Has the Fed Reacted Asymmetrically to Stock Prices?

Søren Hove Ravn
University of Copenhagen and Danmarks Nationalbank
January 8, 2010

Abstract

Yes. To the extent that monetary policy is assumed to react to asset prices, this reaction is usually assumed to be linear. This paper offers a new perspective. I augment the model of Rigobon and Sack (2003) to allow for asymmetric reactions to stock price changes. I then demonstrate that the Federal Reserve has been following an asymmetric monetary policy rule over the period 1998-2008. While a 5% drop in the S&P 500 index is shown to increase the probability of a 25 basis point interest rate cut by 1/3, no significant reaction to stock price increases can be identified.

Keywords: Monetary policy, asset prices, interest rate rules.

JEL classification: E44, E52, E58

---

1E-mail: shr@nationalbanken.dk. I would like to thank Henrik Jensen for competent and inspiring supervision, and Emiliano Santoro for many useful comments. Moreover, thanks to Niels Blomquist, Anders Borup Christensen, Hans Christian Kongsted, György Molnar, Virginia Queijo von Heideken, and seminar participants at Danmarks Nationalbank and the University of Copenhagen for relevant comments and suggestions. Finally, thanks to Lars Jul Overby for his help in obtaining the data. The views expressed in the paper are my own, and do not necessarily correspond with those of Danmarks Nationalbank. Any errors or shortcomings are of course my own.
1 Introduction

In the aftermath of the financial crisis, academics and central bankers alike have been trying to enhance their understanding of the link between financial markets and the macroeconomy, as well as the implications for monetary policy. In particular, the stock market is known to have large spillover effects on the macroeconomy. As a result, it has been debated (Bernanke and Gertler (1999, 2001), Cecchetti et al. (2000)) whether central banks should take stock prices explicitly into account when setting the interest rate.

The present paper offers a new perspective on the reaction of monetary policy to stock price movements. Contrary to common practice, it allows for asymmetric monetary policy reactions to stock price increases and decreases, respectively. I investigate the hypothesis that the Federal Reserve (Fed) has been reacting asymmetrically to stock prices; cutting the interest rate in response to stock market drops, but not raising the interest rate correspondingly when stock prices increase. I build on the framework of Rigobon and Sack (2003), who use identification through heteroskedasticity to show that the Fed has been reacting to daily stock price movements. Expanding their model allows me to investigate whether this reaction is symmetric, i.e. whether the reaction to stock price increases and decreases is the same. The results indicate that the Fed has indeed been pursuing an asymmetric policy over the period 1998-2008, as only the reaction to stock price drops turns out to be significant. Specifically, a 5% drop in the S&P 500 index is shown to increase the probability of a 25 basis point interest rate cut by about 1/3, while a rise in stock prices leads to no significant monetary policy reaction. To my knowledge, this paper is the first to identify different reactions to stock price increases and decreases.

Taylor (2007) describes how monetary policy in the US deviated substantially from the policy prescribed by the so-called Taylor rule over the period 2002-2006. During these years, the interest rate was kept too low for too long compared to the Taylor rule. Taylor (2009) goes on to argue that through its effect on the US housing market, and in particular its effect on the subprime segment of the housing market, the lenient monetary policy pursued by the Fed in these years actually caused the financial crisis. In this light, it is extremely important to understand what was driving monetary policy in this period in order to avoid making the same mistakes in the future. The present paper contributes to this understanding. The results do, however, not explain the deviation from the Taylor rule observed in the period 2002-2006. This period was one of substantial increases in stock prices. My results indicate that the Fed does not react to such increases. Hence, other factors than stock prices must have caused the deviation during these years. On the contrary, my results provide yet another reason that the Fed should have been raising interest rates during this period, as this would have implied a symmetric reaction to stock price movements. Pursuing a monetary policy that involves an asymmetric reaction to stock price changes is likely to lead to moral hazard problems, as investors realize that the Fed is covering part of their downside risk. In other words, the present paper lends support to the view that monetary policy was, in the words of Taylor (2009), "too easy". Instead of the housing market, I investigate the expansionary effects of monetary policy

on the stock market.

Former Chairman of the Fed Alan Greenspan has admitted (Greenspan 2007) that on at least one occasion during his reign, in March 1997, the Fed tried to counteract a perceived bubble in the stock market by raising the interest rate. While stock prices did go down initially, the stock market soon recovered and stock prices increased to an even higher level. To quote Greenspan: "In effect, investors were teaching the Fed a lesson". Greenspan goes on to argue that for the remainder of his time as Chairman, the Fed did not react to asset prices. This statement is in contrast to the alleged existence of the so-called Greenspan Put (Financial Times 2000, Miller et al. 2001), i.e. the hypothesis or perception that in case of a large fall in asset prices, the Fed will cut the interest rate significantly, flooding the market with liquidity and in effect bailing out investors. The Greenspan Put, if it ever existed, is one example of the kind of non-linearities in monetary policy that this paper is concerned with, as it only comes into effect in the event of a drop (possibly of a certain size) in stock prices. By reducing the downside risk faced by investors, the existence of such a put option is likely to lead to moral hazard in the investment decisions by investors, as demonstrated by Miller et al. (2001). More generally, as discussed in section 2, an asymmetric policy reaction to stock price drops and hikes, respectively, will distort the behaviour of investors. If the central bank reacts more heavily to the stock market when stock prices are falling, moral hazard issues arise. The present paper takes exactly this slightly more general approach, focusing not only on the Greenspan Put.

It can be argued that (large) stock price drops pose a larger threat to price stability than do similar-sized stock price booms. If the stock market collapses, the financial stability of the entire economy is threatened. For this reason, central bankers might be more concerned with avoiding stock price drops than stock price hikes when working out their crash management policy. However, the distortion of investor behaviour and the creation of moral hazard problems must be taken properly into account.

The paper proceeds as follows: Section 2 discusses the link between stock prices and monetary policy; in particular the hypothesis of an asymmetric reaction function. Section 3 covers methodological issues and describes the identification method. Results are presented in section 4, while section 5 concludes. Three appendices provide further details on mathematical derivations, bootstrap techniques, and robustness checks, respectively.

2 Monetary Policy, Asset Prices and Asymmetries

Since the seminal contribution by Taylor (1993), large parts of the academic debate about monetary policy have taken place within the framework of a rule-based approach to monetary policy; the Taylor rule. Apart from being intuitively appealing, (reasonably parametrized) Taylor rules also appear to give a relatively good description of actual monetary policy since the mid-1980’s, as shown by, among others, Clarida et al. (2000) and Poole (2007). Indeed, according to published transcripts from the meetings of the Federal Open Market Committee (FOMC) from the period
1993-2001, several FOMC members openly referred to the Taylor rule when discussing current and future directions for monetary policy, as described by Asso, Kahn and Leeson (2007).

As mentioned in the introduction, however, during the period 2002-2006 the Federal Funds rate deviated significantly from the interest rate prescribed by the Taylor rule. In other words, the movements of the Federal Funds rate in this period cannot be explained by macroeconomic conditions, specifically the output gap and the inflation outlook. This deviation has motivated researchers to investigate whether other factors play a role in the monetary policy process. More formally, various augmented Taylor rules have been suggested that incorporate a monetary policy reaction to other variables than the output gap and (expected) inflation. One of these possible augmentations of the simple Taylor rule is to include a term that allows the central bank to react to asset prices. In particular, it has often been discussed whether central banks should actively and directly react to movements in the stock market.

2.1 Asset Prices and Central Bank Policy

The discussion about the the link between asset prices and monetary policy can essentially be broken down into two parts; a normative and a positive discussion. The normative part is concerned with the question of whether central banks should react to movements in asset prices, while the positive discussion is about whether they actually do react.

As for the normative part, Bernanke and Gertler (1999, 2001) argue that central banks should not react to asset prices as such, except when asset prices are believed to carry additional information about future inflation. This conclusion is reached in a framework where stock prices can be driven by technology shocks or by pure "bubbles" which emerge and burst stochastically. Bini Smaghi (2009) has labelled this "the pre-crisis consensus view". As described by Bini Smaghi, part of the reason for this consensus is that the New-Keynesian framework, which has become dominant over the last ten years, has (at least until recently) failed to pay sufficient attention to financial markets. When financial markets play only a small role in the model economy, the potential gains from reacting to asset prices are reduced markedly.

In opposition, Cecchetti et al. (2000) reach the opposite conclusion, as they find that the optimal monetary policy rule includes a reaction to the stock market, though often a very small reaction is optimal. As Cecchetti et al. work within the same framework as Bernanke and Gertler, this finding may seem puzzling. According to Bernanke and Gertler (2001), the explanation is that the rule suggested by Cecchetti et al. is optimal only if the central bank immediately realizes that the movements in asset prices are due to a "bubble", as well as when this bubble is going to burst.

While a number of contributions have sided with each of these views, no consensus seems to have been reached. It is tempting to draw a general lesson for when stock prices should be included in the monetary policy reaction function and when not. As also stressed by Cecchetti et al., with perfect capital markets there is no scope for a monetary policy reaction to stock prices. In other words, the presence of financial market imperfections seems to be a necessary condition for a reaction to stock prices to be optimal. However, it is not a sufficient condition, as the contributions by Bernanke and Gertler illustrate.
Other researchers have focused on the positive part of the discussion. Rigobon and Sack (2003) carry out an empirical analysis of US monetary policy in the period 1985-1999, and find that a 5% drop in the S&P 500 index increases the probability of a 25 basis point interest rate cut by 57%. D’Agostino, Sala and Surico (2005) also find that the Fed does indeed react to stock prices, although in their paper, the size of the reaction depends on the concurrent volatility of the stock market. They find that the reaction of the Fed to movements in the stock market is substantially larger in periods of high volatility in the stock market than when volatility is low. Recent contributions to this literature include Castro (2008), Fuhrer and Tootell (2008) and Finocchiaro and Queijo von Heideken (2009).

One might argue that apart from the normative part (should central banks react to stock prices) and the positive part (do they react), there is a third and more practical part of the discussion concerned with the question "can they react", i.e. how an active policy of "leaning against the wind" can be carried out in practice. One issue which I shall return to is the problem of separating fundamental and non-fundamental movements in asset prices. Even if this is possible, the problem of how to "prick" an identified bubble still remains. As Bernanke (2002) argues, raising the interest rate in an attempt to prick an asset price bubble gives no guarantee that the bubble will in fact burst. At the same time, however, the monetary contraction is sure to have other, less attractive effects on the macroeconomy. Summing up, even if there was any consensus about the optimality of reacting to asset prices, the practical conduct of such a policy would be extremely difficult.

2.2 Asymmetric Monetary Policy

As noted in the introduction, monetary policy might not always be perfectly linear or symmetric. The debate about the existence of a Greenspan Put is an illustration of this. Miller et al. (2001) demonstrate how market perception about the existence of a Greenspan Put will push up stock prices (for given levels of risk aversion), as investors’ perceived downside risk is reduced considerably. A more general asymmetry in monetary policy is likely to create similar moral hazard problems. Consider a central bank that systematically reacts to stock price increases with one factor, and to stock price decreases with a different and numerically larger factor. In effect, the central bank will be covering part of the downside risk of any investment. Investors, realizing this, will then be inclined to engage in more risky investment, as their potential gain is only reduced slightly, while they will be able to share large parts of any losses with the central bank. In this way, the monetary policy conducted by the central bank will cause moral hazard problems.

If an asymmetric monetary policy is observed, at least two possible explanations exist. The asymmetry could arise from monetary policy itself, or from inherent asymmetries in the stock market. One example of the latter is that while large drops in the stock market sometimes happen very suddenly, increases usually take place over extended periods of time. As an example, during the 5-year period from October 2002 to October 2007 the S&P 500 index more than doubled. However, it took only little over a year, from October 2007 to November 2008, for the S&P 500 to fall back to its October 2002-level. And on October 19, 1987, the S&P 500 dropped by over 20% in one day, cancelling out more than 19 months of accumulated growth. If the Greenspan Put implies that the
Fed would cut interest rates in response to a sudden drop in asset prices, and a "Greenspan Call" exists, working the other way round, only the Put would have been relevant in these two cases. In other words, it might actually be that not only a Greenspan Put, but also a Greenspan Call exists; only the conditions for the Call to be exercised have never been satisfied. If this explanation holds, there is no asymmetry in the way monetary policy is conducted by the Fed, but only in the way the stock market moves.

Other sources of asymmetry could exist: While economists disagree on whether central banks should react to asset prices at all, there seems to be widespread agreement that central banks certainly not react to movements in stock prices that reflect changes in their fundamental value. The fundamental value of a share is given by the discounted value of expected future dividends. Technological progress increases firms' earnings potential, and hence the fundamental value of their shares. As firms continuously put new and better machines to use, it seems reasonable that most technology shocks are positive in nature. A company might switch from one machine to a new and better model that enhances productivity, while a switch to a poorer machine that lowers productivity is not very likely. Hence, the possibility of a drop in the fundamental stock price caused by a technological step backwards seems quite small.

Consider a central bank that has decided to react symmetrically to non-fundamental movements in stock prices. Whenever the stock market index is increasing, the central bank has to decide whether this is due to non-fundamentals, or reflects a fundamental increase based on continuous technology improvements and productivity growth. Separating fundamental and non-fundamental increases in stock prices is extremely difficult, especially when conducted real-time. On the other hand, when the central bank observes a fall in stock prices, there is a larger probability that this movement is due to non-fundamentals, as the probability of this drop being caused by technological regress (i.e., a fundamental technology-driven change) is not very large. As a consequence, while the central bank in principle follows a symmetric reaction function, its reaction to stock prices might appear asymmetric to the market, as policymakers are more likely to identify as non-fundamental (and hence react to) a stock market drop than a jump.\(^3\)

Note that this explanation does not state that fundamental stock price decreases are less frequent in general. Rather, it only considers fundamental changes driven by technology shocks. Some might even argue that this inherent bias in technology-driven movements provides a theoretical rationale for central banks to react asymmetrically to the stock market in order to correct for the bias. However, in this case the asymmetric policy would not be addressing an imperfect as such, but rather a fundamental feature of the functioning of the economy. Whether this is desirable is highly debatable.

In short, there might be good reasons that a perfectly linear monetary policy rule might come off as non-linear. On the other hand, monetary policy itself might be the source of the asymmetry. Indeed, Bini Smaghi (2009) argues that the pre-crisis consensus view about monetary policy reactions to asset prices involves an inherent asymmetry. Specifically, if the central bank does not lean against

---

\(^3\) Exceptions from this tendency are the drops in stock prices that occur after the bursting of a bubble, as these are likely to reflect movements towards the fundamental stock value. Moreover, one might argue that if the market expects continuous technological progress, a period of slower progress than expected might be sufficient to cause a (fundamental) stock price drop.
asset price bubbles, it might be forced to intervene after the bursting of such a bubble, since otherwise the stability of the financial system, and hence the entire economy, might be threatened. Bini Smaghi points out how such a policy in practice is likely to lead to "excessive accomodation" after the bursting of the bubble, as the central bank is afraid that an interest rate hike might damage the initially very fragile economic recovery, and that as a result, interest rates will often be kept too low for too long.

If monetary policy is the source of the asymmetry, the reason could also be that the central bank actually does follow a non-linear rule, where increases and decreases in stock prices are systematically met with distinct reactions. This approach is, to my knowledge, novel. As already mentioned, Miller et al. (2001) also operate with an asymmetric policy rule. But in their purely theoretical paper, they model the Greenspan Put more like an actual put, as they assume that the central bank will only intervene if stock prices fall by more than 25% with respect to their previous peak. They then simulate the model. As the purpose of the present paper is not to investigate the Greenspan Put, but asymmetric reactions more generally, I believe that my approach is more appropriate here.

Some empirical papers are closer to my setup. D’Agostino, Sala and Surico (2005) look for asymmetric monetary policy reactions to the stock market, but allow the concurrent volatility of the stock market to determine the policy reaction. Moreover, they apply a slightly different identification method. Cecchetti and Li (2005) identify an asymmetry in the Fed’s reaction to stress in the banking sector. Their findings suggest that during economic downturns, the Fed will react to stress in the banking sector by cutting interest rates, while in upturns, the reaction will be the opposite. Taylor and Davradakis (2006) model an asymmetric (piecewise linear) Taylor rule for the UK, but do not include asset prices. They find that the Bank of England only reacts to inflation whenever actual inflation is too far above the target level.

3 Methodology

Identifying econometrically the response of the central bank to changes in stock prices involves a number of difficulties. Due to endogeneity problems, it is not immediately possible to estimate a monetary policy rule with a distinct reaction to stock prices. As stock prices and interest rates are determined simultaneously, the "ceteris paribus"-interpretation of the parameters breaks down, and the results are likely to be misleading, as also illustrated by Rigobon and Sack (2003). One technique that is often used to avoid endogeneity problems is the instrumental variable (IV) method. However, it is hard to find an appropriate instrumental variable for this problem, since it is extremely difficult to think of any variable that is correlated with stock prices but uncorrelated with the interest rate.\footnote{One candidate instrument for the stock price could be the lagged value of the stock price, as this is likely to be correlated with the current value of the stock price, but uncorrelated with the current interest rate. Thanks to Henrik Jensen for pointing this out. However, in this paper I use daily changes in stock prices, in which case the correlation between two successive observations is likely to be a lot smaller.}

Other papers have used GMM to estimate the parameters in the Taylor rule, including Clarida et al. (2000).
In the present paper, however, I follow instead the identification method proposed by Rigobon and Sack (2003). This involves working with daily data.\footnote{At least, using lower-frequency data, e.g. monthly averages of stock prices and interest rates, would exclude many of the rich patterns in the comovement between these variables that is found using daily data and is used for identification.} Hence, it is not meaningful to estimate a standard monetary policy rule as proposed by, among others, Clarida et al. (2000), as these rules involve variables as output and inflation, for which no daily observations exist. Instead, the asymmetry hypothesis is incorporated into a structural vector autoregressive (SVAR) system describing the dynamics and the interaction between the interest rate and stock prices on a daily basis. This system closely resembles the setup in Rigobon and Sack, except for the asymmetric part. The system consists of the following equations:

\[
\begin{align*}
  i_t &= \begin{cases} 
    \beta_1 s_t + \lambda_1 x_t + \gamma z_t + \varepsilon_t & \text{if } s_t \geq 0 \\
    \beta_2 s_t + \lambda_2 x_t + \gamma z_t + \varepsilon_t & \text{if } s_t < 0
  \end{cases} \\
  s_t &= \alpha i_t + \phi x_t + z_t + \eta_t
\end{align*}
\] (1)

The variables are the following: \(i_t\) represents daily observations of the 3-month Treasury Bill rate. As discussed by Rigobon and Sack (2003), this rate will adjust on a daily basis to reflect expectations of future monetary policy decisions. Moreover, as the identification method relies on the use of daily data, the Federal Funds Target rate would be an inappropriate measure, as it is changed less frequently. The Federal Funds rate does change on a daily basis, but only fluctuates within a small band around the target rate. To confirm the validity of using the 3-month T-Bill rate, I calculated the correlation between the Federal Funds rate and the T-Bill rate lagged by 3 months. This gives a correlation coefficient as high as 0.97. In other words, the market does seem to forecast very precisely the short-term policy rate.

It can be discussed whether the 3-month T-Bill rate is the most appropriate interest rate to use in the current context. One might argue that given the relatively high degree of transparency in US monetary policy, changes in expected future monetary policy will not affect the 3-month T-Bill rate, since such changes are not likely to materialize within only 3 months. If monetary policy is believed to be known for the next 3 months, a longer interest rate is needed to capture changes in expected future monetary policy. Hence, the 6-month or even the 12-month T-Bill rate could be used instead. On the other hand, longer interest rates are likely to be less influenced by monetary policy. Thus, while I maintain the 3-month rate as the interest rate in my baseline scenario, the 6-month rate is used as a robustness check.\footnote{Rigobon & Sack (2003) use the 3-month T-Bill rate, but it can be argued that the transparency of US monetary policy is higher in my sample period (1998-2008) than in theirs (1985-1999). For instance, since 1994 most decisions about interest rate changes have been made at regularly scheduled FOMC meetings.}

\(s_t\) is the daily percentage change in the closing value of the S&P 500 index. \(x_t\) is a matrix capturing news about key macroeconomic indicators. More specifically, for each of the variables in \(x_t\), the daily observation is set to zero on days when no news about this variable is released.
On release dates, the value equals the surprise in the news, measured as the actual release minus the market expectation of the given release, which is collected from Bloomberg. The following six variables are included in $x_t$: Output growth (GDP), consumer price index (CPI), nonfarm payrolls (NFPAY), producer price index (PPI), retail sales (RETL) and the purchasing managers index (ISM).

Finally, the system contains three shock parameters, which are assumed to be mutually uncorrelated. $z_t$ is a common shock to both equations and can be interpreted as macroeconomic shocks not captured by the six variables in $x_t$. Thus, this shock will affect both the interest rate and the stock market. $\varepsilon_t$ captures shocks to monetary policy. Rigobon and Sack suggest that this variable could, among other things, be driven by changes in the preferences of individual FOMC members. Such changes are, however, likely to materialize only on days of FOMC meetings, which are scheduled every six weeks. Hence, when working with daily data, this interpretation is too narrow in my opinion. As an example of other factors driving $\varepsilon_t$, one could think of speeches, interviews or comments in the media by leading Fed officials that could surprise the public. The final shock parameter, $\eta_t$, measures shocks to the stock market. These shocks include shocks to the risk appetite of investors as well as "bubbles" or "fads" in the stock market.

While the assumption of mutually uncorrelated shocks seems reasonable with the interpretation of the monetary policy shock given by Rigobon and Sack (i.e., that this shock reflects changes in the preferences of FOMC members), it may be less so with the more general interpretation of the shock suggested above. One could imagine that signals sent by Fed officials in speeches or interviews affect not only the monetary policy process, but also the stock market. However, as long as $\varepsilon_t$ is uncorrelated with the shock to the stock market, $\eta_t$, the assumption is satisfied. Similarly, one might suspect that shocks to the stock market could be correlated with macroeconomic shocks $z_t$ through its influence on macroeconomic conditions. However, a shock to the stock market is unlikely to affect macroeconomic announcements on the same day. These announcements refer to the macroeconomic development in earlier periods, for instance the unemployment rate of last month. Indeed, the choice of announcement day is in itself arbitrary. The stock market shock might affect subsequent macroeconomic announcements, but this will then also be reflected in the market’s expectations of these announcements. In other words, this amounts to assuming that a given shock can only surprise the market once. I therefore follow Rigobon and Sack in assuming that the shocks are uncorrelated.

Basically, (1) is supposed to capture any daily movements in the 3-month T-Bill rate. The equation states that these movements could be driven by macroeconomic news, monetary policy shocks or stock price changes. In particular, stock price increases and decreases are allowed to have different effects on the interest rate. If the central bank reacts in an asymmetric way to the stock market, market participants will realize this and act accordingly. Thus, daily drops or jumps in stock prices will lead to asymmetric effects on the daily 3-month T-Bill rate, as this is in essence driven by expectations to monetary policy decisions over the next three months.

Note that the parameters multiplying the shocks in (1) are the same under both regimes. By assuming that the shocks hitting the interest rate are the same no matter if the stock market is rising or falling, I exclude the shocks as a possible source of asymmetry in the monetary policy reaction.
Remember that the shocks are unobserved and thus not included in the regression below. On the other hand, the parameters for the (observed) macroeconomic announcements \( x_t \) are not restricted to be the same. The reason is that it is possible to regress the daily changes in the interest rate and stock prices on \( x_t \). This is done in order to extract the part of the movements in \( i_t \) and \( s_t \) that is driven by macroeconomic news. If the restriction \( \lambda_1 = \lambda_2 \) was imposed, the residuals from this regression would still to some (probably small) extent be driven by these news. Allowing \( \lambda_1 \neq \lambda_2 \) extracts as much "macro-driven" movement as possible, leaving the residuals to contain only the movements in \( i_t \) that is driven by stock prices and unobserved shocks.

Similarly, (2) implies that daily stock price changes are driven by macroeconomic factors, interest rate movements and shocks. Rigobon and Sack show that this equation is in essence a version of Gordon’s growth formula if it is assumed that expectations of future dividends are driven by macroeconomic news, and that expectations of future interest rates are shaped by this news as well as the current interest rate. Thus, (2) is derived from the fundamental value of an asset.

3.1 Identification through heteroskedasticity

To obtain identification, Rigobon and Sack (2003) apply the method of identification through heteroskedasticity described below. As it turns out, this method is also applicable in order to address the question of this paper. This paper builds heavily on their work, but Rigobon and Sack do not allow for any asymmetries in the monetary policy rule.

To understand the method of identification through heteroskedasticity, consider Figure 1a. The upward sloping schedule illustrates the hypothesis that the Central Bank reacts to a stock price increase by raising the interest rate, giving rise to a positive relation between the two variables. The downward sloping curve, labelled Stock Market Response (SMR), captures the effect that a rise in the interest rate will cause a drop in stock prices, as future dividends are discounted more heavily. Imagine that the volatility of the stock market goes up. After this, the Monetary Policy Response (MPR) will account for a larger part of the comovement between stock prices and interest rates than before, since if stock prices are more volatile, so will be the monetary policy response to them.
Correspondingly, the SMR will now have relatively less explanatory power. Graphically, this means that the observations will now to a larger extent than before be distributed along the MPR-schedule, as illustrated in Figure 1b. Hence, the observations now trace out the slope of the MPR-curve. The slope is exactly the parameter of interest, as it measures the reaction of monetary policy to stock prices.

In other words, the identification method exploits the fact that when the variance of stock prices changes, so does the covariance between stock prices and interest rates. This is supported by empirical observations, as illustrated in Figure 2, which displays the volatility of stock prices and the correlation between stock price changes and interest rate changes. The correlation coefficient between the two is 0.60. The displayed period (1998-2008) is the period used in the analysis later on.

In economic terms, the reaction of monetary policy to the stock market accounts for a larger share of the comovement between asset prices and interest rates in periods of high volatility in the stock market. The identification method of this paper uses this insight to estimate the reaction by comparing the covariance matrix between stock market changes and interest rate changes in periods of high and low volatility. The method is developed by Rigobon (2003) and applied in order to estimate the reaction of monetary policy to stock prices in Rigobon and Sack (2003). In that paper, monetary policy is assumed to react linearly to stock prices, so that the response to a 1 % rise in stock prices is the exact opposite of the response to a 1 % fall. However, as explained below, the same method allows me to relax this assumption and investigate if there is any asymmetry in the reaction to stock market jumps and drops, respectively.

3.2 Obtaining Identification

The dataset includes daily observations from the period January 1998 to December 2008. The reason for not using a longer sample is lack of data, as I did not have access to Bloomberg data on
market expectations older than 1998. Due to the endogeneity problems pointed out above, as well as the presence of unobserved shocks, the system (1)-(2) cannot be regressed. Instead the following regression is carried out:

\[
\begin{pmatrix}
  i_t \\
  s_t
\end{pmatrix} = \Theta x_t + \begin{pmatrix}
  v^i_t \\
  v^s_t
\end{pmatrix}
\]

(3)

By inserting (1) and (2) into each other and solving for \(i_t\) and \(s_t\), it follows that the residuals \(v^i_t\) and \(v^s_t\) are given by the following system:

\[
\begin{pmatrix}
  v^i_t \\
  v^s_t
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{1-\alpha \beta_1} [(\beta_1 + \gamma) z_t + \beta_1 \eta_t + \varepsilon_t] \\
  \frac{1}{1-\alpha \beta_1} [(1 + \alpha \gamma) z_t + \eta_t + \alpha \varepsilon_t]
\end{pmatrix}
\]

if \(s_t \geq 0\)

\[
\begin{pmatrix}
  v^i_t \\
  v^s_t
\end{pmatrix} = \begin{pmatrix}
  \frac{1}{1-\alpha \beta_2} [(\beta_2 + \gamma) z_t + \beta_2 \eta_t + \varepsilon_t] \\
  \frac{1}{1-\alpha \beta_2} [(1 + \alpha \gamma) z_t + \eta_t + \alpha \varepsilon_t]
\end{pmatrix}
\]

if \(s_t < 0\)

(4)

Note that \(\lambda_1\) and \(\lambda_2\) do not appear in these expressions, as they are included in the expression for the matrix \(\Theta\) multiplying \(x_t\) in (3). Hence, the only difference arises from \(\beta_1\) or \(\beta_2\) appearing. Therefore, in the following analysis I will work only with the system with \(\beta_1\), as the analysis with \(\beta_2\) is entirely analogous.

Running the regression in (3) gives me the residuals which must satisfy (4). This is the structural part of the SVAR analysis. As stressed above, the identification method relies on changes in the variance and covariance of stock prices and interest rates. Hence, the covariance matrix of \(v^i_t\) and \(v^s_t\) is calculated. This matrix looks as follows:

\[
\Omega = \frac{1}{(1-\alpha \beta_1)^2}.
\]

\[
\begin{bmatrix}
(\beta_1 + \gamma)^2 \sigma^2_z + \beta_1^2 \sigma^2_{\eta} + \sigma^2_{\varepsilon} & (1 + \alpha \gamma) (\beta_1 + \gamma) \sigma_z^2 + \beta_1 \sigma^2_{\eta} + \alpha \sigma^2_{\varepsilon} \\
(1 + \alpha \gamma) (\beta_1 + \gamma) \sigma_z^2 + \beta_1 \sigma^2_{\eta} + \alpha \sigma^2_{\varepsilon} & (1 + \alpha \gamma)^2 \sigma_z^2 + \sigma^2_{\eta} + \alpha^2 \sigma^2_{\varepsilon}
\end{bmatrix}
\]

(5)

In calculating the covariance matrix, it is a key assumption that the shocks \(z_t\), \(\eta_t\) and \(\varepsilon_t\) are assumed to be mutually uncorrelated, as this means that the covariance terms cancel out.

As pointed out in Rigobon and Sack (2003), the covariance matrix is not enough to identify the variables, as it provides a system of three equations in six unknowns \((\alpha, \beta_1, \gamma\) and the variances of the three shocks). However, dividing the observations into four variance-covariance regimes based on their variance yields four covariance matrices. I then follow Rigobon and Sack in assuming that

---

\(^7\)When running (3), 5 lags of each of \(i_t\) and \(s_t\) are included. See section 4 for a discussion of the choice of lags.
while the variance of $z_t$ and $\eta_t$ is allowed to vary across regimes, the variance of the monetary policy shock $\varepsilon_t$ is constant over time and across regimes. This can be motivated in the following way: remember that $z_t$ and $\eta_t$ measure macroeconomic shocks and stock market shocks, respectively. It seems unlikely that the variances of these shocks remain constant as the variance of $v^i_t$ and $v^s_t$ shifts. Indeed, shifts in the variance of $v^i_t$ and $v^s_t$ are likely to be driven in large part by shifts in the variance of the stock market shock $\eta_t$ as well as the macroeconomic shock $z_t$. On the contrary, the monetary policy shock $\varepsilon_t$ reflects changes in or deviations from the process of conducting rule-based monetary policy, as argued above. These types of institutional disturbances are likely not to change over time. Hence, it is assumed that $\sigma^2_\varepsilon$ is constant across all regimes.

With this assumption, each new covariance matrix adds three equations and two unknowns ($\sigma^2_\varepsilon$ and $\sigma^2_\eta$) to the system. Thus, starting out with one covariance matrix (i.e. three equations) and six unknowns, the system will be just identified with four covariance matrices, as this gives 12 equations in 12 unknowns. However, as it turns out, the parameter of interest ($\beta_1$) can actually be identified from just three covariance matrices. In this case, while the system as such is underidentified, $\beta_1$ is just identified as the system of equations can be shown to collapse into two equations in two unknowns due to the symmetry of the equations. This is shown explicitly in Appendix 1.

Once the system is broken down into two equations in two unknowns, $\beta_1$ can be solved for. As shown in Appendix 1, $\beta_1$ will solve the following equation:

$$a\beta_1^2 - b\beta_1 + c = 0$$

where

$$a = \Delta \Omega_{41,22}\Delta \Omega_{21,12} - \Delta \Omega_{21,22}\Delta \Omega_{41,12}$$

$$b = \Delta \Omega_{41,22}\Delta \Omega_{21,11} - \Delta \Omega_{21,22}\Delta \Omega_{41,11}$$

$$c = \Delta \Omega_{41,12}\Delta \Omega_{21,11} - \Delta \Omega_{21,12}\Delta \Omega_{41,11}$$

In this system, $\Delta \Omega_{xy,zv}$ denotes the difference between element $zv$ in covariance matrices $x$ and $y$, with $x, y = \{1, 2, 3, 4\}$ and $zv = \{11, 12, 22\}$.

4 Results

The residuals are obtained by running regression (4). Rigobon and Sack (2003) include five lags in their regression, but do not give any reasons for their choice of this number of lags. To address this...
issue, I carry out an analysis of the optimal number of lags in the VAR-model. Using a likelihood
ratio test as described in Hamilton (1994) allows me to reject the hypothesis that three or four lags
are sufficient. On the other hand, the null hypothesis that five lags are enough cannot be rejected
against the alternative that six lags are needed. When the alternative is that seven lags are necessary,
the null of five lags being enough is just barely rejected at the 5\% level. Hence, without giving a
clear answer, this method lends some support to the choice of five lags.

To provide more evidence on the appropriate choice of lags, Schwarz’s Bayesian Information
Criterion (Schwarz 1978) can be calculated for the model with p lags, where p = \{1, 2, \ldots, 10\}. p
can then be chosen to minimize this information criterion. As it turns out, this criterion is in fact
minimized for p = 5. Thus, the choice of five lags seems to be supported by the data in various
ways. Changing the number of lags is then used as a robustness check.

The next step is to divide the residuals into four different covariance regimes. For \(v_{it}\) and \(v_{st}\), the
30-day rolling variance is calculated throughout the sample. I then define periods of high variance
as periods in which this rolling variance exceeds its sample average by more than one standard
deviation. This results in four regimes: When the variance of both \(v_{it}\) and \(v_{st}\) are high, when
one is high and one is low, and when both are low. The share of observations falling under each
regime is shown in Table 1, which clearly shows that the large majority of observations are in the
"low,low"-regime.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Share of obs., (s_t &lt; 0)</th>
<th>Share of obs., (s_t \geq 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1 (l,l)</td>
<td>88.3 %</td>
<td>82.6 %</td>
</tr>
<tr>
<td>Regime 2 (l,h)</td>
<td>4.3 %</td>
<td>5.3 %</td>
</tr>
<tr>
<td>Regime 3 (h,l)</td>
<td>5.1 %</td>
<td>10.0 %</td>
</tr>
<tr>
<td>Regime 4 (h,h)</td>
<td>2.3 %</td>
<td>2.1 %</td>
</tr>
</tbody>
</table>

Having separated the observations into these four regimes, the covariance matrix of each regime
is then calculated. Subtracting the elements in these from one another as illustrated in the previous
section then yields an estimate of \(\beta_1\) (resp., \(\beta_2\)). As it is not possible to calculate their standard
deviations and perform regular statistical inference, the raw estimates of \(\beta_1\) and \(\beta_2\) are difficult to
interpret as such. Instead, bootstrap methods can be applied (see Appendix 2) in order to obtain
10,000 estimates for \(\beta_1\) and \(\beta_2\). The distribution of these can then be used to draw more robust
conclusions about the parameters.

Tables 2 and 3 display the results of the estimation. The parameter estimate for \(\beta_1\) (the parameter
governing the reaction to stock price increases) is -0.0134 when calculated using regimes 1, 2 and
3. While the sign is surprising, it is important to note that this parameter is clearly insignificant,
as illustrated by the distribution of the probability mass. 16.68 \% of the probability mass falls to
the right of zero. On the other hand, \(\beta_2\) is rather precisely estimated at 0.0123. With 96.75 \% of
the probability mass to the right of zero, this parameter is significant and has the expected sign.
Interpreting these results in economic terms, it seems that the Fed has indeed reacted asymmetrically

\footnote{This definition of high variance is the same as the one proposed by Rigobon & Sack (2003).}
to stock price changes. When stock prices go up, no significant reaction from the Fed is found. On the other hand, as stock prices fall, the Fed reacts by cutting the interest rate. I also tested whether the two parameter estimates are significantly different from each other. This turns out to be the case, though only at the 10 % level.

$\beta_2$ is significant not only statistically but also economically. If stock prices drop by 5 %, the 3-month T-Bill rate drops by 6.15 basis points (in expectation of a future interest rate cut by the Fed). As demonstrated elegantly by Rigobon and Sack, it is possible to convert this into a more intuitive result. As the FOMC meets every six weeks, there is on average three weeks until the next meeting. The 3-month T-Bill rate expresses the expectations to monetary policy over the next 12 weeks, but since the Federal Funds Target rate will on average stay unchanged for the next three weeks (until the next FOMC meeting), only 3/4 of the expected change in the Federal Funds Target rate will carry through to the 3-month T-Bill rate. Thus, the reaction of the T-Bill rate (6.15 basis points) equals only 3/4 of the reaction of the Federal Funds Target rate, which then must equal 8.20 basis points. This is equivalent to a 5 % daily drop in the S&P 500 index increasing the probability of an interest rate cut of 25 basis points by 32.8 %, or roughly one third.\footnote{In other words, if the perceived probability of a 25 basis point interest rate cut was initially 25%, the probability will then increase to almost 58% after a 5 % drop in the S&P 500 index.} This result is comparable with the result of Rigobon and Sack, who find that a 5 % daily drop in stock prices increases the probability of a 25 basis point interest rate cut by about a half.

In principle, the results above also imply that if the S&P 500 index drops by 50 %, the 3-month T-Bill rate goes down by 61.5 basis points. This might seem like a very small reaction to a stock market crash of this magnitude. The problem is that with the specification of an asymmetric policy rule chosen in this paper, the monetary policy response to a 50 % drop in stock prices equals ten times the reaction to a 5 % drop. In practice, this is not very likely. As argued in the introduction, large stock price drops pose a threat to the entire financial stability of the economy. In response to stock price decreases of this magnitude, central banks are likely to cut the interest rate promptly and aggressively. In the present paper, the destabilizing effects of very large stock price drops are not taken into account. As a consequence, the results are not able to explain the monetary policy reactions to this kind of drops.\footnote{Also, one might argue that in the case of, say, a 50 % drop in stock prices, monetary policy is not reacting to the stock price drop as such, but to the financial instability caused by this drop.}

Hence, the results of this paper should be interpreted as describing the response of the Fed to stock price changes of a reasonable size. For these moderate stock price movements, the result above is economically significant.

### Table 2: Estimates for $\beta_1$; the parameter measuring the reaction to stock price increases

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>Regime 1,2,3</th>
<th>Regime 1,2,4</th>
<th>Regime 1,3,4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-0.0134</td>
<td>-0.0387</td>
<td>0.0050</td>
</tr>
<tr>
<td>Median</td>
<td>-0.0144</td>
<td>-0.0616</td>
<td>-0.0038</td>
</tr>
<tr>
<td>Probability mass above 0</td>
<td>16.68 %</td>
<td>25.73 %</td>
<td>44.97 %</td>
</tr>
</tbody>
</table>

### Table 3: Estimates for $\beta_2$; the parameter measuring the reaction to stock price decreases
If instead $\beta_1$ and $\beta_2$ are calculated using regimes 1, 2 and 4, the results change quantitatively, but not qualitatively. While the parameter estimate for $\beta_1$ is now -0.0387, it is still highly insignificant. On the contrary, $\beta_2$ is still positive and significant, though now only at the 10% level. The parameter estimate is as high as 0.0737, but this is mainly due to a few extremely large observations. Because of the extreme observations, the median might be a more correct measure in this case. The median of $\beta_2$ is estimated at 0.019, which seems a lot more plausible. Hence, this regime also lends support to the hypothesis of an asymmetric policy rule.

The results do change, however, when regimes 1, 3 and 4 are used. As can be seen from the table, the estimate for $\beta_2$ becomes very small numerically and highly insignificant. $\beta_1$ is still small and insignificant. Thus, while this regime is still not able to find any reaction to stock price increases, it is now also impossible to identify any reaction to stock price drops, and hence also any asymmetry in the policy rule.\textsuperscript{12}

### 4.1 Robustness

Calculating $\beta_1$ and $\beta_2$ using different covariance regimes can be seen as a robustness check. Specifically, as regime 4 has only a small number of observations, it seems natural to view the specification under regimes 1, 2 and 3 as a baseline scenario. In the following, some other assumptions made during the analysis above are altered in order to check the robustness of the results.

As already discussed, one could argue in favour of using the 6-month T-Bill rate instead of the 3-month rate. The entire analysis is therefore conducted with the 6-month rate entering the VAR equation. The results are presented in Appendix 3. As can be seen from Table 4 in the appendix, altering the choice of interest rate does not change the result that no significant reaction to stock price increases can be found. Indeed, the parameter estimates in all three regime combinations are highly insignificant. On the other hand, Table 5 suggests that a reaction to stock price drops is present. When evaluated at the 10% significance level, two of the three combinations of regimes identify a significant drop in the interest rate when stock prices fall. This is similar to the results using the 3-month T-Bill rate. Notice that the parameter estimate in the baseline scenario changes only slightly when changing the choice of interest rate, making the economic significance of the results more robust. In other words, it appears that the asymmetry result is relatively robust to the choice of interest rate.

As the choice of the number of lags in the VAR was not obvious, it is interesting to change the number of lags and investigate how this changes the results. Remember that while six lags did not

\textsuperscript{12}The results using regimes 2, 3 and 4 are not shown. Remember that around 85% of all observations were counted under regime 1. Thus, when discarding this regime, the analysis builds on very few observations, which in general makes it very difficult to obtain any significant or useful results. Indeed, none of the parameters could be precisely estimated under this regime.
seem to improve the model, the hypothesis that five lags are sufficient was just rejected against the alternative that seven lags are needed. Thus, I run the system with seven lags. In short, this does not change the results in any important way. \( \beta_2 \) is now estimated at 0.0133, i.e. quite close to the estimate with five lags. This number is significant at the 5\% level. On the contrary, the parameter estimate for \( \beta_1 \) is small (-0.007) and insignificant. Using the other regimes, the results from the five lag specification carry over quantitatively, with the parameter estimates changing only slightly. Running the regression with four lags also leads to no major changes.

When dividing the observations into different covariance regimes, it is not at all obvious that "high variance" should be defined as when the rolling variance exceeds its sample average by more than one standard deviation. As a robustness check, this threshold is changed to the sample average plus 0.5 and 1.25 times the standard deviation, respectively. Once again, the results (not reported) seem robust to this change. Specifically, the asymmetry in the policy reaction to stock prices is still present in the baseline scenario. Changing the threshold to 0.5 times the standard deviation leads to only minor changes in the parameter estimates, whereas setting it to 1.25 times the standard deviation increases the numerical value of the parameter estimates somewhat. In terms of statistical significance, the results are the same as in the baseline specification. Setting the threshold to two times the standard deviation, however, does change the results. In this case, only very few observations fall outside regime "low,low", leaving too few observations in the other regimes for the results to become significant.

Even though many lags were included in the original VAR, it might be interesting to test for unit roots in the dependent variables. As the variable \( s_t \) measures daily changes in the S&P 500 index, one would expect this series to be stationary. This is confirmed when testing for a unit root. The null hypothesis of non-stationarity is easily rejected at all conventional significance levels. On the other hand, the above analysis was done with \( i_t \) measured in levels, i.e. the daily observation of the interest rate. For this variable, the null hypothesis that the series has a unit root cannot be rejected. I therefore carry out the analysis with \( i_t \) measured in daily changes instead. Testing for a unit root in this series also leads to a rejection of the null of non-stationarity. Using regimes 1, 2 and 3, the estimate for \( \beta_1 \) is once again insignificant, while \( \beta_2 \) is now borderline insignificant. However, the difference between \( \beta_1 \) and \( \beta_2 \) is still significant at the 10\% level, lending some support to the hypothesis of an asymmetric policy response.

Finally, as also discussed in the conclusion, one can think of other specifications of asymmetric monetary policy. For instance, one might think that as long as stock prices do not increase or decrease by "too much", the Fed does not react. In other words, stock prices are allowed to fluctuate within a band around zero without leading to monetary policy reactions.\(^{13}\) However, this type of reaction function cannot be investigated in the setup of this paper. For instance, if it is assumed that the Fed only reacts to daily stock price changes exceeding, say, 2\%, then almost all of the observations would fall under the same covariance regime. Obviously, on days when the S&P 500 index increases or decreases by more than 2\%, the volatility of the stock market is also relatively high, placing this

\(^{13}\)This specification would also eliminate the risk that the asymmetric reaction found in this paper is mainly caused by large reactions to large stock price drops.
observation in the "high" covariance regime. When almost all of the observations fall in the same regime, the identification method becomes unreliable. Thus, testing the robustness of the results under this definition of asymmetric monetary policy is not possible in the current setup.

In conclusion, while the asymmetric result breaks down when using regimes 1, 3 and 4 to identify $\beta_1$ and $\beta_2$, the apparent robustness of the asymmetric monetary policy reaction to various other model specifications and assumptions is reassuring.

5 Conclusion

This paper takes a new perspective on the link between monetary policy and the stock market, relaxing the usual assumption that monetary policy reactions to stock price increases and decreases are of the same magnitude. The paper provides an extension of the model of Rigobon and Sack (2003) in the sense that it sets up a framework for analyzing whether a central bank reacts to movements in stock prices in an asymmetric way. Within this framework, I investigate the hypothesis that the Fed has reacted asymmetrically to stock prices in the period 1998-2008. The empirical results show that the Fed has indeed followed such a policy. While a drop in stock prices is met with an interest rate cut that is both statistically and economically significant, no significant reaction of monetary policy to stock price increases can be identified. The asymmetry result collapses under one of the covariance regime combinations, but otherwise seems robust to changes in the model specification.

While other authors have identified a reaction to the stock market, this paper is to my knowledge the first to identify different reactions to stock price drops and hikes, respectively.

Bini Smaghi (2009) stated that "it seems that some of the financial imbalances which built up prior to the crisis resulted from monetary policies which were not fully in line with the objective of price stability". The empirical results of this paper and the discussion of the implied moral hazard problems lend support to this view and provides a possible explanation of the build-up of these imbalances. One might suspect that the monetary policy of the Fed in the years 1998-2008 has helped build up asset price "bubbles" where the price of a share is out of line with its fundamental value. Such misalignments seem to have been among the causes of the recent financial crisis, as argued by, among others, Taylor (2009). In order to avoid creating moral hazard problems, the Fed should have conducted its activist policy towards the stock market in a symmetric way.

Several leading Fed officials have declared that central banks should abstain from reacting to asset prices (Bernanke 2002, Kohn 2006, Mishkin 2008). The message in these speeches is that "leaning against the wind" with respect to stock prices is not and should not be undertaken by the Fed. Specifically, Kohn (2006) explicitly states that "US monetary policy has responded symmetrically to the implications of asset price movements for actual and projected developments in output and inflation". The results in this paper stand in contrast to this statement.

The specification of asymmetric monetary policy used in this paper is just one of many possible specifications. The simple asymmetric rule I propose probably does not capture all aspects of asymmetric monetary policy. One candidate definition was discussed in section 4.1. Another option
capturing much the same idea would be to include quadratic terms in the monetary policy reaction to stock prices, allowing large stock market fluctuations to cause a much larger monetary policy reaction than small fluctuations. This is left for future research. While the definition of asymmetric policy in this paper might not be particularly subtle, its simplicity allows me to look for (and identify) a very general type of asymmetry in US monetary policy. Thus, the results in this paper should be considered not as much a final answer as a first step in identifying asymmetries in monetary policy.

Other directions for future research include conducting a similar analysis on the monetary policy of the ECB and other central banks. For the US, both the S&P 500 index and the Dow Jones Industrial Index are obvious candidates to use in this type of analysis. For the ECB, it is less obvious which stock market index to consider. It would also be interesting to see if the results of this paper could be reproduced using the Federal Funds Futures rate instead of the 3-month T-Bill rate. Finally, the analysis could be carried out in the framework of a threshold VAR (TVAR) model. In this paper, the threshold separating the monetary policy reactions (i.e., a zero change in stock prices) was imposed by the researcher. Within a TVAR model, it would be possible to estimate this threshold from the data. This approach would, however, be subject to some of the same difficulties discussed at the end of section 4.1, and is thus beyond the scope of the present paper.
6 References


Bernanke, B., 2002, Asset-Price "Bubbles" and Monetary Policy, speech held at the New York Chapter of the National Association for Business Economics, New York, New York.


Bini Smaghi, L., 2009. Monetary Policy and Asset Prices. Speech held at the University of Freiburg, Germany.


7 Appendix 1: Mathematical derivations

As in the main text, the calculations in this appendix are shown for $\beta_1$. Solving for $\beta_2$ proceeds in the exact same way.

In section 3, I showed what the covariance matrix for $v_i^t$ and $v_s^t$ looked like for a given regime. The covariance matrix for regime $i$ is repeated here for convenience:

$$
\Omega_i = \frac{1}{(1-\alpha \beta_1)^2} \begin{bmatrix}
(\beta_1 + \gamma)^2 \sigma_{i,z}^2 + \beta_1^2 \sigma_{i,y}^2 + \sigma_z^2 & (1 + \alpha \gamma) (\beta_1 + \gamma) \sigma_{i,z}^2 + \beta_1 \sigma_{i,y}^2 + \alpha \sigma_z^2 \\
(1 + \alpha \gamma) (\beta_1 + \gamma) \sigma_{i,z}^2 + \beta_1 \sigma_{i,y}^2 + \alpha \sigma_z^2 & (1 + \alpha \gamma)^2 \sigma_{i,z}^2 + \sigma_{i,y}^2 + \alpha^2 \sigma_z^2
\end{bmatrix}
$$  \hfill (A1)

As already described, the identification involves subtracting the covariance matrices of different regimes from each other. Subtracting covariance matrices $i$ and $j$ from each other yields:

$$
\Delta \Omega_{ij} = \frac{1}{(1-\alpha \beta_1)^2} \begin{bmatrix}
(\beta_1 + \gamma)^2 \Delta \sigma_{i,z}^2 + \beta_1^2 \Delta \sigma_{i,y}^2 & (1 + \alpha \gamma) (\beta_1 + \gamma) \Delta \sigma_{i,z}^2 + \beta_1 \Delta \sigma_{i,y}^2 \\
(1 + \alpha \gamma) (\beta_1 + \gamma) \Delta \sigma_{i,z}^2 + \beta_1 \Delta \sigma_{i,y}^2 & (1 + \alpha \gamma)^2 \Delta \sigma_{i,z}^2 + \Delta \sigma_{i,y}^2
\end{bmatrix}
$$  \hfill (A2)

Note in this step how, due to the assumption of homoskedasticity of the monetary policy shock $\varepsilon_t$ across regimes, the terms involving $\sigma_z^2$ cancel out.

As noted in the main text, all four covariance regimes are needed for the system to be fully identified. However, for my purposes, identifying $\beta_1$ is enough. For this, only three different regimes are needed, as shown below. Therefore, fix $j = 1$ and let $i = \{2, 3\}$. Moreover, I follow Rigobon and Sack (2003) in rewriting the covariance matrix in the following way:

Define:

$$
\theta = \frac{(1+\alpha \gamma)}{(\beta_1 + \gamma)} \text{ and } \varpi_{z,i} = (\beta_1 + \gamma) \Delta \sigma_{i,z}^2
$$

Using this notation, (A2) can be rewritten as:

$$
\Delta \Omega_{i1} = \frac{1}{(1-\alpha \beta_1)^2} \begin{bmatrix}
\varpi_{z,i} + \beta_1^2 \Delta \sigma_{i,y}^2 & \theta \varpi_{z,i} + \beta_1 \Delta \sigma_{i,y}^2 \\
\theta \varpi_{z,i} + \beta_1 \Delta \sigma_{i,y}^2 & \theta^2 \varpi_{z,i} + \Delta \sigma_{i,y}^2
\end{bmatrix}
$$  \hfill (A3)

Writing out the equations contained in (A3) for $i = 2$ explicitly yields:

$$
\Delta \Omega_{21,11} = \frac{1}{(1-\alpha \beta_1)^2} \left[ \varpi_{z,2} + \beta_1^2 \Delta \sigma_{21,y}^2 \right] \quad \text{ (A4)}
$$

$$
\Delta \Omega_{21,12} = \frac{1}{(1-\alpha \beta_1)^2} \left[ \theta \varpi_{z,2} + \beta_1 \Delta \sigma_{21,y}^2 \right] \quad \text{ (A5)}
$$

$$
\Delta \Omega_{21,22} = \frac{1}{(1-\alpha \beta_1)^2} \left[ \theta^2 \varpi_{z,2} + \Delta \sigma_{21,y}^2 \right] \quad \text{ (A6)}
$$

A similar system of three equations can be written for $i = 3$. Together, these are six equations in the following seven unknowns: $\alpha, \beta_1, \gamma, \varpi_{z,2}, \Delta \sigma_{21,y}^2, \varpi_{z,3}$ and $\Delta \sigma_{31,y}^2$. Rewriting the system...
(A4) – (A6) in the following way, I am able to exploit the obvious symmetry in these three equations. First, insert (A4) into (A5):

\[ \theta (1 - \alpha \beta_1)^2 \Delta \Omega_{21,11} - \theta \beta_1^2 \Delta \sigma^2_{21,\eta} + \beta_1 \Delta \sigma^2_{21,\eta} = (1 - \alpha \beta_1)^2 \Delta \Omega_{21,12} \iff \]

\[ \Delta \Omega_{21,12} - \theta \Delta \Omega_{21,11} = \frac{\beta_1 (1 - \theta \beta_1)}{(1 - \alpha \beta_1)^2} \Delta \sigma^2_{21,\eta} \quad (A7) \]

Similarly, insert (A5) into (A6):

\[ \theta (1 - \alpha \beta_1)^2 \Delta \Omega_{21,12} - \theta \beta_1 \Delta \sigma^2_{21,\eta} + \Delta \sigma^2_{21,\eta} = (1 - \alpha \beta_1)^2 \Delta \Omega_{21,22} \iff \]

\[ \Delta \Omega_{21,22} - \theta \Delta \Omega_{21,12} = \frac{(1 - \theta \beta_1)}{(1 - \alpha \beta_1)^2} \Delta \sigma^2_{21,\eta} \quad (A8) \]

Next, divide \( \frac{(A7)}{(A8)} \) \[ \frac{\Delta \Omega_{21,12} - \theta \Delta \Omega_{21,11}}{\Delta \Omega_{21,22} - \theta \Delta \Omega_{21,12}} = \beta_1 \iff \theta = \frac{\Delta \Omega_{21,12} - \beta_1 \Delta \Omega_{21,22}}{\Delta \Omega_{21,11} - \beta_1 \Delta \Omega_{21,12}} \quad (A9) \]

Remember that a system similar to (A4) – (A6) can be written for \( i = 3 \). Solving that system for \( \theta \) then yields:

\[ \theta = \frac{\Delta \Omega_{21,12} - \beta_1 \Delta \Omega_{21,22}}{\Delta \Omega_{21,11} - \beta_1 \Delta \Omega_{21,12}} \quad (A10) \]

As it turns out, (A9) and (A10) are two equations in just two unknowns, \( \beta_1 \) and \( \theta \). This illustrates how the underidentified system of six equations collapses to a smaller system where \( \beta_1 \) is now identified. To solve the system for \( \beta_1 \), equalize the right hand sides of (A9) and (A10) and cross-multiply:

\[ \Delta \Omega_{21,12} \Delta \Omega_{31,11} - \beta_1 \Delta \Omega_{21,12} \Delta \Omega_{31,12} - \beta_1 \Delta \Omega_{21,22} \Delta \Omega_{31,11} + \beta_1^2 \Delta \Omega_{21,22} \Delta \Omega_{31,12} = \]

\[ \Delta \Omega_{31,12} \Delta \Omega_{21,11} - \beta_1 \Delta \Omega_{31,12} \Delta \Omega_{21,12} - \beta_1 \Delta \Omega_{31,22} \Delta \Omega_{21,11} + \beta_1^2 \Delta \Omega_{31,22} \Delta \Omega_{21,12} \]

\[ \iff 0 = \beta_1^2 [\Delta \Omega_{31,22} \Delta \Omega_{21,12} - \Delta \Omega_{21,22} \Delta \Omega_{31,12}] - \beta_1 [\Delta \Omega_{31,22} \Delta \Omega_{21,11} - \Delta \Omega_{21,22} \Delta \Omega_{31,11}] + [\Delta \Omega_{31,12} \Delta \Omega_{21,11} - \Delta \Omega_{21,12} \Delta \Omega_{31,11}] \]

\[ \iff 0 = a \beta_1^2 - b \beta_1 + c \quad (A11) \]

- where:

\[ a = [\Delta \Omega_{31,22} \Delta \Omega_{21,12} - \Delta \Omega_{21,22} \Delta \Omega_{31,12}] \]

\[ b = [\Delta \Omega_{31,22} \Delta \Omega_{21,11} - \Delta \Omega_{21,22} \Delta \Omega_{31,11}] \]

\[ c = [\Delta \Omega_{31,12} \Delta \Omega_{21,11} - \Delta \Omega_{21,12} \Delta \Omega_{31,11}] \]

This solves the system for the parameter of interest; \( \beta_1 \). As noted above, the exact same method is used to solve for \( \beta_2 \).
It should be noted that the quadratic equation (A11) has two roots. Rigobon and Sack (2003) describe how the system of two equations in two unknowns (A9) and (A10) is solvable for $\beta$ and $\theta$ whenever one of these roots is real. This condition is ensured by the positive definiteness of the covariance matrices. Rigobon and Sack then show that one set of solutions to the system gives the correct values of $\beta$ and $\theta$, while the other set gives the inverse of these values.
8 Appendix 2: The Bootstrap

For the purpose of this paper, I do not have to bootstrap the actual observations that enter the original VAR. (Remember that this VAR has 2 dependent variables and 16 regressors). Instead, I can bootstrap the residuals from the VAR (see Efron and Tibshirani (1994,) or Johnston and DiNardo (1997) for a treatment of bootstrapping residuals). Usually, in order to bootstrap the residuals, these first need to be standardized, as emphasized by Johnston and DiNardo (1997). However, this is only necessary when the residuals are used for computing fitted values of the dependent variable in the original regression. The fitted values can then be regressed on the regressors to obtain a large number of estimates of the regression coefficients.

However, estimating the regression coefficients of the VAR is not the primary purpose of this paper. Instead, I am interested in the residuals from the VAR themselves, as I want to impose theoretical restrictions on these. Therefore, standardizing the residuals before implementing the bootstrap is not appropriate in the current context.

Following the above discussion, I use the raw residuals from the VAR to do the bootstrap. This gives me 10,000 realizations of the covariance matrix for each regime. With these in hand, it is easy to obtain 10,000 estimates of $\beta_1$ and $\beta_2$, the parameters of interest.
9 Appendix 3: Robustness checks

Table 4: Estimates for $\beta_1$: the parameter measuring the reaction to stock price increases; using the 6-month T-Bill rate.

<table>
<thead>
<tr>
<th>$\beta_1$</th>
<th>Regime 1,2,3</th>
<th>Regime 1,2,4</th>
<th>Regime 1,3,4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$-0.0077$</td>
<td>$0.0553$</td>
<td>$-0.0080$</td>
</tr>
<tr>
<td>Median</td>
<td>$-0.0083$</td>
<td>$0.0013$</td>
<td>$-0.0163$</td>
</tr>
<tr>
<td>Probability mass above 0</td>
<td>29.97 %</td>
<td>50.41 %</td>
<td>32.14 %</td>
</tr>
</tbody>
</table>

Table 5: Estimates for $\beta_2$: the parameter measuring the reaction to stock price decreases; using the 6-month T-Bill rate.

<table>
<thead>
<tr>
<th>$\beta_2$</th>
<th>Regime 1,2,3</th>
<th>Regime 1,2,4</th>
<th>Regime 1,3,4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>$0.0131$</td>
<td>$0.0768$</td>
<td>$0.0109$</td>
</tr>
<tr>
<td>Median</td>
<td>$0.0122$</td>
<td>$0.0276$</td>
<td>$0.0105$</td>
</tr>
<tr>
<td>Probability mass above 0</td>
<td>91.90 %</td>
<td>73.46 %</td>
<td>90.10 %</td>
</tr>
</tbody>
</table>