Chapter 20
Explaining Business Cycles: Aggregate Supply and Aggregate Demand in Action

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The previous chapter showed how our model of aggregate supply and aggregate demand determines the levels of total output and inflation in the short run and in the long run. In this chapter we will use the AS-AD model to investigate the causes of the fluctuations in economic activity which we observe in the real world. We will illustrate how business fluctuations may be seen as the economy's reaction to various shocks which tend to shift the aggregate supply and demand curves. We will also study the extent to which our AS-AD model is able to reproduce the most important stylized facts of the business cycle.

The perspective on business cycles adopted here is sometimes referred to as the *Frisch-Slutsky paradigm*, named after the Norwegian economist and Nobel Prize winner Ragnar Frisch and the Italian statistician Eugen Slutzky who first introduced this way of interpreting business cycles[1]. The Frisch-Slutzky paradigm distinguishes between the *impulse* which *initiates* a movement in economic activity, and the *propagation mechanism* which subsequently transmits the shock through the economic system *over time*. In our AS-AD

[1] See Ragnar Frisch, 'Propagation Problems and Impulse Problems in Dynamic Economics', in *Economic Essays in Honour of Gustav Cassel*, London, Allen and Unwin, 1933; and Eugen Slutzky, 'The Summation of Random Causes as the Source of Cyclic Processes', *Econometrica*, vol. 5, April 1937, pp. 105-146.

framework, the impulse is a sudden exogenous change in one of the 'shock' variables determining the position of the aggregate supply and demand curves. The propagation mechanism is the endogenous economic mechanism which converts the impulse into *persistent* business fluctuations. The propagation mechanism reflects the structure of the economy and determines the manner in which it reacts to shocks and how long it takes for it to adjust to a shock. Ragnar Frisch stressed that even though shocks to the economy may follow an unsystematic pattern, the structure of the economy may imply that it reacts to disturbances in a systematic way which is very different from the pattern of the shocks themselves. Frisch was inspired by the famous Swedish economist Knut Wicksell who used the following metaphor to explain the difference between the unsystematic impulse to the economy and the systematic business cycle response implied by the propagation mechanism: "If you hit a wooden rocking chair with a club, the movement of the chair will be more or less regular because of its form, even if the hits are quite irregular"[2].

In this chapter we will raise three basic questions: 1) What are the most important shocks causing economic activity to fluctuate over time? 2) Why do movements in economic activity display persistence, and 3) Why do these movements tend to follow a cyclical pattern?

We start out in Part I by briefly restating the various potential sources of shocks to aggregate supply and demand. In Part II we then use the AS-AD model to illustrate how the economy reacts to such shocks in a so-called deterministic world. In this *deterministic* version of our AS-AD model, the demand and supply shocks are non-random, occuring either within a limited time span, or representing a permanent level shift in some exogenous variable. Following a qualitative graphical analysis, we will set up a quantitative version of the deterministic AS-AD model to study the *impulse-response functions* which show how

[2]This statement by Wicksell was made in a discussion at a meeting of the Swedish Economic Association in Uppsala in 1924. See 'Nationalekonomiska Föreningens Förhandlingar 1924', Uppsala 1925.

the economy responds to various shocks over time. As we shall see, the deterministic AS-AD model is capable of explaining the observed *persistence* of the movements in economic activity following a shock, but it cannot really explain why business fluctuations tend to follow a *cyclical* pattern. To deal with this problem, Part III sets up a *stochastic* version of the AS-AD model in which the exogenous demand and supply shock variables are *random variables*. As we shall see, this model turns out to be able to reproduce the most important stylized business cycle facts reasonably well.

1 Sources of shocks to aggregate supply and demand

In Chapter 19 we have already identified a number of factors which may cause the aggregate supply and demand curves to shift up and down, thereby initiating movements in business activity. On the supply side, any structural change which increases the natural unemployment rate will shift the SRAS curve to the left, thus representing a negative supply shock. As you recall from Chapters 18 and 19, the natural unemployment rate will increase in case of a rise in the profit margin of firms, a rise in the real wage aspirations of workers, or a rise in unemployment benefits. An unusually low rate of productivity growth - a so-called negative productivity shock - will also shift the SRAS curve to the left.

If a supply shock is only temporary, it will not affect the position of the long run aggregate supply (LRAS) curve. By contrast, if the negative supply shock is permanent, it will shift the LRAS curve to the left. Hence the distinction between temporary and permanent supply shocks is important, especially for the analysis of the long-run effects.

Note that several types of supply shocks may be modeled as productivity shocks. For example, a loss of output due to industrial conflict may be interpreted as a temporary fall in labour productivity. An unusually bad harvest due to bad weather conditions may likewise be seen as a temporary drop in productivity. An exogenous increase in the real price of imported raw materials such as oil will also work very much like a negative productivity shock. If the price of oil increases relative to the general price level, an economy dependent on imported oil will have to reserve a greater fraction of domestic output for exports to maintain a given volume of oil imports. Thus, for given inputs of domestic labour and capital, a lower amount of domestic output will be available for domestic consumption, just as if factor productivity had declined. More generally, any exogenous change in the economy's international terms of trade (a shift in import prices relative to export prices) may be modeled as a productivity shock in our AS-AD model.

Over the last three decades, the real price of energy inputs has fluctuated considerably, as illustrated in Figure 20.1. For example, following political turmoil in the Middle East, the OPEC cartel of oil-exporting countries was able to raise the real price of oil quite dramatically in 1973-74 and again in 1979-80. Because most OECD economies were large net importers of oil at the time, these oil price shocks worked like a significant negative productivity shock for the OECD area. On the other hand, the collapse of oil prices from around 1985 tended to boost real incomes in the OECD, just like a positive productivity shock.

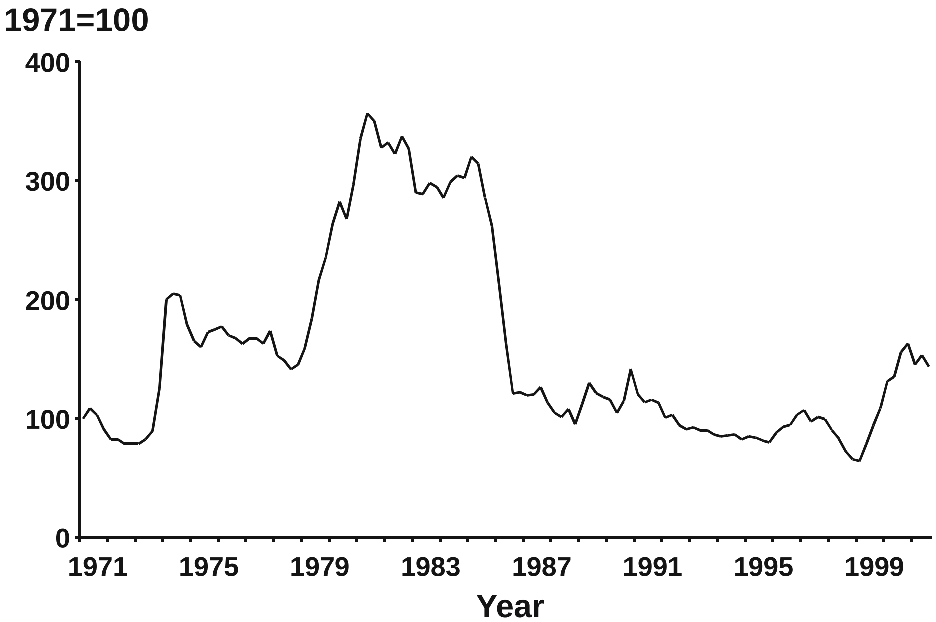

Figure 20.1: The real price of fuel imports in Denmark, 1971-2001

Source: MONA database

Turning to the economy's demand side, we remember from Chapter 19 that shifts in government spending or shifts in the state of private sector confidence (shifts in expected future income growth) will cause shifts in the aggregate demand curve. While demand shifts due to changes in private sector confidence are usually temporary in nature, the AD curve may also shift permanently, say, in case of a permanent change in government spending, or if there is a structural shift in the private propensity to consume due to a lasting change in consumer preferences. Finally, the aggregate demand curve will shift if the central bank adopts a new target for the inflation rate.

Figure 20.2 shows the evolution of the cyclical components of real GDP and real government demand for goods and services in Denmark in recent decades. We see that output has often been above trend when government spending was above trend, and vice versa. Statistically, the coefficient of correlation between the cyclical components of GDP and government absorption was 0.384 over the period considered. This suggests that exogenous shifts in aggregate demand resulting from shifts in public spending may have been a driver behind some of the output fluctuations observed in Denmark.

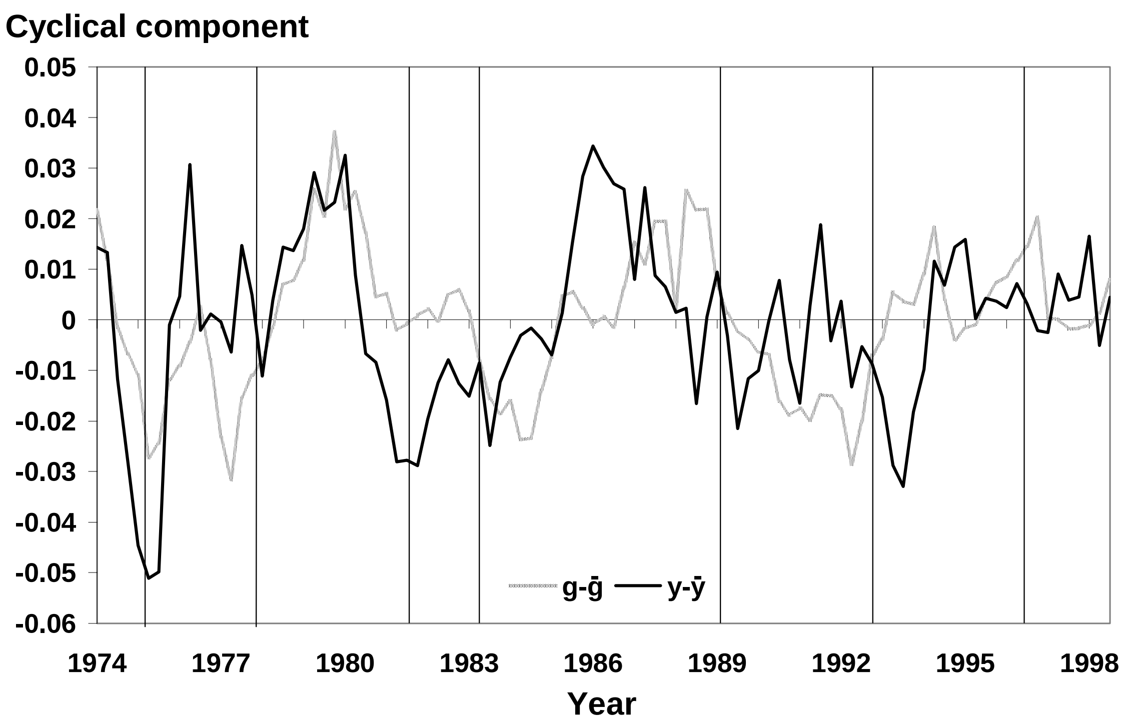

Figure 20.2: The cyclical components of real GDP and real government demand for goods and services in Denmark, 1974-98

Source: MONA database

In practice the disturbances to aggregate demand and supply may sometimes be related. For example, a positive productivity shock stemming from a wave of innovation may boost private sector expectations of future real income growth, thereby causing a positive shock to demand as well as supply. In statistical terms, we may thus observe a positive correlation between supply and demand shocks. However, for analytical purposes it is useful to study the two types of shocks separately, as we do below.

2 Business fluctuations in a deterministic world

How does the economy react on impact and over time to shocks to aggregate demand and aggregate supply? We will now use our AS-AD model to discuss this question in qualitative and quantitative terms. As a starting point, it will be useful to restate our model of aggregate supply and aggregate demand in the following form, where the subscript t indicates that we consider period t:

$$y_t - \overline{y}_o = \widetilde{v}_t - \alpha_2 \left(r_t - \overline{r}_o \right), \qquad \widetilde{v}_t \equiv v_t + \alpha_1 \left(g_t - \overline{g}_o \right) \tag{1}$$

$$r_t = \overline{r} + h \left(\pi_t - \pi^* \right) + b \left(y_t - \overline{y}_o \right) \tag{2}$$

$$\pi_t = \pi_{t-1} + \gamma \left(y_t - \overline{y}_o \right) - \gamma s_t \tag{3}$$

Equation (1) is the aggregate demand curve, restated from equation (19) in Chapter 19. The left-hand side measures the relative deviation of output from its initial trend level $\overline{y}_o$. We thus assume that the economy starts out in period zero at the 'normal' trend level of output. As you recall from Chapter 19, the variable v_t reflects shocks to private sector demand. Since $g_t - \overline{g}_o$ is the percentage deviation of public spending from trend, the shock variable $\widetilde{v}_t$ in (1) thus captures demand shocks originating from the public as well the private sector.

The variable $\overline{r}_o$ in (1) is the real interest rate prevailing in the *initial* long run equilibrium (in period 0), whereas the variable $\overline{r}$ in the monetary policy rule (2) is the central bank's estimate of the *current* equilibrium real interest rate (in period t). Equation (2) follows directly from the Taylor rule (29) in Chapter 19 by inserting $r_t = i_t - \pi_t$[3]. As long as

[3] As noted in Chapter 19, the ex ante expected real interest rate is really given by $r_t = i_t - \pi^e_{t+1}$, but since we are assuming static inflation expectations ($\pi^e_{t+1} = \pi_t$), we get $r_t = i_t - \pi_t$.

the economy has not been hit by a *permanent* shock, the current equilibrium real interest rate will remain equal to $\overline{r}_o$. However, when a permanent shock occurs, the equilibrium real interest rate will change, as we shall see below. When the central bank recognizes the permanency of the shock, it will then revise its estimate of the equilibrium real interest rate, and after that time the variable $\overline{r}$ in (2) will deviate from the initial equilibrium real interest rate $\overline{r}_o$ in (1).

Equation (3) is just a restatement of the aggregate supply curve derived in equation (13) in the previous chapter. As you remember, this equation for the SRAS curve assumes static inflation expectations, so the expected inflation rate for the current period equals last period's actual inflation rate π_{t-1}. The exogenous variable s takes a positive (negative) value in case of a positive (negative) supply shock.

In the initial period 0 we assume $\widetilde{v} = s = 0$ and $\pi_t = \pi_{t-1}$. It then follows from (1) through (3) that $y_t = \overline{y}_o$, $r_t = \overline{r}_o$ and $\pi_t = \pi^*$ in period 0. This means that the economy starts out from an initial long run equilibrium point on its trend growth path. Note that the inflation rate corresponds to the central bank's inflation target π^* in the initial equilibrium.

A temporary negative demand shock

Now suppose that, after having been in long-run equilibrium in period 0, the economy is hit by a *temporary* negative demand shock in period 1, say, because private agents temporarily become more pessimistic about the economy's growth potential. Suppose further that the central bank correctly expects this drop in private sector confidence to be short-lived, having no impact on the long run equilibrium real interest rate. In equation (2) we may then set $\overline{r} = \overline{r}_o$ and insert the resulting expression into equation (1) to obtain the following equation for the aggregate demand curve:

$$y_t - \overline{y}_o = \widetilde{v}_t - \alpha_2 h (\pi_t - \pi^*) - \alpha_2 b (y_t - \overline{y}_o) \qquad \Longleftrightarrow$$

$$\pi_t = \pi^* + \frac{\widetilde{v}_t}{\alpha_2 h} - \left(\frac{1 + \alpha_2 b}{\alpha_2 h}\right)(y_t - \overline{y}_o) \tag{4}$$

Using this equation, Figure 20.3 illustrates how the economy will react to the temporary weakening of private sector confidence. Between period 0 and period 1 the shock variable $\widetilde{v}_t$ changes from zero to some negative number. According to (4) the AD curve therefore shifts down by the distance $|\widetilde{v}_t/\alpha_2 h|$, from AD_o to AD_1 in Figure 20.3. This drives the economy from the initial long-run equilibrium $\overline{E}_o$ to the new short-run equilibrium E_1 where output as well as inflation are lower.

However, in period 2 private sector confidence is restored, pushing the aggregate demand curve back to its original position AD_o as the variable $\widetilde{v}_t$ in (4) returns to its original value of zero. One might think that this would immediately pull the economy back to its initial equilibrium $\overline{E}_o$. Yet this is not what happens, since the observed fall in inflation during period 1 causes a fall in expected inflation from $\overline{\pi}^*$ to the lower level π_1 as the economy moves from period 1 to period 2. Hence the short-run aggregate supply curve shifts down to SRAS_2 in period 2, generating a new short-run equilibrium at point E_2. Remarkably, we see that output in period 2 *overshoots* its long-run equilibrium value $\overline{y}_o$. Real GDP will only gradually return to its normal trend level as the above-normal level of activity gradually drives up actual and expected inflation. As expected inflation goes up, the SRAS curve will gradually shift back towards its original position SRAS_o, and the economy will move back along the AD curve to the initial long-run equilibrium $\overline{E}_o$. The interesting point is that the initial recession generated by the temporary demand shock is followed by an extended economic boom. This shows how the economy's propagation mechanism may generate a pattern of adjustment which is rather different from the time pattern of the driving shock itself.

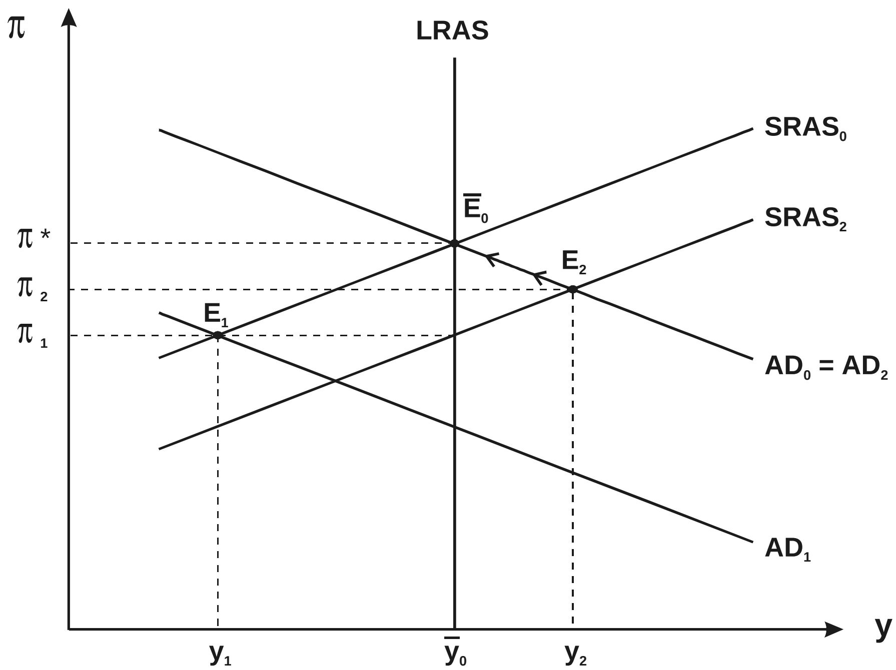

Figure 20.3: Effects of a negative demand shock

Note from equation (4) that the more aggressively the central bank cuts the interest rate in response to a fall in inflation (the higher the value of h), and the stronger the response of aggregate demand to this fall in the interest rate (the larger the value of α_2), the smaller will be the downward shift $|\widetilde{v}_t/\alpha_2 h|$ of the AD curve in period 1, so the smaller the fluctuations in output and inflation will be. Hence a strong policy reaction from the central bank can help to keep the initial recession mild.

A permanent negative demand shock

Suppose alternatively that the negative demand shock hitting the economy in period 1 is *permanent*. As an example, we may think of a lasting fall in private sector growth expectations in reaction to a period with very optimistic expectations of the economy's long-term growth potential. Such a permanent demand shock will affect the equilibrium real interest rate. To see this, recall from Chapter 19 that the equilibrium real interest rate is the level of interest ensuring that the goods market clears at the natural rate of output. Hence we may find the new equilibrium real interest rate $\overline{r}$ by setting actual output y_t equal to the natural rate of output $\overline{y}_o$ in (1) and solving for $r_t = \overline{r}$ to get

$$\overline{r} = \overline{r}_o + \frac{\widetilde{v}}{\alpha_2} \tag{5}$$

In this equation we have dropped the time subscript to $\widetilde{v}$ since the shock is now assumed to be the same for all $t \geq 1$. Because the permanent demand shock is negative ($\widetilde{v} < 0$), we see from (5) that the equilibrium real interest rate will fall. Intuitively, if the private sector's propensity to consume or invest goes down (or if the government reduces its spending), it takes a lower real interest rate to maintain a total level of demand equal to the natural rate of output.

While it is clear that a permanent negative demand shock must reduce the equilibrium real interest rate, it is not clear how long it will take the central bank to discover the permanent character of the shock. As a benchmark, suppose that it takes just one period for the central bank to find out that the shock is permanent. In period 1 the AD curve will then shift down by the distance $|\widetilde{v}/\alpha_2 h|$, from AD_o to AD_1, and the new short run equilibrium in period 1 will be given by the point E_1 in Figure 20.3, just as before. In period 2 the central bank realizes that the demand shock is permanent. Until that time the bank estimated that $\overline{r} = \overline{r}_o$, but now it revises its estimate of the equilibrium real interest rate to the new level given by (5), recognizing that it will have to pursue a less restrictive interest rate policy to prevent the inflation rate from falling permanently below the target level. From period 2 and onwards the central bank therefore follows a modified monetary policy rule which may be found by substituting (5) into (2), yielding

$$r_t = \overline{r}_o + \frac{\widetilde{v}}{\alpha_2} + h(\pi_t - \pi^*) + b(y_t - \overline{y}_o) \quad (6)$$

Inserting this into the goods market equilibrium condition (1) and rearranging, we get the equation for the AD-curve from period 2 and onwards:

$$\pi_t = \pi^* - \left(\frac{1 + \alpha_2 b}{\alpha_2 h}\right)(y_t - \overline{y}_o), \qquad t = 2, 3, ... \quad (7)$$

Now compare (7) to (4) and recall that, in the case of a *temporary* demand shock, we had $\widetilde{v}_t = 0$ from period 2 and onwards. For all $t \geq 2$ the position of the AD curve will thus be *exactly the same* under the permanent and under the temporary demand shock. By an appropriate downward adjustment of the interest rate, the central bank simply neutralizes the impact of the permanent demand shock from the time it recognizes the permanency of the shock. Hence the AD curve shifts back from AD_1 to AD_o from period 2 and onwards, and the economy starts adjusting from the period 2 equilibrium E_2 back towards the initial long run equilibrium $\overline{E}_o$ in Figure 20.3, exactly as it did in the scenario with a temporary demand shock. By modifying its interest rate policy in accordance with equation (6), the central bank ensures that the inflation rate returns to the target level π^* in the long run, despite the permanent drop in the private sector's spending propensity.

Since the effects in period 1 were also the same, we seem to have the striking result that *there is no difference between the effects of temporary and permanent demand shocks.* However, this holds only under the strong assumption that the central bank is able to identify a permanent shock already after one period. In practice, it will typically take several periods for the central bank to find out whether a shock is permanent or not (and even then it can never be quite sure). Thus the central bank will normally maintain its previous estimate of the equilibrium real interest rate $\overline{r} = \overline{r}_o$ for quite a while after the economy has been hit by a permanent demand shock.

In Figure 20.4 below this means that, after period 1, the economy will start to move down along the new aggregate demand curve AD_1 from point E_1, as the SRAS curve gradually shifts down due to the fall in expected inflation caused by the fall in the actual inflation rate. At some point such as E_n in Figure 20.4, the observation of steadily falling inflation may convince the central bank that a permanent negative demand shock has occurred. When this happens, the AD curve will shift upwards to its original position AD_o as the central bank revises its estimate of the equilibrium real interest rate. But since this takes place at a time $t > 2$ when the position of the SRAS curve is below the supply curve $SRAS_2$ for period 2, the new short run equilibrium E_{n+1} illustrated in Figure 20.4 implies that output is driven further above its long run equilibrium value than would be the case if the central bank had realized the permanency of the shock already in period 2.

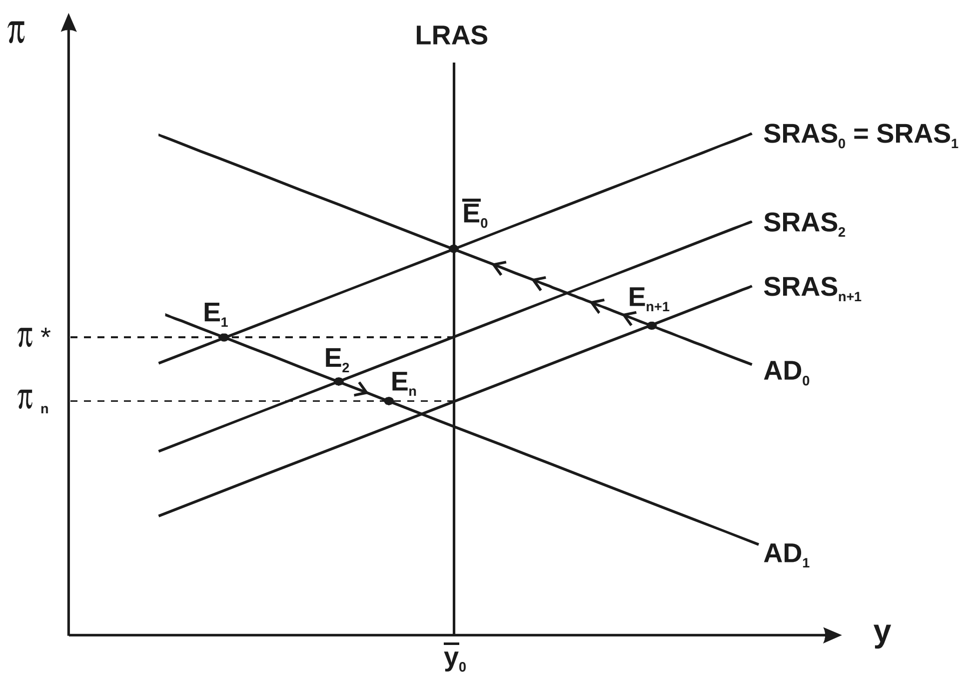

Figure 20.4: A permanent negative demand shock which is recognized in period n ($n > 2$)

In other words, the longer it takes the central bank to uncover the character of the shock, the greater is the danger that output and inflation will fluctuate considerably around their long run equilibrium levels. If the central bank is smart, it will probably recognize this danger and modify the haste with which it changes its interest rate policy rule, once it has discovered that a permanent shock has occurred. Indeed, there is strong empirical evidence that central banks do in fact "smooth" changes in interest rates, adjusting their interest rates only gradually towards their target levels, as we explained in the previous chapter. In practice such interest rate smoothing may help to ensure that the overshooting of output relative to its long run equilibrium level is reduced compared to the scenario illustrated in Figure 20.4.

A temporary negative supply shock

Let us now study the economy's reaction to supply shocks. Figure 20.5 shows the effects of a *temporary* negative supply shock such as an industrial conflict or a temporary rise in the real price of oil. Because of the temporary nature of the shock, the long-run aggregate supply curve is not affected, but the short-run aggregate supply curve moves up from SRAS_o to SRAS_1 during period 1 as the economy is hit by the shock. In formal terms, our supply shock variable s drops from zero to some negative value which is numerically equal to s_1, and according to equation (3) this causes the SRAS curve to shift upwards by the amount γs_1. The result is a period of *stagflation* characterized by a rise in inflation combined with a fall in output. In period 2, the source of the shock disappears, but in the meantime expected inflation has risen due to the rise in actual inflation in period 1. As a consequence of the rise in expected inflation, the SRAS curve does not shift down by the full amount γs_1 in period 2, even though productivity is now back at its normal level[4]. Hence output only moves part of the way back towards the original level $\overline{y}_o$. However, since the disappearance of the supply shock in period 2 reduces inflation to the lower level

[4]To be specific, the downward shift in the SRAS curve from period 1 to period 2 (the vertical distance between SRAS_1 and SRAS_2) is only equal to γs_1 *minus* the rise in expected inflation $\pi_1 - \overline{\pi}^*$.

π_2, expected inflation falls from π_1 to π_2 as we move from period 2 to period 3, causing a further downward shift in the SRAS curve in the latter period, and so on. The continued downward revision of the expected inflation rate enables the economy to move gradually back to the original long-run equilibrium $\overline{E}_o$.

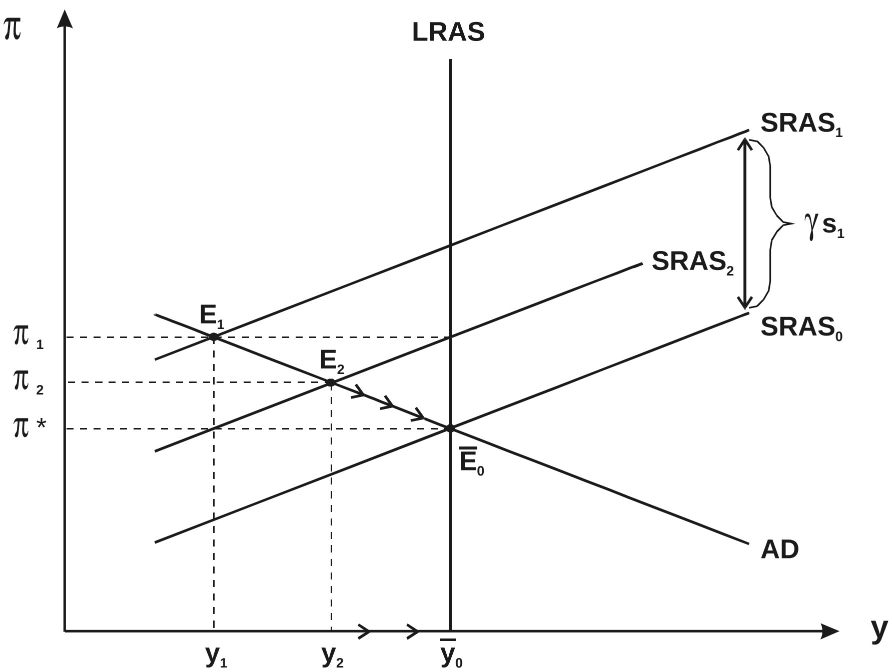

Figure 20.5: Effects of a temporary negative supply shock

A permanent positive supply shock

In contrast to a temporary supply shock, a *permanent* supply shock will have a lasting effect on output. To see this, insert the long-run equilibrium condition $\pi_t = \pi_{t-1}$ into the supply curve (2) and solve for the long-run equilibrium value of output $\overline{y}_t$ to get

$$\overline{y}_t = \overline{y}_o + s \tag{8}$$

where $s > 0$ is the magnitude of the permanent *positive* supply shock assumed to occur in period 1. Because it changes the natural rate of output $\overline{y}_t$, a permanent supply shock will also affect the equilibrium real interest rate. When the central bank recognizes the permanent character of the shock, it will therefore revise its estimate of $\overline{r}$ in the monetary policy rule (2). We may use equation (1) to derive the central bank's new estimate of the equilibrium real interest rate by setting $y_t = \overline{y}_t = \overline{y}_o + s$ and $\widetilde{v}_t = 0$ (since we are now focusing on supply shocks), and solving for $r_t = \overline{r}$ to get

$$\overline{r} = \overline{r}_o - \frac{s}{\alpha_2} \tag{9}$$

Thus the equilibrium real interest rate will fall when a permanent positive supply shock occurs. This is intuitive: when natural output goes up, it takes a lower real interest rate to ensure a level of aggregate demand equal to the natural rate of output.

To prepare the ground for our graphical analysis, let us now derive the equations for the AD curve in the various periods. Like before, let us start by assuming that it takes only one period for the central bank to learn that the shock is permanent. In period 0 the shock has not yet occurred, so $\overline{r} = \overline{r}_o$. In period 1 when the shock hits, the central bank does not yet know that the disturbance is permanent, so the bank maintains its estimate that $\overline{r} = \overline{r}_o$. In both of these periods the monetary policy rule is thus given by the equation $r_t = \overline{r}_o + h(\pi_t - \pi^*) + b(y_t - \overline{y}_o)$. Inserting this into (1), we obtain the AD curve for periods 0 and 1 (remembering that $\widetilde{v}_t = 0$):

$$\pi_t = \pi^* - \left(\frac{1 + \alpha_2 b}{\alpha_2 h}\right)(y_t - \overline{y}_o), \qquad t = 0, 1 \tag{10}$$

From period 2 and onwards, the central bank revises its estimate of $\overline{r}$ in accordance with (9), so the monetary policy rule modifies to $r_t = \overline{r}_o - \frac{s}{\alpha_2} + h(\pi_t - \pi^*) + b(y_t - \overline{y}_o)$. Substituting this into (1), we get the AD curve from period 2 and onwards:

$$\pi_t = \pi^* - \left(\frac{1+\alpha_2 b}{\alpha_2 h}\right)[y_t - (\overline{y}_o + s)], \qquad t = 2, 3, ... \tag{11}$$

We are now ready to illustrate the effects of the permanent positive supply shock. In period 0 the economy is in the long run equilibrium $\overline{E}_o$ in Figure 20.6. When the permanent supply shock s hits in period 1, the long run aggregate supply curve (LRAS) and the short run aggregate supply curve (SRAS) both shift to the right by the horizontal distance s, as shown in Figure 20.6. However, since the central bank has not yet realized that the shock is permanent, it still sticks to its original estimate of the equilibrium real interest rate, so the AD curve remains in the position AD_o during period 1. Hence a new short run equilibrium is established in point E_1 in Figure 20.6. In period 2 the central bank reduces its estimate of $\overline{r}$, and according to (11) this causes an upward shift of the AD curve from AD_o to AD_1 in Figure 20.6. As the figure illustrates, and as you can see from (11), the magnitude of the shift in the AD curve ensures that inflation will return to its target rate π^* when the economy reaches the new and higher natural rate of output. One might think that the economy will then reach the new long run equilibrium $\overline{E}_1$ already in period 2. Yet this is not the case, since the fall in inflation in period 1 will reduce the expected inflation rate for period 2 to π_1, causing the SRAS curve to shift down to the level SRAS_2 in period 2. Hence the economy temporarily settles in the new short run equilibrium E_2 where output is driven *above* the new and higher natural rate. But since the movement from E_1 to E_2 also involves a rise in the rate of inflation, the expected inflation rate will start to rise again from period 3 and onwards. This will induce successive upward shifts in the SRAS curve, pushing the economy up along the AD_1 curve towards the new long run equilibrium $\overline{E}_1$, as indicated in Figure 20.6.

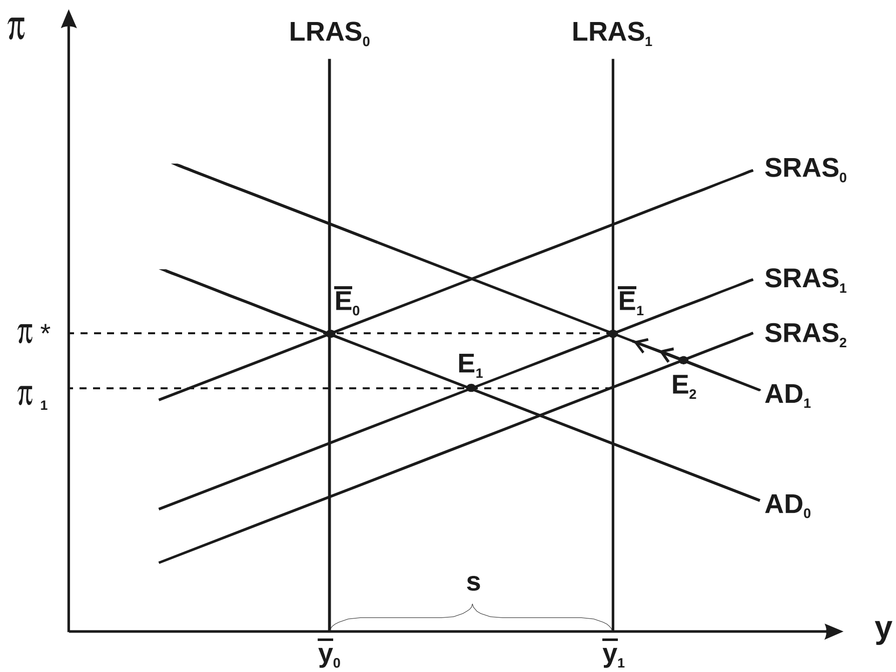

Figure 20.6: A positive permanent supply shock which is recognized after one period

Once again we thus find that a shock - in this case a permanent supply shock - may cause the economy to *overshoot* its long run equilibrium during the adjustment process, due to the economic mechanisms propagating the shock. In our model it is the delayed adjustment of inflation expectations combined with the delay in the central bank's ability to correctly diagnose the shock which are responsible for the overshooting. If the central bank had immediately revised its estimate of $\overline{r}$, the AD curve would have shifted to the position AD_1 already in period 1, and the economy would have jumped immediately to the new long run equilibrium. And even if the shift in the AD curve does not take place until period 2, the economy could still have moved directly from E_1 to $\overline{E}_1$ without overshooting if the delayed adjustment of inflation expectations had not shifted the SRAS curve down to SRAS_2.

In practice the overshooting of output may be even larger than illustrated in Figure 20.6, since it will typically take several periods before the central bank feels able to conclude that the supply shock is truly permanent. In the meantime the economy will move further down the original aggregate demand curve AD_o, beyond point E_1,. When the AD curve finally shifts up as a result of the central bank's revision of its estimate of $\overline{r}$, the new short run equilibrium will therefore lie further down the AD_1 curve than the point E_2 in Figure 20.6, implying more overshooting of output, unless the central bank decides to smooth the change in the interest rate.

It is worth emphazing the role of monetary policy in the economy's adjustment to a positive supply shock: as the boost to productivity gradually forces down the rate of inflation, the central bank gradually reduces the real rate of interest, thereby allowing aggregate demand to increase in line with potential output. According to many observers, something like this happened in the United States in the second half of the 1990s where accelerating productivity growth due to improvements in information technology was followed up by a supportive monetary policy which allowed the U.S. economy to utilize its potential for higher non-inflationary growth. It is also worth noting that, just as a positive supply shock was probably an important factor behind the record long U.S. economic expansion of the 1990s, a negative supply shock may have contributed to the economic recession in 2001. As illustrated in Figure 20.1, the real price of energy rose substantially right before the turn of the century, and as we have seen, such a negative shock will generate a fall in economic activity (although there were also important demand factors behind the 2001 recession, including the collapse in stock prices after March 2000).

Calibrating the model: how long is the long run?

So far our analysis has been purely qualitative, but it is also of interest to study a *quantitative* version of our AS-AD model. For example, by assigning plausible values to the parameters of the model, we can investigate how fast the economy is likely to move from its short-run to its long-run equilibrium after having been hit by a shock, and how strongly output and inflation are likely to react to various shocks.

To identify the parameters which determine the economy's speed of adjustment, we will solve the model for the endogenous variables in terms of the exogenous variables. We start by considering the case of *permanent shocks which have already been recognized by the central bank*. From our previous analysis we then know that the AD curve will be given by equation (11) which does not include permanent *demand* shocks, since such shocks do not affect the AD curve once the central bank has adjusted its estimate of the equilibrium real interest rate. Let $\widehat{y}_t \equiv y_t - \overline{y}_o$ denote the relative deviation of output from trend, and let $\widehat{\pi}_t \equiv \pi_t - \pi^*$ indicate the deviation of inflation from the target inflation rate. We may then restate our AS-AD model with permanent shocks in the following form which will turn out to be convenient:

$$\text{AD curve:} \qquad \widehat{\pi}_{t+1} = \alpha^{-1}\left(s - \widehat{y}_{t+1}\right), \qquad \alpha \equiv \frac{\alpha_2 h}{1+\alpha_2 b} \tag{12}$$

$$\text{SRAS curve:} \qquad \widehat{\pi}_{t+1} = \widehat{\pi}_t + \gamma\left(\widehat{y}_{t+1} - s\right) \tag{13}$$

Equation (12) is just a restatement of (11), where the parameter α has already been introduced in Chapter 19, and (13) is a simple restatement of (3). From (12) we have $\widehat{\pi}_t = \alpha^{-1}\left(s - \widehat{y}_t\right)$, which may be inserted into (13) along with (12) to give

$$\widehat{y}_{t+1} = \beta\widehat{y}_t + \alpha\gamma\beta s, \qquad \beta \equiv \frac{1}{1+\alpha\gamma} \tag{14}$$

It also follows from (12) that $\widehat{y}_{t+1} - s = -\alpha\widehat{\pi}_{t+1}$, which may be substituted into (13) to yield

$$\widehat{\pi}_{t+1} = \beta\widehat{\pi}_t \tag{15}$$

The linear first-order difference equations in (14) and (15) have the solutions

$$\widehat{y}_t = s + (\widehat{y}_o - s)\,\beta^t, \qquad t = 0, 1, 2, ... \tag{16}$$

$$\widehat{\pi}_t = \widehat{\pi}_o \beta^t, \qquad t = 0, 1, 2, ... \tag{17}$$

where $\widehat{y}_o$ and $\widehat{\pi}_o$ are the initial values of $\widehat{y}$ and $\widehat{\pi}$, respectively[5]. According to the definition given in (14), $\beta \equiv 1/(1+\alpha\gamma)$ is less than one, so the terms involving β on the right-hand sides of (16) and (17) will tend to zero as time t tends to infinity. This is the formal proof that the economy is *stable* in the sense that it tends towards its long-run equilibrium. Literally speaking, it will take infinitely long for the economy to reach the long-run equilibrium, but we may ask how long it will take before, say, half the adjustment to equilibrium has been completed. Let t_h denote the number of time periods which must elapse before half of the initial gap $\widehat{y}_o - s$ between actual output and long-run equilibrium output has been closed. According to (16), the value of t_h may be found from the equation

$$\widehat{y}_t - s = (\widehat{y}_o - s)\,\beta^{t_h} \equiv \frac{1}{2}(\widehat{y}_o - s) \Longleftrightarrow$$

$$\beta^{t_h} = \frac{1}{2} \Longleftrightarrow t_h \ln\beta = \ln(1/2) \Longleftrightarrow$$

$$t_h = -\frac{\ln 2}{\ln\beta} = -\frac{0.693}{\ln\beta}, \qquad \beta \equiv \frac{1}{1+\alpha\gamma} \tag{18}$$

[5]To see that (16) is indeed the solution to (14), recall from (14) that $\beta = 1/(1+\alpha\gamma)$ and note that (16) implies

$$\widehat{y}_{t+1} = s + (\widehat{y}_o - s)\,\beta^{t+1} = s + \beta(\widehat{y}_o - s)\,\beta^t = s + \beta(\widehat{y}_t - s)$$

$$= \beta\widehat{y}_t + (1-\beta)\,s = \beta\widehat{y}_t + \left(\frac{1+\alpha\gamma-1}{1+\alpha\gamma}\right)s = \beta\widehat{y}_t + \alpha\gamma\beta s$$

We see that the last expression corresponds to the right-hand side of (14). In a similar way you may verify that (17) represents the solution to (15).

Hence the economy's speed of adjustment is uniquely determined by the value of the parameter β which in turn depends on the values of γ and α. If one time period corresponds to one quarter of a year, a value of γ around 0.05 is usually considered to be realistic[6]. According to equation (A.4) in the appendix to Chapter 19, the parameter α_2 entering the expression for α in (12) can be written as

$$\alpha_2 \equiv \frac{-D_r}{\overline{Y}_o\,(1-D_Y)} = \left(\frac{1-\tau}{1-D_Y}\right)\eta, \qquad \eta \equiv \frac{-D_r}{\overline{Y}_o\,(1-\tau)} \tag{19}$$

where D_r is the marginal effect of a rise in the real interest rate on private goods demand, D_Y is the marginal private propensity to spend income on consumption and investment goods, and τ is the net tax rate (taxes net of transfers) levied on the private sector. The parameter η indicates the effect of a one percentage point rise in the real interest rate on the private sector's savings surplus (savings minus investment), measured relative to private disposable income. For Denmark, this parameter has been estimated to be roughly 3.6[7], while plausible values for τ and D_Y would be $\tau = 0.2$ and $D_Y = 0.8$, implying $\alpha_2 = (0.8/0.2) \times 3.6 = 14.4$. If we use this value of α_2 and set the monetary policy parameters h and b appearing in (12) equal to 0.5 as proposed by John Taylor[8], we get $\alpha = 0.878$, implying a value of β equal to 0.958. Inserting this into (18), we obtain $t_h \approx 16$. In other words, for reasonable parameter values our model implies that it will take roughly 16 quarters=4 years for the economy to complete half of the adjustment to its new long-run equilibrium after it has been hit by a shock. In a similar way one can show that it will take a little less than 13 years before the economy has completed 90 percent of the total adjustment to long-run equilibrium. Thus our model implies that it will take quite a

[6] This frequently used estimate for the U.S. economy is reported on p. 82 of Robert G. King, 'The New IS-LM Model: Language, Logic, and Limits', *Federal Reserve Bank of Richmond Economic Quarterly,* vol. 86/3, Summer 2000, pp. 45-103.

[7] See equation (4) on p. 85 in Erik Haller Pedersen, 'Udvikling i og Måling af Realrenten', Danmarks Nationalbank, Kvartalsoversigt, 3. kvartal, 40. å rgang, nr. 3, 2001, pp. 69-88.

[8] See the reference in footnote 11 of Chapter 19. In that article Taylor argued that $b = h = 0.5$ was a reasonably good description of actual U.S. monetary policy since the early 1980s.

long time for the output gap to be closed if the economy is exposed to a permanent shock. This is just another way of saying that there is considerable *persistence* in the deviations of output from trend. The reason for this persistence it that actual and expected inflation adjust only slowly over time, so in the short and medium run output and employment have to bear a large part of the burden of adjusting to a shock.

Equations (12) and (13) only allowed for permanent shocks which have already been recognized by the central bank. In the case of temporary shocks, or in a situation with permanent shocks which have not yet been discovered by the central bank, our previous analysis has shown that the AD curve will be given by equation (4) which may be rewritten as

$$\widehat{\pi}_t = \alpha^{-1}\left(z_t - \widehat{y}_t\right), \qquad z_t \equiv \frac{\widetilde{v}_t}{1+\alpha_2 b} \tag{20}$$

Since (20) implies $\widehat{y}_t = z_t - \alpha\widehat{\pi}_t$ and output is measured in logs, our modified demand shock variable z expresses the demand shock in percent of initial GDP, just as our supply shock variable s measures the shock in percent of GDP. By definition, temporary shocks vary over time, so we must now incorporate a time subscript to the shock variable in the SRAS curve:

$$\widehat{\pi}_{t+1} = \widehat{\pi}_t + \gamma\left(\widehat{y}_{t+1} - s_{t+1}\right) \tag{21}$$

From (20) we have $\widehat{\pi}_{t+1} = \alpha^{-1}\left(z_{t+1} - \widehat{y}_{t+1}\right)$. Inserting this plus (20) into (21), we find

$$\widehat{y}_{t+1} = \beta\widehat{y}_t + \beta\left(z_{t+1} - z_t\right) + \alpha\gamma\beta s_{t+1} \tag{22}$$

In a similar way we may use (20) to eliminate $\widehat{y}_{t+1}$ from (21), yielding

$$\widehat{\pi}_{t+1} = \beta\widehat{\pi}_t + \gamma\beta\left(z_{t+1} - s_{t+1}\right) \tag{23}$$

In the next section we will use the difference equations (22) and (23) to simulate the quantitative macroeconomic effects of various shocks. Equations (22) and (23) are directly applicable in the case of *temporary shocks* (which may well last for several periods) *and* in the case of permanent shocks which have not yet been identified by the central bank. However, from the time period when the central bank discovers the permanent character of a *demand* shock, one must set the demand shock variable z in (22) and (23) equal to zero, since the central bank's adjustment of the interest rate will fully neutralize the demand effect of the shock from that time. Moreover, from the time when the central bank recognizes the permanency of a *supply* shock, one must set z equal to s in both equations, because the bank's adjustment of its estimate for $\overline{r}$ at that time will generate a permanent change in demand equal to the exogenous change in supply.[9]

Impulse-response functions

Using the plausible parameter values suggested in the previous section, and modelling the different shocks in the manner just described, we may now simulate equations (22) and (23) from period 1 and onwards to obtain so-called *impulse-response functions* showing how output and inflation react over time to various shocks. Such functions are illustrated in Figures 20.7 through 20.10 where we have set all the exogenous demand and supply shocks equal to 2 percent of initial equilibrium output[10]. The figures complement our earlier graphical analysis. Figure 20.7 shows the effects of a temporary negative demand shock occurring only in period 1, *or* the effects of a permanent demand shock which is discovered by the central bank already in period 2. As we demonstrated earlier, the impact on the economy will be exactly the same in those two scenarios. We see that the demand shock

[9] To see this, note from (9) that the central bank's adjustment of the real interest rate equals s/α_2. According to (1) this will change aggregate demand by the amount $\alpha_2 \cdot (s/\alpha_2) = s$.

[10] The simulation programme (programmed in Microsoft Excel) is available on the internet address *www.econ.ku.dk/pbs/diversefiler/AdaptivCh20.xls* where you can gain further insight into the model by performing your own simulations.

causes output to overshoot its long-run equilibrium value after the shock has disappeared, or after its demand effect has been neutralized by the central bank, but the degree of overshooting is modest. For comparison, Figure 20.8 illustrates the effects of a permanent negative demand shock which is not recognized by the central bank until after period 12 (that is, after 3 years, given that each period is a quarter). In that case we see that the effects on output and inflation are more considerable, just as our previous graphical analysis in Figure 20.4 predicted.

Figure 20.9 shows that the effects of a temporary 2 percent negative supply shock are quantitatively modest, although it takes a long time before the effects on output and inflation fully fade away. In Figure 20.10a we have simulated a 2 percent permanent positive supply shock which is recognized by the central bank already in period 2, that is, only one period.after the shock occurs. In this scenario we see that the overshooting in output is quite limited. By contrast, Figure 20.10b assumes that the central bank does not realize the permanent character of the shock until after period 12. Then output will overshoot its new long run level by about three quarters of a percent in period 13 when the central bank cuts the interest rate in response to the fall in the equilibrium real interest rate.

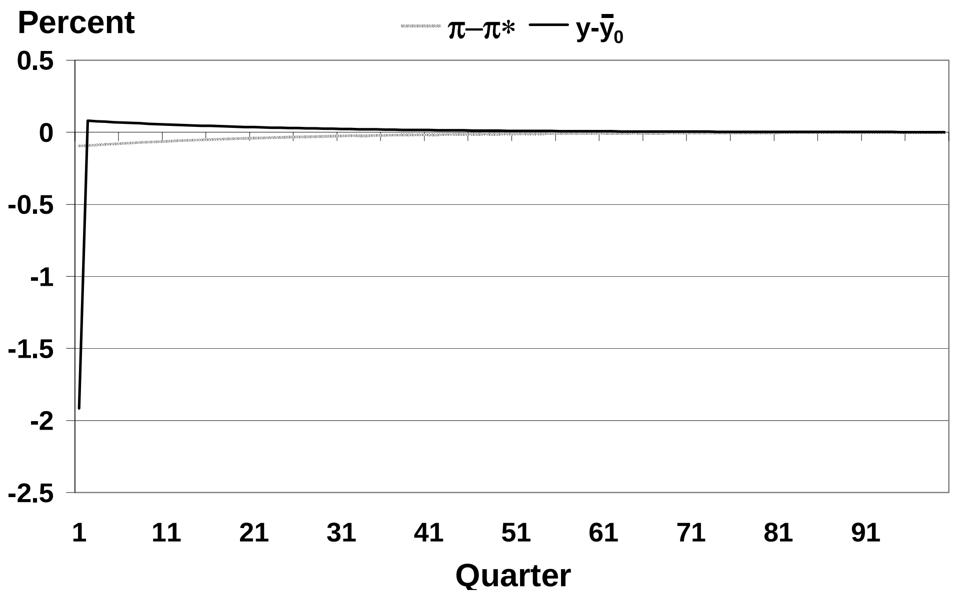

Figure 20.7: The adjustment to a temporary negative demand shock
Parameter values: $\gamma = 0.05$, $\tau = 0.2$, $D_y = 0.8$, $\eta = 3.6$, $h = b = 0.5$

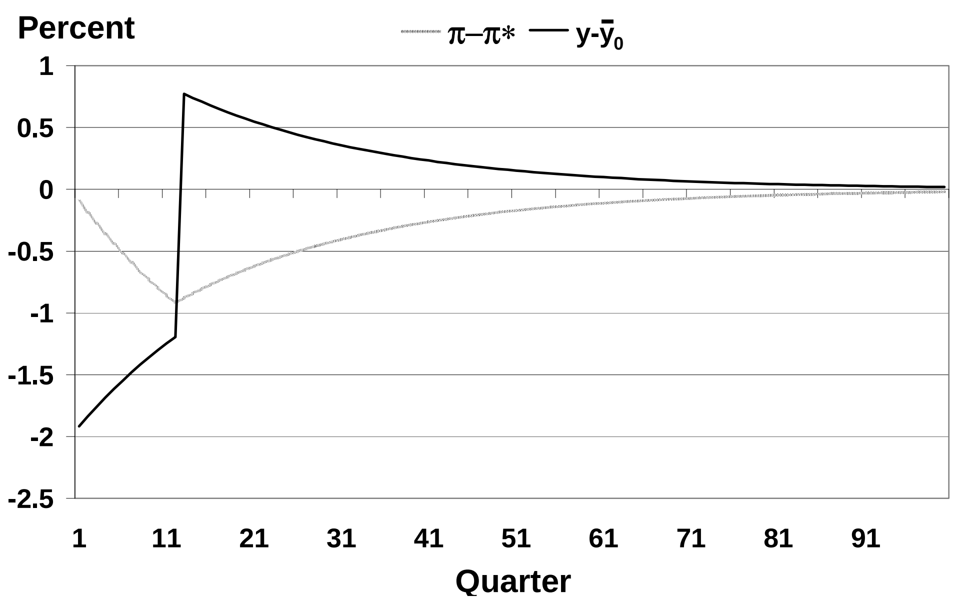

Figure 20.8: A permanent negative demand shock which is recognized after 12 periods
Parameter values: $\gamma = 0.05$, $\tau = 0.2$, $D_y = 0.8$, $\eta = 3.6$, $h = b = 0.5$

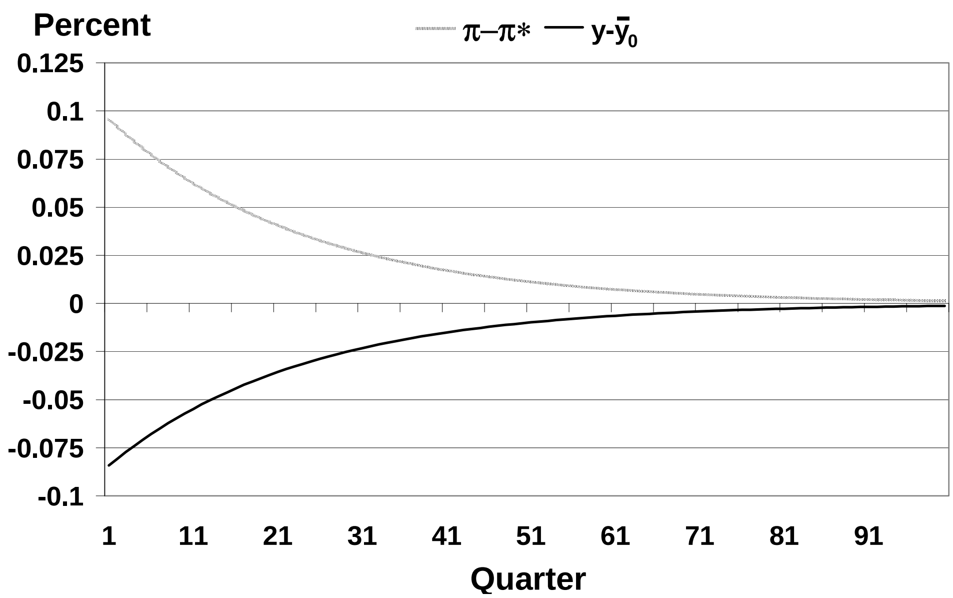

Figure 20.9: The adjustment to a temporary negative supply shock
Parameter values: $\gamma = 0.05$, $\tau = 0.2$, $D_y = 0.8$, $\eta = 3.6$, $h = b = 0.5$

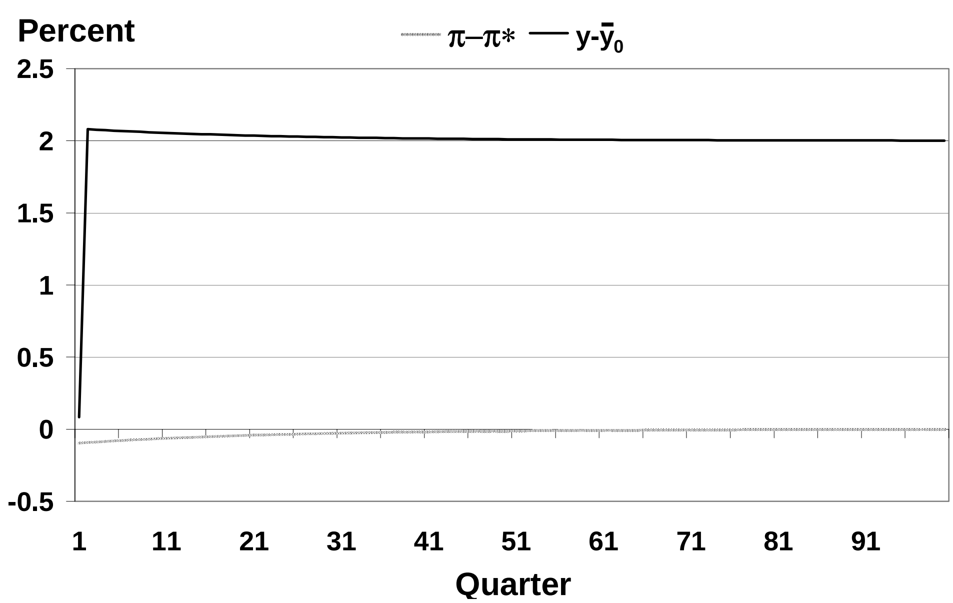

Figure 20.10a: A positive permanent supply shock which is recognized after one period
Parameter values: $\gamma = 0.05$, $\tau = 0.2$, $D_y = 0.8$, $\eta = 3.6$, $h = b = 0.5$

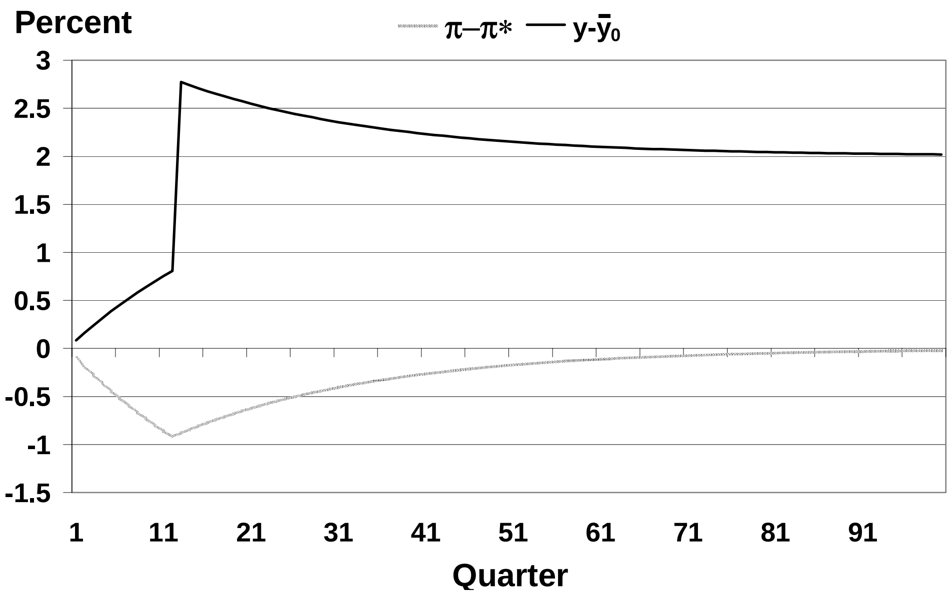

Figure 20.10b: A positive permanent supply shock which is recognized after 12 periods
Parameter values: $\gamma = 0.05$, $\tau = 0.2$, $D_y = 0.8$, $\eta = 3.6$, $h = b = 0.5$

3 Business cycles in a stochastic world

As we have seen, our deterministic AS-AD model does quite a good job in accounting for the observed *persistence* in the movement of output over time. But the deterministic model does not really explain the crucial feature of business cycles that economic booms repeatedly tend to be followed by recessions, and vice versa. A satisfactory model of the business cycle must be able to replicate the *recurrent fluctuations* in output and inflation, like those in Figure 20.11 which illustrates the evolution of the cyclical components of real GDP and domestic inflation in the United States in the most recent 100 quarters.

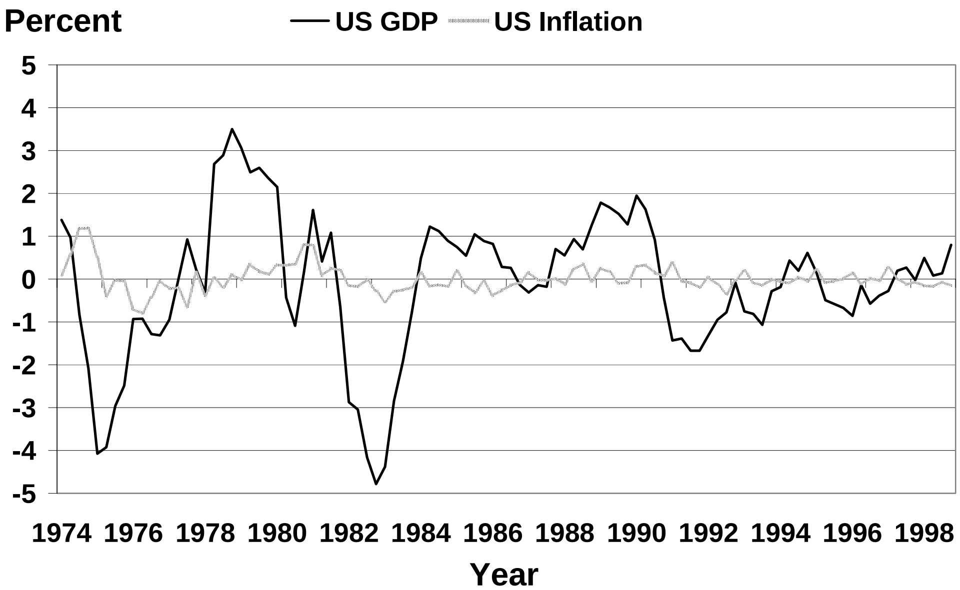

Figure 20.11: The cyclical components of real GDP and domestic inflation in the United States, 1974-98

Source: Bureau of Economic Analysis

To explain the cyclical pattern of output and inflation, we will now set up a *stochastic* version of our AS-AD model in which our demand and supply shock variables z and s are assumed to be *random variables*. In taking this step, we are building on a fundamental discovery made in 1937 by the Italian economist-statistician Eugen Slutzky (se the reference in footnote 1). Slutzky found out that if one adds a stochastic term with a zero mean and a constant variance to a first-order linear difference equation like our equation (18), and if the coefficient on the lagged endogenous variable (our β) is not too far below unity, the resulting stochastic difference equation will generate a time series which looks very muck like the irregular cyclical pattern of output displayed in Figure 20.11!

By nature, the 'shocks' to demand and supply which we have been discussing are very hard to predict. Recall that supply side shocks include phenomena such as industrial conflicts, fluctuations in agricultural output due to changing weather conditions, oil price shocks due perhaps to military conflict or political unrest in oil-producing countries, changes in productivity stemming from new technological breakthroughs, etc. On the demand side, shocks may occur due to sudden shifts in market psychology, or due to political regime shifts involving significant changes in fiscal policies, among other things. Whether events such as these occur with deterministic necessity - that is, whether they 'had' to happen, given the way things had developed - or whether they are fundamentally unpredictable, just as the outcome of the toss of a fair coin, is a deep scientific question. But as long as our understanding of the causes of such events - and hence our ability to predict them - is so limited, it seems to make sense to treat the supply and demand shocks in macroeconomic models as random variables. In doing so, we admit that we can only predict what demand and supply will be 'on average', while acknowledging that the actual levels of demand and supply may deviate from their average positions in a way we cannot anticipate. Let us therefore investigate how far a stochastic version of our AS-AD model can take us towards explaining the stylized facts of business cycles.

The stochastic AS-AD model with static expectations

The stylized business cycle facts which we would like our model to explain are summarized in the bottom row of Table 20.1. These figures are based on quarterly data for the United States from 1955 to 2001. We have chosen to focus on the relatively closed U.S. economy because we still have not extended our AS-AD model to allow for international trade in goods and capital (we will do so in Chapter 22). The data in Table 20.1 show the degree of volatility and persistence in output and inflation, measured by the standard deviations and the coefficients of autocorrelation, respectively. In addition, the third row indicates the degree to which output and inflation move together, measured by the coefficient of correlation. Ideally, simulations of our stochastic AS-AD model should be able to reproduce these statistical measures of the U.S. business cycle as closely as possible.

Table 20.1: The stochastic AS-AD model and the stylized business cycle facts

	Standard deviation (%)		Correlation between output and inflation	Autocorrelation in output				Autocorrelation in inflation			
	Output	Inflation		t-1	t-2	t-3	t-4	t-1	t-2	t-3	t-4
AS-AD model with static expectations and no supply shocks[1]	1.62	0.52	0.08	0.81	0.66	0.47	0.37	0.99	0.96	0.91	0.85
AS-AD model with static expectations and no demand shocks[2]	1.67	1.90	-1.00	0.92	0.86	0.79	0.73	0.92	0.86	0.79	0.73
AS-AD model with adaptive expectations and a combination of demand and supply shocks[3]	1.68	0.29	0.09	0.82	0.67	0.50	0.41	0.46	0.34	0.24	0.26
The U.S economy, 1955:I-2001:IV[4]	1.66	0.29	0.10	0.86	0.65	0.41	0.18	0.50	0.29	0.24	0.17

[1] $\phi = 0$, $\sigma_k = 0$, $\sigma_x = 1$, $\delta = 0.75$, $\varphi = 0$ [2] $\phi = 0$, $\sigma_k = 15$, $\sigma_x = 0$, $\delta = 0$, $\varphi = 0$ [3] $\phi = 0.9$, $\sigma_k = 5$, $\sigma_x = 1$, $\delta = 0.75$, $\varphi = 0.2$

[4] The cyclical components of output and inflation have been estimated via detrending of quarterly data using the HP-filter with $\lambda = 1600$.

Common parameter values in all AS-AD simulations: $\gamma = 0.05$, $\tau = 0.2$, $D_Y = 0.8$, $\eta = 3.6$, $h = b = 0.5$

Economists have often debated whether demand shocks or supply shocks are the most important type of disturbances driving the business cycle. One way of resolving this issue is to investigate whether a model driven by demand shocks is better at replicating the stylized business cycle facts than a model driven by supply shocks, or vice versa. In the first row of Table 20.1 we consider a version of our AS-AD model with static expectations which includes only demand shocks. Thus we have set s equal to zero in all time periods. The demand shocks are assumed to evolve according to the following first-order autoregressive stochastic process:

$$z_{t+1} = \delta z_t + x_{t+1}, \qquad 0 \leq \delta < 1, \qquad x_t \sim N\left(0, \sigma_x^2\right), \qquad x_t \ \text{ i.i.d.} \tag{24}$$

The notation $x_t \sim N\left(0, \sigma_x^2\right), \ x_t$ i.i.d. means that x_t is assumed to follow a normal distribution with a zero mean value and a constant finite variance σ_x^2, and that it is identically and independently distributed over time (i.i.d.). Hence the probability distribution of x_t is the same in all time periods, and the realized value of x_t in any period t is independent of the realized value of x_j in any other time period j[11]. A stochastic process x_t with these 'i.i.d.' properties is called 'white noise'. Note from (24) that, by allowing the parameter δ to be positive, we allow for the possibility that a demand shock occurring in a given quarter may not die out entirely within that same quarter, but may partly be felt in subsequent quarters. At the same time the restriction $\delta < 1$ implies that demand shocks do not last forever[12].

[11] Formally, $E\left[x_t x_j\right] = 0$ for all $t \neq j$, where $E\left[\cdot\right]$ is the expectations operator.

[12] According to equation (38) in Chapter 19 we have

$$z_t = \frac{v_t + \alpha_1 \left(g_t - \overline{g}\right)}{1 + \alpha_2 b}$$

where we remember that v_t captures shifts in private sector confidence. This expression shows that the variance of z_t is affected by the monetary policy parameter b. If this parameter changes, we must allow for the impact on the variance of z_t, as we explain in the next chapter. However, in the present chapter we may ignore this complication, since we keep b fixed throughout the chapter.

To arrive at the figures shown in the top row of Table 20.1, we go through the following steps: 1) Insert (24) into equations (16) and (17). 2) Set $\widehat{y}_o = \widehat{\pi}_o = z_o = 0$ for $t = 0$ and $s_t = 0$ for $\forall t$. 3) Let the computer pick a sample of 100 observations from the standardized normal distribution $N(0, 1)$. 4) Use these observations as realizations of x_t and feed these values of x into equations (16) and (17), thus simulating the AS-AD model over the interval $t = 1, 2, 3, ..., 100$. 5) Use the resulting simulated values of $\widehat{y}_t$ and $\widehat{\pi}_t$ to calculate the standard deviations, cross-correlation, and coefficients of autocorrelation for the two endogenous variables over the 100 time periods.

In the simulations we use the same parameter values as those used to generate the impulse-response functions in the deterministic AS-AD model. These parameter values are restated in the bottom note in Table 20.1. In addition, we set the value of the parameter δ in (24) equal to 0.75. This value was chosen such that the model simulation generates a standard deviation of output roughly equal to the one observed in the U.S. economy. Comparing the top and bottom rows in Table 20.1, we see that our simulation without supply shocks reproduces the observed correlation between output and inflation and the persistence (autocorrelation) in output reasonably well. However, the model simulation exaggerates the volatility (standard deviation) of inflation and particularly the degree of persistence (autocorrelation) in inflation. This impression is confirmed by a glance at Figure 20.12 which plots the simulated values of $\widehat{y}_t$ and $\widehat{\pi}_t$ in our scenario without supply shocks. Comparing this figure to the actual U.S. business cycle pictured in Figure 20.11, we see that while our model generates a reasonably realistic cyclical variability of output, it produces much too sluggish movements in the inflation rate. This suggests that a model driven solely by demand shocks cannot give a fully satisfactory account of the business cycle.

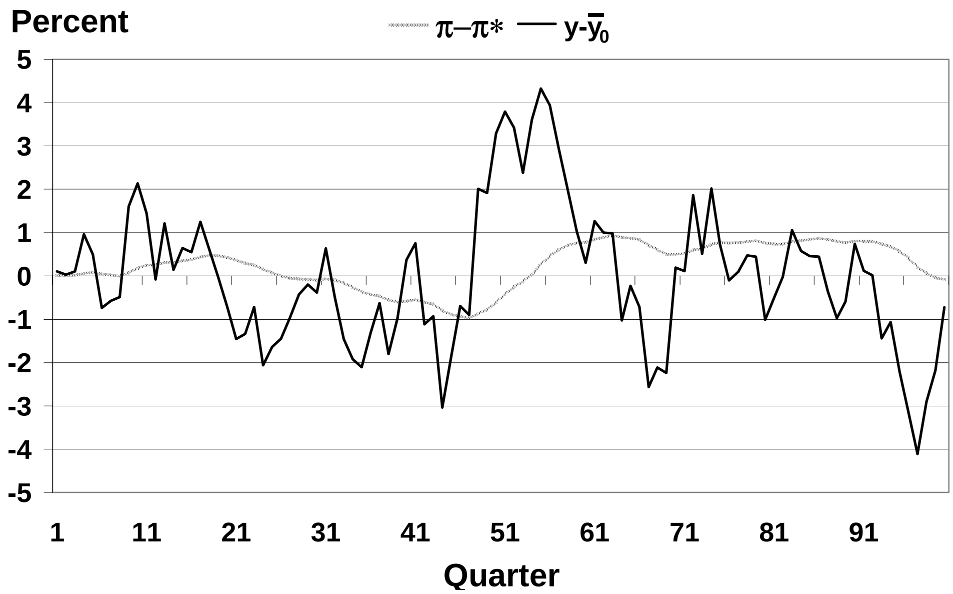

Figure 20.12: Simulation of the stochastic AS-AD model with static expectations and no supply shocks

Parameter values: See notes to table 20.1

In the second row of Table 20.1 we therefore focus on the opposite benchmark case where the stochastic disturbances occur only on the economy's supply side. By analogy to (24), we assume that supply shocks follow a stochastic process of the form

$$s_{t+1} = \varphi s_t + k_{t+1}, \qquad 0 \leq \varphi < 1, \qquad k_t \sim N\left(0, \sigma_k^2\right), \qquad k_t \text{ i.i.d.} \tag{25}$$

To derive the figures in the second row of the table, we have followed the same steps as those explained above, except that we now set the demand shock variable $z = 0$ for $\forall t$. The variance of the white noise variable k_t was chosen to ensure a simulated standard deviation of output in line with the empirical standard deviation. The 'persistence' parameter φ in (25) was set to zero, since positive values of φ only generate an even poorer fit to the data than that displayed in the second row of Table 20.1. Even so, we see that the purely supply-driven AS-AD model is inconsistent with the stylized business cycle fact in the bottom row.

The model generates far too much persistence in output and particularly in inflation, far too much volatility of inflation, and a counterfactual perfect negative correlation between output and inflation. Of course, this negative correlation is not surprising, since we have previously seen that a positive supply shock which shifts the SRAS curve downwards will drive down inflation at the same time as it raises output. By contrast, in the U.S. economy the correlation between output and inflation has been positive in recent decades, suggesting that supply shocks cannot have been a major driver of the business cycle. This conclusion stands out clearly from a comparison of Figure 20.11 to Figure 20.13 where we have plotted the results of our simulation of the AS-AD model without demand shocks. Neither output nor inflation displays a realistic pattern in Figure 20.13.

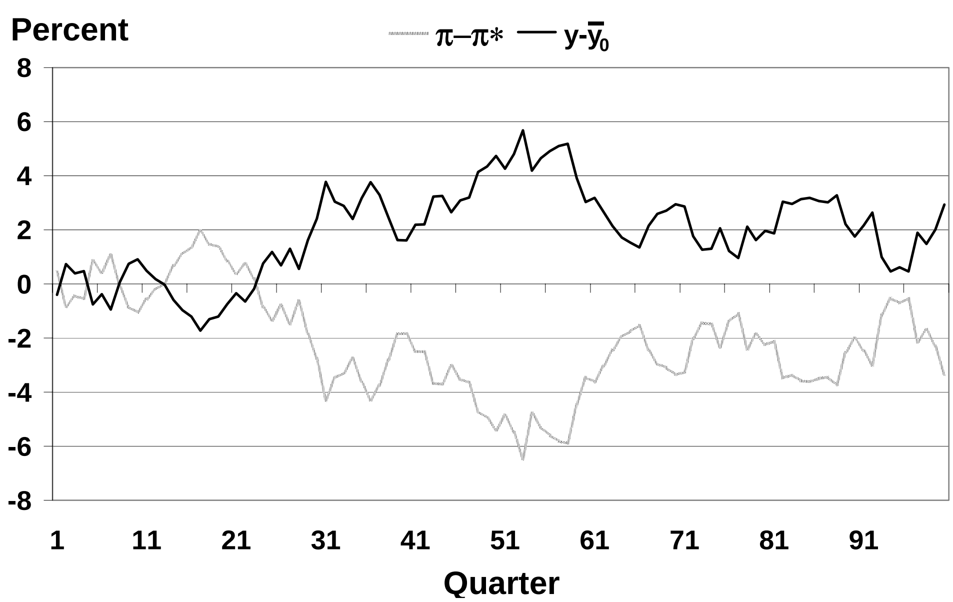

Figure 20.13: Simulation of the stochastic AS-AD model with static expectations and no demand shocks

Parameter values: See notes to table 20.1

It should be stressed that the simulation results reported in Table 20.1 are *sample specific*, relying on particular samples from the normal distribution. If we feed different samples of x_t or k_t into the model, we get somewhat different sample statistics, but the general picture remains that neither a purely demand-driven nor a purely supply-driven AS-AD model can fully account for the stylized facts of the business cycle. In the next section we will therefore consider an extended model allowing both types of shocks to occur at the same time.

The stochastic AS-AD model with adaptive expectations

Apart from allowing for simultaneous demand and supply shocks, we will also generalize our description of the formation of expectations, since this will improve the ability of our AS-AD model to reproduce the empirical business cycle. We have so far assumed that expectations of inflation are *static*, meaning that this period's expected inflation rate is simply equal to last period's observed inflation rate, $\pi_t^e = \pi_{t-1}$. A more general hypothesis is that expectations are *adaptive*, adjusting in accordance with the formula

$$\overbrace{\pi_t^e - \pi_{t-1}^e}^{\substack{\text{revision of expected} \\ \text{inflation rate}}} = (1-\phi) \overbrace{\left(\pi_{t-1} - \pi_{t-1}^e\right)}^{\substack{\text{last period's inflation} \\ \text{forecast error}}}, \qquad 0 \leq \phi < 1 \tag{26}$$

Equation (26) says that the expected inflation rate is adjusted upwards (downwards) over time if last period's actual inflation rate exceeded (fell short of) its expected level. From (26) we get

$$\pi_t^e = \phi \pi_{t-1}^e + (1-\phi)\,\pi_{t-1} \tag{27.a}$$

$$\pi_{t-1}^e = \phi \pi_{t-2}^e + (1-\phi)\,\pi_{t-2} \tag{27.b}$$

$$\pi_{t-2}^e = \phi \pi_{t-3}^e + (1-\phi)\,\pi_{t-3} \tag{27.c}$$

•

•

•

and so on. From (27.a) we see that this period's expected inflation rate is a weighted average of last period's expected and actual inflation rates. We also see that static expectations ($\pi_t^e = \pi_{t-1}$) is that special case of adaptive expectations where the parameter ϕ is equal to zero. If we include the adaptive expectations hypothesis (26) in our AS-AD model, we can therefore easily reproduce all our previous results by simply setting $\phi = 0$. Note that ϕ is a measure of the 'stickiness' of expectations: a relatively high value of ϕ means that people tend to be conservative in their expectations formation, being reluctant to revise their expected inflation rate in response to previous inflation forecast errors. We can gain further insight into the implications of adaptive expectations if we use the expressions for π_{t-1}^e, π_{t-2}^e etc. to eliminate π_{t-1}^e from the right-hand side of (27.a). Via such a series of successive substitutions we obtain

$$\begin{aligned}
\pi_t^e &= \phi^2\pi_{t-2}^e + (1-\phi)\,\pi_{t-1} + \phi\,(1-\phi)\,\pi_{t-2} \\
&= \phi^3\pi_{t-3}^e + (1-\phi)\,\pi_{t-1} + \phi\,(1-\phi)\,\pi_{t-2} + \phi^2\,(1-\phi)\,\pi_{t-3} \\
&= \phi^4\pi_{t-4}^e + (1-\phi)\,\pi_{t-1} + \phi\,(1-\phi)\,\pi_{t-2} + \phi^2\,(1-\phi)\,\pi_{t-3} + \phi^3\,(1-\phi)\,\pi_{t-4} \\
&\quad . \\
&= \phi^n\pi_{t-n}^e + (1-\phi)\,\pi_{t-1} + \phi\,(1-\phi)\,\pi_{t-2} + \phi^2\,(1-\phi)\,\pi_{t-3} + \phi^3\,(1-\phi)\,\pi_{t-4} + .. \\
&\quad +\phi^{n-1}\,(1-\phi)\,\pi_{t-n}
\end{aligned}$$

Since $\phi < 1$, the term $\phi^n\pi_{t-n}^e$ will vanish as we let n tend to infinity. Hence we get

$$\pi_t^e = \sum_{n=1}^{\infty} \phi^{n-1} (1-\phi)\, \pi_{t-n}, \qquad 0 \leq \phi < 1 \tag{28}$$

Equation (28) shows that the expected inflation rate for the current period is a weighted average of all inflation rates observed in the past, with geometrically declining weights as we move further back into history. Thus adaptive expectations put more weight on the experience of the recent past than on the more distant past. But unlike the special case of static expectations, adaptive expectations imply that people do not base themselves only on the experience of the most recent period. The higher the value of ϕ, the longer are people's memories, that is, the greater is the impact of the more distant inflation history on current expectations.

Our AS-AD model with adaptive expectations may now be summarized as follows[13]:

$$\text{AD curve:} \qquad y_t - \overline{y}_o = \alpha (\pi^* - \pi_t) + z_t \tag{29}$$

$$\text{SRAS curve:} \qquad \pi_t = \pi_t^e + \gamma (y_t - \overline{y}_o) - \gamma s_t \tag{30}$$

$$\text{Adaptive expectations:} \qquad \pi_t^e = \phi \pi_{t-1}^e + (1-\phi)\, \pi_{t-1} \tag{31}$$

Moving the SRAS curve (30) one period back in time and rearranging, we get

[13] You may wonder if the AD curve is unaffected by the switch from static to adaptive expectations. The answer is 'yes', provided the central bank has a good estimate of the expected inflation rate. Recall that the AD curve derives from the goods market equilibrium condition

$$y_t - \overline{y}_o = \alpha_1 (g_t - \overline{g}) - \alpha_2 (r_t - \overline{r}) + v_t \tag{i}$$

Suppose that the central bank can estimate the expected inflation rate π_{t+1}^e (for example, via consumer surveys, or by comparing the market interest rates on indexed and non-indexed bonds), and that it sets the nominal interest rate according to the Taylor rule

$$i_t = \overline{r} + \pi_{t+1}^e + h (\pi_t - \pi^*) + b (y_t - \overline{y}_o) \tag{ii}$$

Inserting (ii) along with the definition $r_t \approx i_t - \pi_{t+1}^e$ into (i), you may verify that we get an AD curve of the form (29).

$$\pi_{t-1}^{e} = \pi_{t-1} - \gamma\left(y_{t-1} - \overline{y}_{o}\right) + \gamma s_{t-1} \tag{32}$$

which may be inserted into (31) to give

$$\pi_{t}^{e} = \pi_{t-1} - \phi\gamma\left(y_{t-1} - \overline{y}_{o}\right) + \phi\gamma s_{t-1} \tag{33}$$

Substituting (33) into (30) and using our previous definitions $\widehat{y}_t \equiv y_t - \overline{y}_o$ and $\widehat{\pi}_t \equiv \pi_t - \pi^*$, we may state the AS-AD model with adaptive expectations in the compact form

$$\widehat{y}_t = z_t - \alpha\widehat{\pi}_t \tag{34}$$

$$\widehat{\pi}_t = \widehat{\pi}_{t-1} + \gamma\left(\widehat{y}_t - s_t\right) - \phi\gamma\left(\widehat{y}_{t-1} - s_{t-1}\right) \tag{35}$$

where (34) is the AD curve and (35) is a restatement of the SRAS curve. As you may check, (34) and (35) imply

$$\widehat{y}_{t+1} = a\widehat{y}_t + \beta\left(z_{t+1} - z_t\right) + \alpha\gamma\beta\left(s_{t+1} - \phi s_t\right) \tag{36}$$

$$\widehat{\pi}_{t+1} = a\widehat{\pi}_t + \gamma\beta\left[z_{t+1} - s_{t+1} + \phi\left(s_t - z_t\right)\right] \tag{37}$$

$$a \equiv \frac{1+\alpha\gamma\phi}{1+\alpha\gamma} < 1, \qquad \beta \equiv \frac{1}{1+\alpha\gamma} < 1 \tag{38}$$

In the third row of Table 20.1 we show the business cycle statistics generated by a simulation of the model (36) and (37). We assume that z and s are uncorrelated and that they follow the stochastic processes (24) and (25), respectively, with $\delta = 0.75$, $\varphi = 0.2$, $\sigma_x = 1$ and $\sigma_k = 5$. To obtain realizations of the white noise variables x_t and k_t, we take the same samples from the standardized normal distribution as we used in the first and second columns of the table. The parameter ϕ has been set at 0.9, while the other parameter values are the same as those used in the AS-AD model with static expectations.

Comparing the third and fourth rows of Table 20.1, we see that the extended AS-AD model with adaptive expectations and simultaneous demand and supply shocks fits the empirical business cycle data quite well, given the parameter values we have chosen (of course, we chose the parameters to match the data as well as possible). The volatility of output and inflation and the correlation between the two variables as well as their degree of persistence seem reasonably realistic. The model-generated time series for $\widehat{y}_t$ and $\widehat{\pi}_t$ are plotted in Figure 20.14. For convenience we have also reproduced the actual U.S. business cycle for a recent time interval covering 100 quarters so the reader can compare the model-generated data to reality. Of course, since the timing of the random shocks hitting our model economy does not coincide with the timing of the historical shocks to the U.S. economy, our model cannot be expected to reproduce historical turning points of the American business cycle. But a glance at figures 20.14.a and 20.14.b suggests that the variance and persistence of output and inflation in our calibrated AS-AD model with adaptive expectations is fairly realistic.

We have tried to show that a simple stochastic AS-AD model allowing for shocks to supply as well as demand can provide a reasonably good account of the cyclical movements in output and inflation. In so doing, we have also tried to illustrate the basic methodology of modern business cycle analysis, showing how macro economists build dynamic stochastic general equilibrium models and calibrate these models to reproduce the stylized statistical facts of the business cycle.

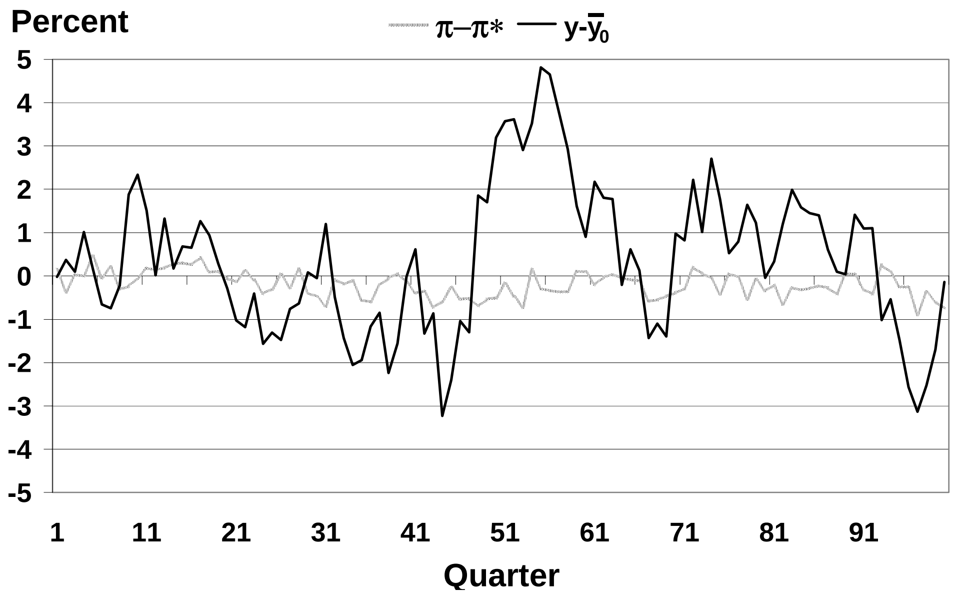

Figure 20.14a: Simulation of the stochastic AS-AD model with adaptive expectations and a combination of demand and supply shocks

Parameter values: See notes to table 20.1

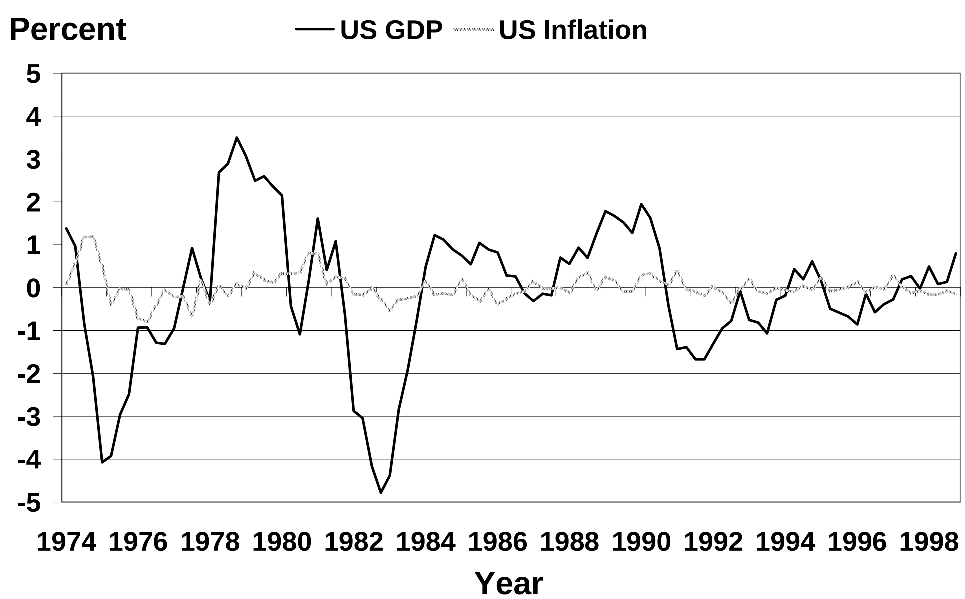

Figure 20.14b: The cyclical components of real GDP and domestic inflation in the United States, 1974-98

Source: Bureau of Economic Analysis

We must emphasize once again that the simulation reported in the third row of Table 20.1 has the character of a numerical example, serving to illustrate that our AS-AD model may be able to fit the data for an appropriate choice of parameter values. But the example was based on two particular samples from the normal distribution. To analyze the model's ability to fit the data more systematically, one should either simulate the model over a very large number of periods, or one should run a very large number of simulations based on a correspondingly large number of realizations from the probability distributions assumed for the stochastic shocks. In Table 20.2 we have taken the latter route. The upper row in the table shows the mean values of the results from 1000 simulations of our stochastic AS-AD model with adaptive expectations, where each simulation covers 100 time periods. The standard deviations (that is, the average deviation from the mean) are indicated in brackets below the mean values. The parameter values in the simulation model are exactly the same as those assumed for the corresponding model in the previous Table 20.1. For comparison, the bottom row in Table 20.2 repeats the stylized facts of the U.S. business cycle between 1955 and 2000. We see that our AS-AD model seems to underestimate the standard deviation and the autocorrelation of output a bit when the model is simulated a large number of times with the parameter values that we previously used. Nevertheless, the overall impression remains that the model fits the data reasonably well, once we consider how simple it really is compared to the staggering complexity of the real world economy.

Yet we must keep in mind that although our AS-AD model with adaptive expectations seems roughly consistent with the data on output and inflation, this does not imply that we have found *the* explanation for business cycles. It is possible to construct other types of models which match the data on output and inflation equally well. Hence we cannot claim to have found the only correct theory of the business cycle. All we can say is that our theory does not seem to be clearly rejected by the data.

Table 20.2: The stochastic AS-AD model and the stylized business cycle facts

	Standard deviation (%)		Correlation between output and inflation	Autocorrelation in output				Autocorrelation in inflation			
	Output	Inflation		t-1	t-2	t-3	t-4	t-1	t-2	t-3	t-4
AS-AD model with adaptive expectations and a combination of demand and supply shocks[1] (mean values of 1000 simulations, standard deviations in brackets)	1.41 (0.18)	0.27 (0.04)	0.09 (0.15)	0.70 (0.07)	0.49 (0.11)	0.34 (0.14)	0.23 (0.15)	0.38 (0.14)	0.24 (0.16)	0.20 (0.17)	0.18 (0.17)
The U.S economy, 1955:I-2001:IV[2]	1.66	0.29	0.10	0.86	0.65	0.41	0.18	0.50	0.29	0.24	0.17

[1] $\gamma = 0.05$, $\tau = 0.2$, $D_Y = 0.8$, $\eta = 3.6$, $h = b = 0.5$, $\phi = 0.9$, $\sigma_x = 1$, $\sigma_k = 5$, $\delta = 0.75$, $\varphi = 0.2$

[2] The cyclical components of output and inflation have been estimated via detrending of quarterly data using the HP-filter with $\lambda = 1600$.

4 Exercises

Exercise 20.1. Experiments with the deterministic AS-AD model

On the internet address *www.econ.ku.dk/pbs/diversefiler/AdaptivCh20.xls* you will find an Excel spreadsheet which enables you to simulate the deterministic as well as the stochastic AS-AD model analyzed in this chapter. You will also find a brief manual instructing you how to carry out the simulations.

In the present exercise we will focus on the deterministic version of the model, and we will assume static expectations, setting the parameter $\phi = 0$. You will be asked to perform some simulations intended to give you further insight into the properties of the model. In all simulations it is assumed that the economy starts out in long run equilibrium in period 0. Furthermore, it is assumed that the central bank considers all the shocks to be temporary and does not revise its estimate of the equilibrium real interest rate.

Question 1: Figure 20.1 illustrated how the real oil price rose sharply in 1973-74 and again in 1979-1980, followed by a sharp drop after 1985 which took the real oil price roughly back to the level of the early 1970s. Recalling that the time period of our AS-AD model is one quarter, we can simulate these supply shocks in a very stylized manner by choosing the following values of our supply shock variable s_t: $s = -3$ in periods 1 through 22; $s = -8$ in periods 23 through 44; $s = 0$ from period 45 and onwards (the simulation stops after period 100). Feed these values of s into the deterministic AS-AD model with $\phi = 0$ and set the remaining parameter values equal to the values given in the main text of the chapter. Simulate the model and give an economic explanation for the results illustrated in the graphs for the evolution of the output gap and the inflation gap.

Question 2: During the 1970s many commentators criticized the fact that governments and central banks allowed a substantial increase in the rate of inflation following the oil price shocks. These critics argued that monetary policy should have reacted more aggressively to the rising rate of inflation. To study the implications of a tougher antiinflationary monetary policy, set the parameter $h = 1.0$ and simulate the supply shocks described in Question 1. Compare the economy's reactions to the supply shocks in the two scenarios with $h = 0.5$ and $h = 1.0$ and try to explain the differences. Discuss which type of monetary policy is most desirable.

Question 3: The second half of the 1990s was a period of great optimism, stimulated by the rapid spread of the new information technologies. Expectations of future growth rates were high, so in terms of our AS-AD model we would say that our demand shock variable z_t (which includes private sector expectations of future economic growth) was positive, reflecting unusually strong confidence in future economic developments. After the turn of the century the IT stock market bubble burst, and at least for a while, private sector growth expectations seem to have become more sober. To study such a scenario where very optimistic expectations are followed by a more normal state of confidence, suppose that $z = 2$ for periods 1 through 20 and $z = 0$ for all subsequent periods. Simulate the AS-AD model with these values of z. Explain the evolution of output and inflation. In particular, you should explain why output falls below trend after period 20 even though z does not turn negative.

Question 4: Assume that z follows the pattern described in question 3, but suppose that monetary policy reacts more strongly to inflation so that $h = 1.0$ rather than 0.5. Simulate the model with this value of h and compare the simulation results to the ones you found in Question 3. Try to explain the differences. Discuss which monetary policy is more desirable.

Question 5: Suppose again that $h = 0.5$, but assume that nominal wages and prices are quite flexible, responding strongly to the output gap so that our parameter $\gamma = 0.5$ rather than 0.05. Simulate the effects of the sequence of z-values described in Question 3 and compare the simulation results in the scenarios $\gamma = 0.5$ and $\gamma = 0.05$. Give an economic explanation for the different results.

Exercise 20.2. Developing and implementing a stochastic AS-AD model

In this exercise you are asked to set up a stochastic AS-AD model, implement it on the computer, and undertake some simulations to illustrate the effects of monetary policy. In this way you will become familiar with the modern methodology for business cycle analysis which was described in Part 3 of this chapter.

Our starting point is a generalized version of the short-run aggregate supply curve. Many econometricians studying the labour market have found that wage inflation is moderated not only by the *level* of unemployment (u_t), but also by the *increase* in the unemployment rate between the previous and the current period ($u_t - u_{t-1}$). The reason is that, ceteris paribus, it is more difficult for a dismissed worker to find an alternative job when unemployment is rising than when it is falling. Hence a rising unemployment rate reduces the value of the representative worker's outside option. Assuming static inflation expectations ($\pi_t^e = \pi_{t-1}$), we therefore get the following generalized version of the expectations-augmented Phillips curve:

$$\pi_{t-1} = \pi_{t-1} + \gamma\left(\overline{u} - u_t\right) - \gamma\theta\left(u_t - u_{t-1}\right), \qquad \gamma > 0, \quad \theta > 0 \tag{1}$$

where $\overline{u}$ is the constant natural rate of unemployment, and the parameter θ indicates the degree to which the wage claims of workers are moderated by rising unemployment. As we explained in Chapter 19, the logs of actual and natural output (y_t and $\overline{y}_t$) and the log of average labour productivity ($\ln a_t$) may be specified as

$$y_t = \ln a_t + \ln N - u_t \tag{2}$$

$$\overline{y}_t = \ln a_t + \ln N - \overline{u}_t \tag{3}$$

$$\ln a_t = \ln a^* + s_t \tag{4}$$

$$\overline{y}_t = \overline{y}_o + s_t \tag{5}$$

where N is the constant labour force, a^* is the constant 'normal' level of labour productivity, s_t is a stochastic supply shock with a mean value of zero, and $\overline{y}_o$ is the trend value of output.

Question 1: Use equations (1) through (5) to demonstrate that the economy's short run aggregate supply curve may be written as

$$\pi_t = \pi_{t-1} + \gamma (1+\theta) \widehat{y}_t - \gamma\theta\widehat{y}_{t-1} - \gamma (1+\theta) s_t + \gamma\theta s_{t-1}, \qquad \widehat{y}_t \equiv y_t - \overline{y}_o \tag{6}$$

How does equation (6) deviate from the SRAS curve in the model in the main text? Explain briefly why the lagged output gap $\widehat{y}_{t-1}$ appears with a negative coefficient on the right-hand side of (6).

As usual, the economy's aggregate demand curve is given by

$$\widehat{y}_t = z_t - \alpha\widehat{\pi}_t, \qquad \widehat{\pi}_t \equiv \pi_t - \pi^* \tag{7}$$

where π^* is the central bank's inflation target.

Question 2: Show that the solutions for the output gap and the inflation gap take the form

$$\widehat{y}_{t+1} = a_1 \widehat{y}_t + \beta \left(z_{t+1} - z_t\right) + a_2 s_{t+1} - a_3 s_t \tag{8}$$

$$\widehat{\pi}_{t+1} = a_1 \widehat{\pi}_t + c_1 \left(z_{t+1} - s_{t+1}\right) + c_2 \left(s_t - z_t\right) \tag{9}$$

and derive the expressions for the coefficients a_1, a_2, a_3, c_1, c_2 and β.

The demand and supply shock variables are assumed to follow stochastic processes of the form

$$z_{t+1} = \delta z_t + x_{t+1}, \qquad 0 \leq \delta < 1, \qquad x_t \sim N\left(0, \sigma_x^2\right), \qquad x_t \ \text{ i.i.d.} \tag{10}$$

$$s_{t+1} = \varphi s_t + k_{t+1}, \qquad 0 \leq \varphi < 1, \qquad k_t \sim N\left(0, \sigma_k^2\right), \qquad k_t \ \text{ i.i.d.} \tag{11}$$

We also remember that

$$\alpha_2 = \frac{\eta\left(1-\tau\right)}{1-D_Y} \qquad\qquad \alpha = \frac{\alpha_2 h}{1+\alpha_2 b} \tag{12}$$

where the parameters are as defined and explained in the main text of this chapter.

Question 3: Implementing the model. In order to undertake simulation exercises, you are now asked to program the model consisting of equations (8), (9), (10), (11) and (12) on the computer, using Microsoft Excel. For hints how to do this, you can check how the stochastic AS-AD model described in the main text has been implemented, consulting the internet address *www.econ.ku.dk/pbs/diversefiler.xls*. From this address you can download an Excel spreadsheet with two different 100-period samples taken from the standardized normal distribution. You should choose the first sample to represent the stochastic shock variable x_t, and the second sample to represent the shock variable k_t. In your first Excel spreadsheet you should list the parameters of the model as well as the variances and covariances of output and inflation and the coefficients of autocorrelation emerging from your simulations. It will also be useful to include diagrams illustrating the simulated values of the output gap and the inflation gap. (You do not have to include the interest rate variable i_t in your model, so you need not worry about the parameters $\overline{r}$ and π^*). You should use the parameter values

$$\gamma = 0.05 \qquad \tau = 0.2 \qquad \eta = 3.6 \qquad D_Y = 0.8$$

$$h = b = 0.5 \qquad\qquad \theta = 0.5$$

To calibrate the magnitude of the demand and supply shocks x_t and k_t, you must choose their respective standard deviations σ_x and σ_k and multiply the samples taken from the standardized normal distribution by these standard deviations. As a starting point, you may simply choose

$$\sigma_x = 1 \qquad\qquad \sigma_k = 1$$

You also have to choose the value of the parameters δ and φ. For a start, just set

$$\delta = 0.5 \qquad\qquad \varphi = 0.5$$

Finally, you must choose the initial values of the endogenous variables in period 0. We assume that the economy is in a long run equilibrium in period 0 so that

$$\widehat{y}_0 = \widehat{\pi}_0 = z_0 = s_0 = 0 \qquad \text{for } t = 0$$

Now program this model in Excel and undertake a simulation over 100 periods. Compare the model-generated statistics on the standard deviations and the coefficients of correlation and autocorrelation of output and inflation with the corresponding statistics for the United States given in the bottom row of Table 20.1 in the main text. Comment on the differences.

Question 4: Experiment with alternative constellations of the parameters σ_x, σ_k, δ and φ until you find a constellation which enables your AS-AD model to reproduce the U.S. business cycle statistics for output and for the correlation between output and inflation reasonably well (you should not bother too much about the behaviour of inflation implied by the model, since we know from the text that a model with static inflation expectations is not very good at reproducing the statistical behaviour of inflation). State a set of values for σ_x, σ_k, δ and φ which in your view gives a reasonable account of the behaviour of output and of the correlation between output and inflation. What does your choice of parameter values imply for the relative importance of demand and supply shocks? Comment.

Question 5: The simulations above use the monetary policy parameter values suggested by John Taylor, that is, $h = b = 0.5$. Now suppose that the central bank decides to react more aggressively to changes in the rate of inflation by raising the value of h from 0.5 to 1.0. Simulate your model to investigate the effects of such a policy change. Try to explain the effects. Discuss whether the policy change is desirable.

Question 6: Suppose again that the central bank decides to raise h from 0.5 to 1.0, but suppose also that supply shocks are very important so that $\sigma_k = 5$. Is the policy change now desirable? Discuss.