest rate affecting aggregate goods demand is the so-called *ex ante real interest rate* which is based on the rate of inflation π^e_{+1} *expected* to prevail over the next period:

$$r \approx i - \pi^e_{+1}. \tag{34}$$

We will stick to our simplifying assumption of static inflation expectations. At the start of the current period firms know the current prices of the goods they sell, but they do not know for sure the prices they will charge in the next period, since wages for that period have not yet been set. However, with static expectations firms will assume that the rate of price increase over the next period will correspond to the rate of inflation experienced between the previous and the current period:[12]

$$\pi^e_{+1} = \pi. \tag{35}$$

Equation (35) obviously implies that the ex ante real interest rate in (34) becomes (roughly) equal to

$$r = i - \pi. \tag{36}$$

We may now insert (36) plus the monetary policy rule (29) into (19) to get

$$y - \overline{y}_o = \alpha_1 (g - \overline{g}) - \alpha_2 \overbrace{\left[h(\pi - \pi^*) + b(y - \overline{y}_o)\right]}^{r-\overline{r}} + v,$$

equivalent to the aggregate demand curve:

$$y - \overline{y}_o = \alpha(\pi^* - \pi) + z, \tag{37}$$

$$\alpha \equiv \frac{\alpha_2 h}{1 + \alpha_2 b} > 0, \qquad z \equiv \frac{v + \alpha_1 (g - \overline{g})}{1 + \alpha_2 b}. \tag{38}$$

We see from (37) and (38) that *the aggregate demand curve is downward-sloping* in the (y, π)-space: a higher rate of inflation is associated with lower aggregate demand for output. The reason is that higher inflation induces monetary policy makers to raise the nominal

[12] Remember that π is defined by $P \equiv (1 + \pi) P_{-1}$.

interest rate by so much that the *real* interest rate also goes up (given that the parameter h in the central bank's reaction function (29) is positive). The higher real interest rate in turn dampens aggregate private demand for goods and services.

To identify the determinants of the position and the slope of the AD curve in the (y, π) plane, it is convenient to rearrange equation (37) as

$$\pi = \pi^* + (1/\alpha) z - (1/\alpha)(y - \overline{y}_o). \tag{37.a}$$

The variable z on the right-hand side of (37.a) captures *aggregate demand shocks*. From the definition of z given in (38) we see that aggregate demand shocks may come from changes in fiscal policy, reflected in g, or from changes in private sector confidence affecting the variable v (see the definition of v in (20.a)). A more expansionary fiscal policy or more optimistic growth expectations in the private sector will shift the aggregate demand curve *upwards* in the (y, π) plane. Given our definitions of v and z in (20.a) and (38), the value of z will be zero under 'normal' conditions where public spending and private sector growth expectations are at their trend levels.

The position of the aggregate demand curve is also affected by the central bank's inflation target π^*. If the central bank becomes more 'hawkish' in fighting inflation (if π^* falls), the aggregate demand curve will shift *downwards*.

The *slope* of the aggregate demand curve $(1/\alpha)$ is also influenced by the behaviour of monetary policy. If the central bank puts strong emphasis on fighting inflation and little emphasis on stabilizing output, the parameter h in the Taylor Rule will be high, and the parameter b will be low. Since $\alpha \equiv \alpha_2 h/(1 + \alpha_2 b)$, this means that the aggregate demand curve will be flat (α will be high). On the other hand, if monetary policy reacts strongly to the output gap and only weakly to inflation, we have a low value of h and a high value of b, generating a steep aggregate demand curve. These results are illustrated in Figure 19.8, where the aggregate demand curve is denoted by AD.

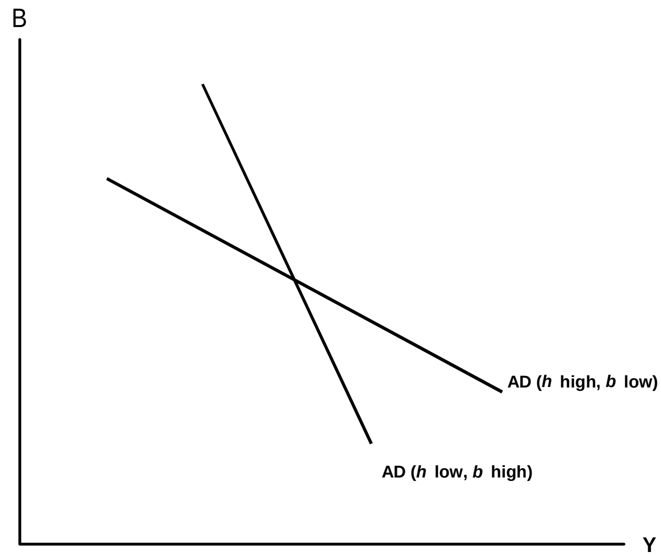

Figure 19.8: The aggregate demand curve

3 Bringing Aggregate Supply and Aggregate Demand Together

After all these preliminaries, we are finally able to determine the levels of output and inflation which will prevail in the short run. In a macroeconomic equilibrium, the aggregate demand for goods and services must match aggregate supply. Output and inflation will therefore adjust to the point E_0 in Figure 19.9 where the SRAS curve intersects the AD curve. Remember that the position of the SRAS curve depends on the expected rate of inflation π^e from the previous to the current period. Thus, the short run equilibrium values π_0 and y_0 for the actual current inflation rate and for current output depend on the expectation π^e about current inflation formed at the end of the previous period.

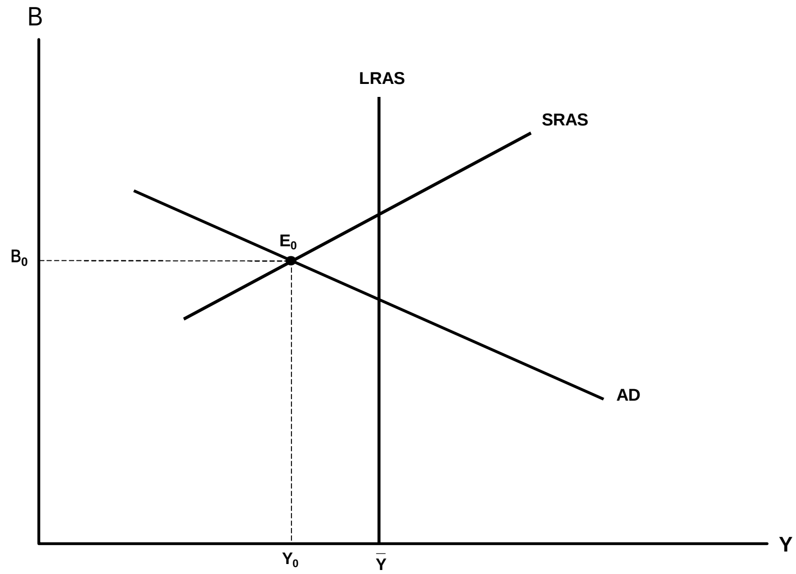

Figure 19.9: Short-run macroeconomic equilibrium with cyclical unemployment

In Figure 19.9 we have also included the long run aggregate supply curve (LRAS). The position of the LRAS curve is determined by the economy's natural rate of output $\overline{y}$ (which will be equal to trend output when our supply shock variable s is zero). The short-run equilibrium illustrated in Figure 19.9 is characterized by cyclical unemployment, since actual output y_0 falls short of natural output. With $y_0 < \overline{y}$, our SRAS curve (7) implies that actual inflation becomes lower than expected inflation. Over time, the overestimation of inflation will motivate people to revise their inflation forecasts, and the expected inflation rate will gradually fall. As a consequence of the fall in π^e, the SRAS curve will gradually shift downwards, and the economy will move down the AD curve towards the long-run equilibrium point $\overline{E}$ illustrated in Figure 19.10. At this point output is at its natural level and expected inflation coincides with actual inflation.

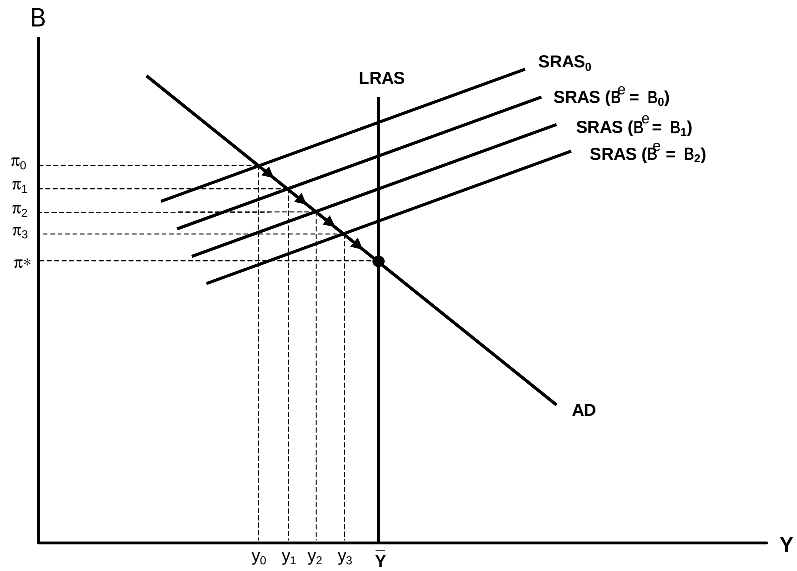

Figure 19.10: The adjustment to long-run macroeconomic equilibrium

The behaviour of the central bank is crucial for this adjustment process. As workers reduce their required rates of nominal wage increase due to the fall in expected inflation, the central bank lowers its interest rate in response to weaker inflationary pressure. The stronger monetary policy reacts to falling inflation, the flatter is the aggregate demand curve, and the faster is the convergence of output to its natural rate. To make sure that falling inflation actually increases aggregate demand, the central bank must cut the nominal interest rate by *more* than one percentage point for each percentage point drop in inflation, that is, our parameter h must be positive. Otherwise falling inflation will cause the real interest rate to rise, exacerbating the initial recession.

In the next chapter we shall study the economy's adjustment to equilibrium in more detail and show how our model of aggregate demand and supply can help to explain the business cycles observed in the data for output and inflation.

4 Summary

1. The short run aggregate supply curve (the SRAS curve) implies that a rise in output relative to trend drives up the rate of inflation, and that actual inflation varies one-to-one with expected inflation. The SRAS curve is derived by combining the expectations-augmented Phillips curve with Okun's Law according to which there is a negative linear short-run relationship between the percentage output gap and the rate of unemployment.

2. The positive slope of the SRAS curve in (y, π)-space reflects that a rise in output lowers the rate of unemployment which in turn generates a higher rate of wage and price inflation. A rise in the natural rate of unemployment or a fall in labour productivity relative to trend will cause an upward shift in the SRAS curve, and vice versa.

3. The long run aggregate supply curve (the LRAS curve) is vertical in (y, π)-space at the rate of output corresponding to the natural rate of unemployment. A rise in the natural rate of unemployment or a fall in labour productivity relative to trend will cause a leftward shift in the LRAS curve, and vice versa.

4. The aggregate demand curve (the AD curve) is derived by combining the aggregate consumption and investment functions with the goods market equilibrium condition that output aggregate saving must equal aggregate investment. The AD curve assumes that the private sector savings surplus (saving minus investment) is a decreasing function of the real rate of interest. The evidence clearly supports this assumption.

5. Because aggregate demand depends on the real rate of interest, it is crucially influenced by the interest rate policy of the central bank. Historically some central banks have followed Milton Friedman's suggested constant-money-growth rule, setting the short-term interest rate with the purpose of attaining a steady growth rate of the nominal money supply. More recently, the interest rate policy of many important central banks has tended to follow the rule suggested by John Taylor according to which the central bank should

raise the short-term real interest rate when faced with a rise in the rate of inflation or a rise in output. If the money demand function is stable, the constant-money-growth rule has similar qualitative implications for central bank interest rate policy as the Taylor rule.

6. While the central bank can control the short term interest rate, aggregate private demand is mainly influenced by the long-term interest rate, since the bulk of private investment depends on long term finance, reflecting the long-lived character of most capital goods. Hence the ability of the central bank to influence aggregate demand depends to a large extent on its ability to influence the long term interest rate via its control over the short term interest rate.

7. The *expectations hypothesis* states that the long-term interest rate is a simple average of the current and expected future short-term interest rates. If a change in the short-term interest rate has little effect on expected future short-term rates, it will also have little effect on the long-term interest rate. The ability of the central bank to influence the long interest rate therefore depends very much on its ability to affect market expectations.

8. When expectations are *static*, the expected future short-term interest rates are equal to the current short rate. A change in the current short rate will then cause a corresponding change in the long term interest rate, and the *yield curve* showing the interest rates on bonds with different terms to maturity will be completely flat. The AD curve is derived on the simplifying assumption that expectations are static so that the central bank can control long term interest rates via its control over the short rate.

9. Because of its empirical relevance, our theory of the agregate demand curve also assumes that monetary policy follows the Taylor rule.which implies that the central bank raises the real interest rate when the rate of inflation goes up. A higher rate of inflation will therefore be accompanied by a fall in aggregate demand, so the AD curve will be downward-sloping in (y, π)-space. The AD curve will shift down if the central bank lowers its target rate of inflation or if the economy is hit by a negative demand shock, due to a

tightening of fiscal policy or a fall in private sector confidence.

10. A short-run macroeconomic equilibrium is achieved in the point of intersection of the SRAS curve and the AD curve. If the resulting level of output is below the natural level (which will equal the trend level in the absence of supply shocks), actual inflation will be lower than expected inflation. Over time agents will then reduce the expected rate of inflation, causing a gradual downward shift in the SRAS curve. In this way the economy will move down along the AD curve until it reaches a long run equilibrium where the SRAS curve and the AD curve intersect in a point on the LRAS curve. The mechanism ensuring convergence to this long run equilibrium is that falling inflation induces the central bank to lower the real interest rate, thereby stimulating aggregate demand until total demand equals the natural rate of output.

5 Exercises

Exercise 1. Optimal monetary policy

This exercise asks you to show how a monetary policy rule may be derived from the objective function of monetary policy makers. We assume that monetary policy makers wish to stabilize output and to avoid inflation (implying an inflation target of zero). We may formalize this assumption by postulating that the central bank wishes to minimize the 'social loss function'

$$SL = \frac{1}{2}(y - \overline{y})^2 + \frac{\lambda}{2}\pi^2, \qquad \lambda > 0. \tag{1}$$

The idea underlying (1) is that society's welfare loss from inflation and output instability increases more than proportionately (in fact, quadratically) with the output gap $y - \overline{y}$ and with the inflation rate. In other words, small fluctuations in output and low rates of inflation do not hurt very much, but large output fluctuations and high inflation cause considerable social damage. The parameter λ in (1) reflects the weight which the central bank attaches to price stability relative to output stability. A highly inflation averse central bank will have a high value of λ, and vice versa.

The economy's short-run output-inflation trade-off is described by the short-run aggregate supply curve

$$\pi = \gamma(y - \overline{y}) + \pi^e, \tag{2}$$

where we take expected inflation π^e to be predetermined in the short run. Ignoring shifts in private sector confidence, assuming that government spending g equals its trend level $\overline{g}$, and remembering that the real interest rate is $r = i - \pi$, we may write the economy's aggregate demand curve in the simple form

$$y - \overline{y} = -\alpha_2 \left(i - \pi - \overline{r} \right). \tag{3}$$

You are now invited to derive the central bank's optimal rule for setting the nominal interest rate i. As a starting point, suppose that $y < \overline{y}$ initially. According to the social loss function (1), a marginal increase in y will then reduce the social welfare loss by the magnitude $\overline{y} - y$ (which measures the reduction in the term $\frac{1}{2}\left(y - \overline{y}\right)^2$ generated by a marginal rise in y). This reduction of the social welfare loss represents the marginal social *benefit* of higher output. On the other hand it follows from (2) that a marginal rise in output will raise the inflation rate by the amount γ, and according to (1) this will increase the social welfare loss by $\gamma\lambda\pi > 0$, assuming a positive initial inflation rate. This is the marginal social *cost* associated with the rise in output. To minimize the net social welfare loss, the central bank should adjust the interest rate to drive output to the point where the marginal benefit is just equal to the marginal cost, that is

$$\overbrace{\overline{y} - y}^{\substack{\text{marginal} \\ \text{social} \\ \text{benefit}}} = \overbrace{\gamma\lambda\pi}^{} . \; {\substack{\text{marginal} \\ \text{social} \\ \text{cost}}} \tag{4}$$

1. Use (3) and (4) to show that an optimal monetary policy requires the nominal interest rate to be set in accordance with a policy rule of the form

$$i = \overline{r} + \pi + h \cdot \pi, \qquad h > 0. \tag{5}$$

State the expression defining the parameter h, and give an intuitive explanation for the factors determining the size of h.

Suppose now that (the log of) government spending g may deviate from its 'normal' level $\overline{g}$. In that case we know from the main text that the aggregate demand curve will be

$$y - \overline{y} = \alpha_1 \left(g - \overline{g} \right) - \alpha_2 \left(i - \pi - \overline{r} \right). \tag{6}$$

2. Find the optimal monetary policy rule when aggregate demand is given by (6). Explain how monetary policy will react to a fiscal expansion. Could monetary policy makers get into conflict with fiscal policy makers? (hint: will fiscal policy makers succeed in raising output if they try to do so?). Discuss the factors determining the size of α_1 and explain how a higher value of α_1 will influence the response of monetary policy to expansionary fiscal policy.

Exercise 2. An AS-AD model with a classical demand side

The classical (pre-Keynesian) economists tended to assume that the demand for (base) money was insensitive to the interest rate. This idea was captured in the classical *quantity equation*

$$M = k \cdot PY, \qquad k > 0, \tag{1}$$

where the term on the right-hand side is the nominal demand for money, assumed to vary in proportion to nominal income PY, and the left-hand side is the nominal money supply. Equation (1) is thus a simple version of the equilibrium condition for the money market.

The supply side of the economy is described by the SRAS curve

$$\pi = \gamma (y - \overline{y}) + \pi^e \tag{2}$$

where we have used our usual notation, and where we take expected inflation π^e to be predetermined in the short run.

1. Suppose the central bank follows the *constant money growth rule*, allowing the nominal money supply to grow at the constant rate μ. Derive the economy's aggregate demand curve (hint: take logs of (1)) and show that the position of the short-run demand curve depends on last period's output level y_{-1}).

2. Illustrate the determination of the short-run levels of output and inflation in a (y, π)-

diagram. What is the slope of the aggregate demand curve? Explain how the equilibrium interest rate is determined.

3. Derive an expression for the short-run effect on output of a marginal rise in the monetary growth rate μ. Explain the effect and illustrate it in your (y, π)-diagram. How do you expect the economy to react to the rise in μ in the longer run?

Exercise 3: Nominal GDP Targeting

In the main text of this chapter we discussed the constant money growth rule versus the Taylor Rule for the conduct of monetary policy. As a third type of guideline for monetary policy, some economists have proposed that the central bank should adopt a target growth rate for nominal GDP. Such a rule would allow real GDP to grow faster when inflation falls and would require real growth to be dampened when inflation rises. In formal terms, if the target growth rate of nominal GDP is μ, the central bank must adjust the interest rate to ensure that

$$y - y_{-1} + \pi = \mu, \tag{1}$$

where y is the log of GDP so that $y - y_{-1}$ is the growth rate of real GDP. Ignoring fluctuations in confidence and government spending, we have the simple aggregate demand curve

$$y - \overline{y} = -\alpha_2 \left(i - \pi - \overline{r} \right). \tag{2}$$

Derive the policy rule for interest rate setting under nominal GDP targeting. How does the interest rate react to inflation? How does it react to the lagged output gap $y_{-1} - \overline{y}$? Try to explain your results.

Exercise 4: Interest rate setting under a constant money growth rule

When we derived the central bank's interest rate reaction function (28) under the constant money growth rule, we assumed for simplicity that there was no secular growth in trend output. We will now relax this restrictive assumption.

1. Derive an equation showing how the central bank should set the short term interest rate if it wishes to maintain a constant growth rate μ of the monetary base, assuming that trend output grows at the constant rate x (hint: use the suggestions made in footnote 8). Compare your interest rate reaction function to the Taylor rule. Are there any important differences?

2. Discuss the arguments for and against relying on a constant money growth rule rather than a Taylor rule for monetary policy.

Exercise 5. Topics in monetary policy

1. Explain and discuss the arguments underlying the constant money growth rule for monetary policy. Explain the similarities and differences between the constant money growth rule and the Taylor rule. What could be the argument for choosing a Taylor rule rather than a constant money growth rule? Is it possible to determine by empirical analysis whether a central bank follows a constant money growth rule or a Taylor rule?

2. Explain the expectations hypothesis of the link between short term interest rates and long term interest rates. What is the crucial assumption underlying the expectations hypothesis? Is this assumption reasonable? Discuss the central bank's possibility of controlling long term interest rates via its control of the short term interest rate.

3. Suppose that a positive aggregate demand shock has driven actual output y above its trend level $\overline{y}_o$. Use an AS-AD diagram to illustrate how the economy adjusts back towards a long-run equilibrium. Explain the role of monetary policy in the adjustment process. Use the graphical analysis to explain how the value of the policy parameter h affects the speed with which the economy returns to long run equilibrium.

A Appendix: Deriving the Log-Linear Aggregate Demand Curve

This appendix takes you through the steps leading to our preliminary aggregate demand curve (19). We start out from an initial situation in which the economy is on its long-run growth trend so that initial output is equal to $\overline{Y}_o$. We then consider a small deviation from trend. Taking a first-order linear approximation of the goods market equilibrium condition (17), remembering that $D_G = D_T = -C_Y$, and denoting the initial trend values by bars, we get

$$Y - \overline{Y}_o = D_Y \left(Y - \overline{Y}_o \right) - C_Y \left(G - \overline{G} \right) + D_r \left(r - \overline{r} \right) + D_\varepsilon \left(\varepsilon - \overline{\varepsilon} \right) + G - \overline{G} \Longleftrightarrow$$

$$Y - \overline{Y}_o = \left(\frac{1 - C_Y}{1 - D_Y} \right) \left(G - \overline{G} \right) + \left(\frac{D_r}{1 - D_Y} \right) \left(r - \overline{r} \right) + \left(\frac{D_\varepsilon}{1 - D_Y} \right) \left(\varepsilon - \overline{\varepsilon} \right). \qquad \text{(A.1)}$$

Our next step is to rewrite (A.1) in terms of relative changes in Y, G and ε:

$$\frac{Y - \overline{Y}_o}{\overline{Y}_o} = \left(\frac{\overline{G}}{\overline{Y}_o} \right) \left(\frac{1 - C_Y}{1 - D_Y} \right) \left(\frac{G - \overline{G}}{\overline{G}} \right)$$

$$+ \left(\frac{D_r}{\overline{Y}_o \left(1 - D_Y \right)} \right) \left(r - \overline{r} \right) + \left(\frac{\overline{\varepsilon} D_\varepsilon}{\overline{Y}_o \left(1 - D_Y \right)} \right) \left(\frac{\varepsilon - \overline{\varepsilon}}{\overline{\varepsilon}} \right). \qquad \text{(A2)}$$

In the final step we use the fact that the change in the log of some variable is equal to the relative change in that variable. We may then write (A.2) in the form (19), that is

$$y - \overline{y}_o = \alpha_1 \left(g - \overline{g} \right) - \alpha_2 \left(r - \overline{r} \right) + v, \qquad \text{(A.3)}$$

where

$$\alpha_1 \equiv \left(\frac{\overline{G}}{\overline{Y}_o} \right) \left(\frac{1 - C_Y}{1 - D_Y} \right), \qquad \alpha_2 \equiv - \left(\frac{D_r}{\overline{Y}_o \left(1 - D_Y \right)} \right), \qquad \text{(A.4)}$$

$$v \equiv \left(\frac{\overline{\varepsilon} D_\varepsilon}{\overline{Y}_o \left(1 - D_Y \right)} \right) \left(\ln \varepsilon - \ln \overline{\varepsilon} \right), \qquad g \equiv \ln G, \qquad \overline{g} \equiv \ln \overline{G}. \qquad \text{(A.5)}$$