Housing Market Heterogeneity in a Monetary Union

Margarita Rubio*

Bank of Spain

January 2010

Abstract

This paper studies the implications of cross-country housing market heterogeneity in a monetary union both for shock transmission and for welfare. I develop a two-country new Keynesian general equilibrium model with housing and collateral constraints to explore this issue. Results show that in a monetary union, consumption reacts more strongly to monetary policy shocks in countries with high loan-to-value ratios (LTVs), a high proportion of borrowers or variable-rate mortgages. I also find that after country-specific housing price shocks consumption does not only increase in the country where the shock takes place, there is an international transmission. From a normative perspective, I conclude that housing-market homogenization in a monetary union is not beneficial per se, only when it is towards low LTVs or predominantly fixed-rate mortgages.

Keywords: Housing market, collateral constraint, monetary policy, monetary union

JEL Codes: E32, E44, F36

---

*Bank of Spain, Alcalá 48, 28014, Madrid, Spain. margarita.rubio@bde.es. I am truly thankful to Fabio Ghironi, Matteo Iacoviello and Peter Ireland for their invaluable advice. A great part of this project has been undertaken during my stays at the Federal Reserve Board and at the Bundesbank. I thank them for their financial support and helpful comments. I am also grateful to the seminar and conference participants and discussants at the Bank of Spain, the University of Valencia, the Symposium of Economic Analysis, the Infiniti Conference, the IREBS Conference on Real Estate, the CEUS Workshop "10 Years of the EMU", the Encuentro de Economía Aplicada, and the ECB workshop on Housing Markets and the Macroeconomy. Special thanks to Antonio Miralles and Rubén Segura-Cayuela and an anonymous referee for the Bank of Spain Working Paper series for very useful comments. All errors are mine. Usual disclaimer applies.
"Several of the benefits of the euro are already clearly visible, such as the deepening of trade and financial links between euro area countries and the greater resilience of the euro area to external shocks. Today I will discuss both of these accomplishments, and I will also touch on some of the challenges that we continue to face. For instance, there is presently a degree of diversity among euro area countries". Jean-Claude Trichet, October 8, 2007.

1 Introduction

Costs and benefits of monetary unions are a much discussed topic, especially in relation to the Europe’s Economic and Monetary Union (EMU). There are clear arguments in favor. A single currency eliminates exchange rate risk, allows rapid price comparison, lowers transaction costs across countries and favors trade. However, costs can arise if countries are not sufficiently similar. Different national characteristics such as heterogeneous institutions, consumption patterns or financial structures can be a source of different transmission of common shocks. Also, country-specific shocks derived from member heterogeneity can enhance the possible divergence.

In this paper, I focus on housing markets. I consider how heterogeneous housing markets across members affect the transmission of shocks (both symmetric and asymmetric) in a currency area. I also use welfare analysis to evaluate whether housing market homogenization would be beneficial.

Countries in Europe clearly differ in their housing markets. There is evidence of different loan-to-value ratios (LTVs), different proportion of residential debt relative to GDP across countries and heterogeneous mortgage contracts. Also, house price movements do not show the same pattern in every country. MacMennan et al. (1998) point out the importance of such heterogeneity in a monetary union. They conclude that there should be an effort toward institutional homogenization among European countries to alleviate possible tensions. The ECB (2009), in its study "Housing Finance in the Euro area", also remarks the importance of such differences for the EMU.

According to the European Mortgage Federation (EMF), in 2006 LTVs in Europe ranged from 60% in Italy to 73% in Germany or 95% in Sweden. In France, in 2004, the average LTV for first-time buyers reached a low 16% due to house price inflation and low interest rates. European countries also differ in their proportion of borrowers. The residential debt to GDP ratio ranges from values such as 18.7% in Italy to 98.4% in the Netherlands or 100.8% in Denmark. In those countries with a high LTV or a high

1Tables in the Appendix summarize this evidence.
proportion of indebted consumers, housing collateral effects are stronger. Therefore, shocks that affect the value of the collateral constraint could potentially have amplified effects on aggregate variables. This is a clear example of the financial accelerator mechanism, first modeled by Bernanke et al. (1999).

Differences in mortgage contracts across countries are another important source of heterogeneity in Europe. In countries such as Germany or France, the majority of mortgages are fixed rate. On the contrary, the predominant type of mortgages in countries such as the United Kingdom, Spain or Greece is variable rate. Calza et al. (2009) and Rubio (2009) show that the mortgage structure of an economy matters for the transmission of shocks, especially for those shocks that display more persistence.

Asymmetric house price shocks can also pose a problem for monetary unions. Different housing markets can also lead to an asymmetric evolution of house prices. According to the data, European countries in recent years show such asymmetry. There are countries such as Spain, the United Kingdom or France that have experienced large house price increases. However, house prices have been pretty stable and even slightly decreased in Germany. Country-specific house price shocks can create extra divergence across monetary union members. It is important to assess to what extent asymmetric house price movements in a specific region can be transmitted to other areas. House prices increasing in one area increase consumer’s wealth and therefore consumption. Since countries are trading also production in other areas can increase. Furthermore, interest rates respond to inflation creating house price movements in the whole union.

There is an extensive literature discussing differences in the transmission mechanisms between European countries using VARs or large macroeconometric models but little focus on the consequences of housing market heterogeneity from a theoretical standpoint. A microfounded general equilibrium model is needed to understand the implications of housing market differences, explore all the interrelations that take place in the economy and do some normative analysis. My paper can be framed into different strands of the literature. On the one hand, it is related to papers that study the shock transmission under different housing market characteristics such as Calza et al. (2009) and Rubio (2009). I extend their framework to an international version to address these issues in a monetary union. My paper is also related to two-country models with a financial accelerator such as Gilchrist et al. (2002). Contrary to their model, which does not feature a housing market, Iacoviello and Smets (2006) and Aspachs and Rabanal (2008) develop a monetary union model with housing markets and collateral constraints. I add to this literature by considering the role of mortgage contract heterogeneity and providing some

---

2 Aspachs and Rabanal (2008) focus on the case of Spain and the EMU.
normative analysis.\textsuperscript{3} The paper I present has also links with other papers that study welfare for different housing markets features. For instance, Campbell and Hercowitz (2009) study the welfare implications of moving to high LTVs. Rubio (2009) analyzes welfare when mortgages can be fixed or variable rate. I also consider these issues but extending the analysis to a two-country setting.

This paper presents a two-country dynamic stochastic general equilibrium (DSGE) model that features a housing market. There is a group of individuals in each country that are credit constrained and need housing collateral to obtain loans. Countries trade goods and savers in each country have access to foreign assets. Across countries, I allow for differences in LTVs, in the proportion of borrowers and in the structure of mortgage contracts (fixed vs. variable rate). I also consider idiosyncratic house price shocks and compare the case in which the two countries have independent monetary policy and different currencies with the case of a monetary union.

Results show that in a monetary union, common shocks have a different impact across countries when there exists housing market heterogeneity. In particular, consumption reacts more strongly after a shock when the LTV is high, the proportion of borrowers is high, or when mortgages are predominantly variable rate.\textsuperscript{4} Concerning asymmetric house price shocks, I find that consumption increases in the country where a positive house price shock takes place but also in the other country. House price shocks are transmitted internationally, especially in the monetary union regime.

From a normative perspective I find that housing market homogeneity per se is not necessarily beneficial. For instance, total welfare is higher in a situation where LTVs are asymmetric than in a situation where they are equal but very high, in line with the findings in Campbell and Hercowitz (2009). Also, for mortgage contracts, homogenization is welfare improving only if it is towards fixed-rate mortgages. These results have clear policy implications.

The paper is organized as follows. Section 2 presents both the baseline model (two countries with different currencies and independent monetary policies) and the monetary union version. Section 3 presents the model dynamics. Section 4 analyzes welfare. Section 5 concludes. Tables, steady-state relationships and the linearized model are shown in the Appendix.

\textsuperscript{3}Darraaq and Notarpietro (2008) study optimal monetary policy in a two-country model with housing for the US and the EMU.

\textsuperscript{4}For the latter case, results are different depending on the type of shock considered. Aggregate differences are more emphasized for a technology shock. See section 3.2.3.
2 A Two-Country Model with Housing

I develop a two-country general equilibrium model with a housing market. As a starting point I consider the case in which each of the countries implements its own monetary policy, under a flexible exchange rate regime. In each country, the central bank sets the interest rate to respond to domestic inflation. I allow for housing market heterogeneity across countries.

2.1 The Model

I consider an infinite-horizon, two-country economy with a flexible exchange rate regime. Households consume, work and demand real estate. There is a financial intermediary in each country that provides mortgages and accepts deposits from consumers. Each country produces one differentiated intermediate good but households consume goods from both countries. Housing is a non-traded good. I assume that labor is immobile across countries. Firms follow a standard Calvo problem. In this economy, both final and intermediate goods are produced. Prices are sticky in the intermediate goods sector.

2.1.1 The Consumer’s Problem

There are three types of consumers in each country: unconstrained consumers, constrained consumers who borrow at a variable rate and constrained consumers who borrow at a fixed rate. The proportion of each type of borrower is fixed and exogenous. Consumers can be constrained or unconstrained, in the sense that constrained individuals need to collateralize their debt repayments in order to borrow from the financial intermediary. Interest payments next period cannot exceed a proportion of the future value of the current house stock. In this way, the financial intermediary ensures that borrowers are going to be able to fulfill their debt obligations next period. As in Iacoviello (2005), I assume that constrained consumers are more impatient than unconstrained ones.

Unconstrained Consumers (Savers) Unconstrained consumers in Country A maximize:

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t^u + j_t \ln H_t^u - \frac{(L_t^u)^\eta}{\eta} \right),
\]  

(1)

According to the EMF, the type of mortgage contracts across countries responds in a large extent to institutional or cultural factors, out of the scope of this model. In the short run, the proportion of each type of mortgage contracts can fluctuate but typically not implying changing the fixed or variable-rate category of the country. This assumption ensures that the borrowing constraint binds in the steady state and the economy is endogenously split into borrowers and savers.
Here, $E_0$ is the expectation operator, $\beta \in (0, 1)$ is the discount factor, and $C_t^u$, $H_t^u$ and $L_t^u$ are consumption at $t$, the stock of housing and hours worked, respectively. $j_t$ represents the weight of housing in the utility function. I assume that $\log (j_t) = \log(j) + u_{jt}$, where $u_{jt}$ follows an autoregressive process. A shock to $j_t$ represents a shock to the marginal utility of housing. These shocks directly affect housing demand and therefore can be interpreted as a proxy for exogenous disturbances to house prices. $1/\eta - 1$ is the aggregate labor-supply elasticity.

Consumption is a bundle of domestically and foreign produced goods, defined as: $C_t^u = (C_{A_t}^u)^n (C_{B_t}^u)^{1-n}$, where $n$ is the size of Country A.

The budget constraint, in units of Country A’s currency, is:

$$P_{A_t} C_{A_t}^u + P_{B_t} C_{B_t}^u + Q_t H_t^u + R_{A_{t-1}} B_{t-1}^u + e_t R_{B_{t-1}} D_{t-1} + \frac{\psi}{2} e_t D_t^2 \leq Q_t H_{t-1}^u + W_t^u L_t^u + B_t^u + L_t + P_{A_t} F_t + P_{A_t} S_t,$$

(2)

where $P_{A_t}$ and $P_{B_t}$ are the prices of the goods produced in Countries A and B, respectively, $Q_t$ is the housing price in Country A, and $W_t^u$ is the wage for unconstrained consumers. $B_t^u$ represents domestic bonds denominated in home currency. $R_{A_t}$ is the nominal interest rate in Country A. Positive bond holdings mean borrowing and negative mean savings. However, as we will see, this group will choose not to borrow at all, they are the savers in this economy. $D_t$ are foreign bond holdings by savers in Country A. $R_{B_t}$ is the nominal rate of foreign bonds, which are denominated in foreign currency. $e_t$ is the exchange rate between currency in Country A and Country B. As it is common in this literature, to ensure stationarity of net foreign assets, I introduce a small quadratic cost of deviating from zero foreign borrowing, $\frac{\psi}{2} e_t D_t^2$. Savers obtain interests for their savings. $S_t$ and $F_t$ are lump-sum profits received from the firms and the financial intermediary in Country A, respectively.

Dividing by $P_{A_t}$, we can rewrite the budget constraint in terms of good A:

$$C_{A_t}^u + \frac{P_{B_t}}{P_{A_t}} C_{B_t}^u + q_t H_t^u + \frac{R_{A_{t-1}} b_{t-1}^u}{\pi_{A_t}} + \frac{e_t R_{B_{t-1}} D_{t-1}}{P_{A_t}} + \frac{\psi}{2P_{A_t}} e_t D_t^2 \leq q_t H_{t-1}^u + w_t^u L_t^u + b_t^u + \frac{e_t D_t}{P_{A_t}} + F_t + S_t,$$

(3)

---

7 It is assumed that housing services are proportional to the housing stock.
8 Savers have access to international financial markets.
9 See Iacoviello and Smets (2006) for a similar specification of the budget constraint.
where $\pi_{At}$ denotes inflation for the good produced in Country A, defined as $P_{At}/P_{At-1}$.

Maximizing (1) subject to (3), we obtain the first-order conditions for the unconstrained group:

$$\frac{C_{At}^u}{C_{Bt}^u} = \frac{n P_{Bt}}{(1-n) P_{At}}$$

(4)

$$\frac{1}{C_{At}^u} = \beta E_t \left( \frac{R_{At}}{\pi_{At+1} C_{At+1}^u} \right)$$

(5)

$$\frac{1 - \psi D_t}{C_{At}^u} = \beta E_t \left( \frac{R_{Bt} e_{t+1}}{\pi_{At+1} C_{At+1}^u} \right)$$

(6)

$$w^u_t = (L^u_t)^{n-1} \frac{C_{At}^u}{n}$$

(7)

$$\frac{j_t}{H^u_t} = \frac{n}{C_{At}^u} q_t - \beta E_t \frac{n}{C_{At+1}^u} q_{t+1}$$

(8)

Equation (4) equates the marginal rate of substitution between goods to the relative price. Equation (5) is the Euler equation for consumption. Equation (6) is the first-order condition for net foreign assets. Equation (7) is the labor-supply condition. These equations are standard. Equation (8) is the Euler equation for housing and states that at the margin the benefits from consuming housing have to be equal to the costs.

Combining (5) and (6) we obtain a non-arbitrage condition between home and foreign bonds:10

$$R_{At} = \frac{R_{Bt} E_t e_{t+1}}{(1 - \psi D_t) e_t}.$$  

(9)

Since all consumption goods are traded and there are no barriers to trade, I assume in this paper that the law of one price holds:

$$P_{At} = e_t P_{At}^*,$$

(10)

where variables with a star denote foreign variables.

10The log-linearized version of this equation could be interpreted as the uncovered interest rate parity.
Constrained Consumers (Borrowers)  Constrained consumers in Country A are of two types: those who borrow at a variable rate and those who do it at a fixed rate. The difference between them is the interest rate they are charged. The variable-rate constrained consumer faces $R_{At}$, which will coincide with the one set by the central bank. The fixed-rate borrower pays $\bar{R}_{At}$, derived from the financial intermediary’s problem. The proportion of variable-rate consumers in Country A is constant and exogenous and equal to $\alpha_A \in [0, 1]$.

Constrained consumers are more impatient than unconstrained ones, that is $\bar{\beta} < \beta$. Constrained consumers face a collateral constraint; the expected debt repayment next period cannot exceed a proportion of the expectation of tomorrow’s value of today’s stock of housing:

$$E_t R_{At} b_{t+1}^{cv} \leq k_A E_t q_{t+1} H_{t+1}^{cv}, \quad (11)$$

$$E_t \bar{R}_{At} b_{t+1}^{cf} \leq k_A E_t q_{t+1} H_{t+1}^{cf}, \quad (12)$$

where equations (11) and (12) represent the collateral constraint for the variable and the fixed-rate borrower, respectively. $k_A$ can be interpreted as the loan-to-value ratio in Country A. Notice that in this kind of models with collateral constraints, the LTV is typically considered to be exogenous. In reality, the LTV can be a decision variable of the bank, depending on the characteristics of the borrowers. However, this is a macroeconomic model in which borrowers are a representative agent within their type and therefore the LTV is considered to be an exogenous parameter. As it will become clear when I present the problem of the financial intermediary, $\bar{R}_{At}$ is an aggregate interest rate that contains information on all the past fixed interest rates associated with past debt. Each period, this aggregate interest rate is updated with a new interest rate linked to the new amount of debt originated in that period.

Without loss of generality, I present the problem for the variable-rate borrower, since the one for the fixed rate is symmetric. Variable-rate borrowers maximize their lifetime utility function:

---

11. The superscript $cv$ stands for "constrained variable" while $cf$ stands for "constrained fixed".
12. At the macroeconomic level, LTVs partly depend on exogenous factors such as regulation. This parameter is usually calibrated to match the average LTV in the country analyzed.
\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t^{cv} + j_t \ln H_t^{cv} - \frac{(L_t^{cv})^\eta}{\eta} \right),
\]

where \( C_t^{cv} = (C_{At}^{cv})^{n} (C_{Bt}^{cv})^{1-n} \), subject to the budget constraint (in terms of good A):

\[
C_{At}^{cv} + \frac{P_{Bt} C_{Bt}^{cv}}{P_{At}} + q_t H_t^{cv} + \frac{R_{At-1} b_{t-1}^{cv}}{\pi_{At}} \leq q_t H_{t-1}^{cv} + w_t^{cv} L_t^{cv} + b_t^{cv},
\]

and subject to the collateral constraint (11). Notice that variable-rate borrowers repay all debt every period and acquire new one at the current new interest rate. This assumption implies that the interest rate on variable rate mortgages is revised every period for the whole stock of debt and changed according to the policy rate.\(^{13}\) In order to make the problem for fixed-rate borrowers symmetric and analogous to the existing models with borrowing constraints, I assume the same debt-repayment structure for this type of borrowers. Obviously, fixed-rate contracts are not revised every period. However, as we will see below, to make the model more realistic but still tractable, the fixed interest rate will be such that a revised fixed rate will be applied only on new debt, keeping constant the interest rate applied to existing debt. In this way, I reconcile the structure of the model with the fact that fixed-rate contracts are long term.\(^{14}\)

The first-order conditions for these consumers are:

\[
\frac{C_{At}^{cv}}{C_{Bt}^{cv}} = \frac{n P_{Bt}}{(1-n) P_{At}}.
\]

\[
\frac{n}{C_{At}^{cv}} = \tilde{\beta} E_t \left( \frac{n R_{At}}{\pi_{At+1} C_{At+1}^{cv}} \right) + \lambda_{At}^{cv} R_{At},
\]

\[
w_t^{cv} = (L_t^{cv})^{\eta-1} \frac{C_{At}^{cv}}{n},
\]

\[
\frac{j_t}{H_t^{cv}} = \frac{n}{C_{At}^{cv}} q_t - \tilde{\beta} E_t \frac{n}{C_{At+1}^{cv}} q_{t+1} - \lambda_{At}^{cv} k_A E_t q_{t+1} \pi_{At+1}.
\]

These first-order conditions differ from those of the unconstrained individuals. In the case of con-

\(^{13}\)This assumption is consistent with reality, in which variable interest rates are revised very frequently and changed according to an interest rate index tied to the interest rate set by the central bank.

\(^{14}\)Another option would be to have an overlapping generation model in which we are able to keep track of the debt issued each period. However, the model would become more complex and less comparable with the standard collateral constraint DSGE models such as Iacoviello (2005).
strained consumers, the Lagrange multiplier on the borrowing constraint \(\lambda^v_t\) appears in equations (16) and (18). As in Iacoviello (2005), the borrowing constraint is always binding, so that constrained individuals borrow the maximum amount they are allowed to and their saving is zero.\(^{15}\)

The problem for consumers is analogous in Country B.

2.1.2 The Financial Intermediary

There is a financial intermediary in each country. The financial intermediary in Country A accepts deposits from domestic savers, and extends both fixed and variable-rate loans to domestic borrowers. I assume a competitive framework and thus the intermediary takes the variable interest rate as given.\(^{16}\)

The profits of the financial intermediary are defined as:

\[
F_t = \alpha_A R_{At-1} b^c_{t-1} + (1 - \alpha_A) \bar{R}_{At-1} b^f_{t-1} - R_{At-1} b^u_{t-1}. \tag{19}
\]

In equilibrium, aggregate borrowing and saving must be equal, that is,

\[
\alpha_A b^c_t + (1 - \alpha_A) b^f_t = b^u_t. \tag{20}
\]

Substituting (20) into (19), we obtain,

\[
F_t = (1 - \alpha_A) b^f_{t-1} \left( \bar{R}_{At-1} - R_{At-1} \right). \tag{21}
\]

In order for the two types of mortgage to be offered, the fixed interest rate has to be such that the intermediary is indifferent between lending at a variable or fixed rate. Hence, the expected discounted profits that the intermediary obtains by lending new debt in a given period at a fixed interest rate must be equal to the expected discounted profits the intermediary would obtain by lending it at variable rate:

\[
E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{At}^{OPT} = E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{\tau,i} R_{At-1}, \tag{22}
\]

\(^{15}\)From the Euler equations for consumption of the unconstrained consumers, we know that \(R_A = 1/\beta\), where variables without a time subscript denote steady-state variables. If we combine this result with the Euler equation for consumption for the constrained individual we have that \(\lambda^v = n (\beta - \bar{\beta}) / C_A^v > 0\). Given that \(\beta > \bar{\beta}\), the borrowing constraint holds with equality in steady state. Since the model is log-linearized around the steady state and low uncertainty is assumed, this result can be generalized to off-steady-state dynamics.

\(^{16}\)See Andrés and Arce (2008) for a housing model with collateral constraints in which banks are imperfectly competitive and are able to set optimal lending rates.
where $\Lambda_{t,i} = \frac{C_{At}^u}{C_{At+i}^u}$ is the unconstrained consumer relevant discount factor. Since the financial intermediary is owned by the savers, their stochastic discount factor is applied to the financial intermediary’s problem. Notice that, as stated before, variable-rate debt is one period but the portion of new debt acquired at a fixed rate is associated with a long-term contract. Since the agent is infinitely lived, I assume here that the maturity of fixed-rate mortgages is also infinity.

We can obtain the equilibrium value of the fixed rate in period $\tau$ from expression (22):

$$R_{At}^{OPT} = \frac{E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{t,i} R_{At-1}}{E_\tau \sum_{i=\tau+1}^{\infty} \beta^{i-\tau} \Lambda_{t,i}}.$$  

Equation (23) states that, for every new debt issued at date $\tau$, there is a different fixed interest rate that has to be equal to a discounted average of future variable interest rates. Notice that this is not a condition on the stock of debt, but on the new amount obtained in a given period. New debt at a given point in time is associated with a different fixed interest rate. Both the fixed interest rate in period $\tau$ and the new amount of debt in period $\tau$ are fixed for all future periods. However, the fixed interest rate varies with the date the debt was issued, so that in every period there is a new fixed interest rate associated with new debt in this period. If we consider fixed-rate loans to be long-term, the financial intermediary obtains interest payments every period from the whole stock of debt, not only from the new ones. Hence, we can define an aggregate fixed interest rate that is the one the financial intermediary effectively charges every period for the whole stock of mortgages. This aggregate fixed interest rate is composed of all past fixed interest rates and past debt, together with the current period equilibrium fixed interest rate and new amount of debt. Therefore, the effective fixed interest rate that the financial intermediary charges for the stock of fixed-rate debt every period is:

$$R_{At} = \begin{cases} \frac{\pi_{At-1} b_{t-1}^{cf} + R_{At}^{OPT} (b_t^{cf} - b_{t-1}^{cf})}{b_t^{cf}} & \text{if } b_t^{cf} > b_{t-1}^{cf} \\ R_{At-1} & \text{if } b_t^{cf} \leq b_{t-1}^{cf} \end{cases}. $$  

Equation (24) states that the fixed interest rate that the financial intermediary is actually charging today is an average of what it charged last period for the previous stock of mortgages and what it charges this period for the new amount. In the case that there is not new debt, the fixed interest rate will be equal to last period’s. Then, the same way that variable rates are revised every period, fixed-rates are revised by including the new optimal fixed interest rate for the new debt originated in this period. Importantly,
this assumption is not crucial for results. Both $\bar{R}^{OPT}$ and $\bar{R}_{At}$ are practically unaffected by interest rate shocks. This assumption is a way to make the model compatible with the fact that fixed-rate loans are not one-period assets but longer term ones.

As noted above, if any, profits from financial intermediation are rebated to the unconstrained consumers every period. Even if the financial intermediary is competitive and it does not make profits in absence of shocks, if there is a shock at a given point in time, the fact that only the variable interest rate is directly affected can generate non-zero profits.

The financial intermediary problem for Country B is symmetric.

2.1.3 Firms

**Final Goods Producers** In Country A, there is a continuum of final goods producers that aggregate intermediate goods according to the production function

$$Y^{k}_{lt} = \left[ \int_{0}^{1} Y^{k}_{lt}(z)^{\frac{\epsilon + 1}{\epsilon}} dz \right]^\frac{1}{\epsilon - 1} ,$$

where $\epsilon > 1$ is the elasticity of substitution between intermediate goods.

The total demand of intermediate good $z$ is given by $Y_{At}(z) = \left( \frac{P_{At}(z)}{P_{At}(1)} \right)^{-\epsilon} Y_{At}$, and the price index is $P_{At} = \left[ \int_{0}^{1} P_{At}(z)^{1-\epsilon} dz \right]^\frac{1}{1-\epsilon}$.

**Intermediate Goods Producers** The intermediate goods market is monopolistically competitive. Following Iacoviello (2005), intermediate goods are produced according to the following production function:

$$Y_{At}(z) = \xi_{t} (L^{u}_{t}(z)^{\gamma} (L^{c}_{t}(z))^{(1-\gamma)} ,$$

where $\xi_{t}$ represents technology. I assume that $\log \xi_{t} = \rho_{\xi} \log \xi_{t-1} + u_{\xi t}$, where $\rho_{\xi}$ is the autoregressive coefficient and $u_{\xi t}$ is a normally distributed shock to technology. $\gamma_{A} \in [0, 1]$ measures the relative size of each group in terms of labor. $L^{c}_{t}$ is labor supplied by constrained consumers, defined as $\alpha_{A} L^{cv}_{t} + (1 - \alpha_{A}) L^{cf}_{t}$. This Cobb-Douglas production function implies that labor efforts of constrained and unconstrained consumers are not perfect substitutes. This specification is analytically tractable and allows for closed form solutions for the steady state of the model. This assumption can be economically
justified by the fact that savers are the managers of the firms are their wage is higher\textsuperscript{17}. Experimenting with a production function in which hours are substitutes leads to very similar results in terms of model dynamics. These two assumptions are strictly comparable: under the Cobb-Douglas specification each household has mass one and \( \gamma_A \) represents the economic size of the patient household. In the alternative specification, the absolute size of savers in the population would be specified instead.

The first-order conditions for labor demand are the following:\textsuperscript{18}

\[ w_t^u = \frac{\xi_t}{X_t \gamma A} \frac{Y_{At}}{L_t^u}, \tag{27} \]

\[ w_t^{cw}_t = w_t^{cf} = \frac{\xi_t}{X_t} (1 - \gamma A) \frac{Y_{At}}{L_t^c}, \tag{28} \]

where \( X_t \) is the markup, or the inverse of marginal cost.

The price-setting problem for the intermediate goods producers is a standard Calvo-Yun problem. An intermediate good producer sells goods at price \( P_{At} (z) \), and \( 1 - \theta \) is the probability of being able to change the sale price in every period. The optimal reset price \( P_{At}^{OPT} (z) \) solves:

\[ \sum_{k=0}^{\infty} (\theta \beta)^k E_t \left\{ \Lambda_{t,k} \left[ \frac{P_{At}^{OPT} (z)}{P_{At+k}^{OPT}} - \frac{\varepsilon}{(\varepsilon - 1)} \frac{Y_{At+k}^{OPT} (z)}{X_{t+k}^{OPT}} \right] \right\} = 0. \tag{29} \]

The aggregate price level is given by:

\[ P_{At} = \left[ \theta P_{At-1}^\varepsilon + (1 - \theta) \left( P_{At}^{OPT} \right)^{1 - \varepsilon} \right]^{1/(1 - \varepsilon)}. \tag{30} \]

Using (29) and (30) and log-linearizing, we can obtain the standard forward-looking Phillips curve (See equation in the Appendix 2).\textsuperscript{19}

The firm problem is similar in Country B.

\subsection{2.1.4 Aggregate Variables and Market Clearing}

Given \( \alpha_A \), the fraction of variable-rate borrowers in Country A, we can define aggregates across constrained consumers as the sum of variable-rate and fixed-rate aggregates, so that \( C_t^c \equiv \alpha_A C_t^{cv} + \)

\textsuperscript{17}It could also be interpreted as the savers being older than the borrowers, and therefore more experienced.

\textsuperscript{18}Symmetry across firms allows to avoid the index \( z \).

\textsuperscript{19}This Phillips curve is consistent with other two-country models with financial accelerator. See for instance Gilchrist et al (2002) or Iacoviello and Smets (2006).
\[(1 - \alpha_A) C_t^{cf}, \ H_t^{c} \equiv \alpha_A H_t^{cw} + (1 - \alpha_A) H_t^{cf} \text{ and } b_t^{c} \equiv \alpha_A b_t^{cw} + (1 - \alpha_A) b_t^{cf}.\]

Therefore, economy-wide aggregates in Country A are \(C_t \equiv C_t^u + C_t^{c}, \ L_t \equiv L_t^u + L_t^{c}.\) Aggregate supply of housing is fixed, so that market clearing requires \(H_t \equiv H_t^u + H_t^{c} = H.\)\(^{20}\)

The market clearing condition for the final good in Country A is \(nY_{At} = nC_{At} + (1 - n) C_{At}^{*} + n \psi d_t^{u},\) Domestic financial markets clear: \(b_t^{c} = b_t^{u}.\) The world bond market clearing condition is \(n\psi d_t + (1 - n) \frac{P_{At}}{P_{At}^*} d_t^{*} = 0,\) where \(d_t\) denotes the foreign bonds in real terms. Everything is similar in Country B.

2.1.5 Monetary Policy

The model is closed with a Taylor rule with interest-rate smoothing for interest-rate setting by each country’s central bank.\(^{21}\) In Country A,

\[
R_{At} = (R_{At-1})^{\rho_A} \left( \frac{\pi_{At}}{\pi_{At}^{*}} R_A \right)^{1 - \rho_A} \epsilon_{AR,t},
\]

\(0 \leq \rho_A \leq 1\) is the parameter associated with interest-rate inertia. \((1 + \phi_{\pi A})\) measures the sensitivity of interest rates to current inflation. \(\epsilon_{AR,t}\) is a white noise shock process with zero mean and variance \(\sigma_{\epsilon}^2.\)

In Country B, \(R_{Bt}\) is set similarly.

2.2 The Monetary Union Case

Now we can consider the case in which Country A and Country B form a monetary union. The problem for consumers in this case differs from the previous one in that prices are denominated in a common currency and therefore there is no need for the use of the exchange rate. Monetary policy is now conducted by a single central bank that reacts to inflation and output in both countries weighted by its relative size. Equations are presented in the Appendix.

\(^{20}\)An endogenous supply of housing could be easily introduced in a two-sector version of this model. However, the qualitative results would not change for the demand side of the model which is the focus of this paper. For two-sector models see for instance Iacoviello and Smets (2006) or Iacoviello and Neri (2008).

\(^{21}\)This rule is consistent with the primary objective of the ECB being price stability. This type of rule is also used in other monetary union models. See Iacoviello and Smets (2006) or Aspachs and Rabanal (2008)
3 Dynamics

3.1 Parameter Values

The discount factor for savers, \( \beta \), is set to 0.99 so that the annual interest rate is 4% in steady state. The discount factor for borrowers, \( \tilde{\beta} \), is set to 0.98.\(^{22}\) The steady-state weight of housing in the utility function, \( j \), is set to 0.1 in order for the ratio of housing wealth to GDP to be approximately 1.40 in the steady state.\(^{23}\) I set \( \eta = 2 \), implying a value of the labor supply elasticity of 1.\(^{24}\) For the loan-to-value ratio, I pick \( k_A = k_B = 0.8 \) for the baseline calibration, consistent with a weighted average of LTVs in 2004 calculated by the EMF on European countries.\(^{25}\) However, one of the experiments I perform consists of testing the sensitivity of results to this parameter. The labor income share of unconstrained consumers, \( \gamma_A = \gamma_B \), is set to 0.7 as a reference point.\(^{26}\) Nonetheless, as for the LTV ratio, experiments with different values of this parameter will also be performed. I pick a value of 6 for \( \varepsilon \), the elasticity of substitution between intermediate goods. This value implies a steady-state markup of 1.2. The probability of not changing prices, \( \theta \), is set to 0.75, implying that prices change every four quarters on average. For the Taylor Rule parameters I use \( \rho_A = \rho_B = 0.8 \), \( \phi_{\pi A} = \phi_{\pi B} = 0.5 \). The first value reflects a realistic degree of interest-rate smoothing.\(^{27}\) \( \phi_{\pi A} \) and \( \phi_{\pi B} \) are consistent with the original parameters proposed by Taylor in 1993. For the baseline model, I consider \( \alpha_A = \alpha_B = 1 \), that is, all mortgages are variable rate.\(^{28}\) Results for the case of fixed-rate mortgages are also checked. I consider Country B to be a small country so that \( n = 0.9 \).

Monetary policy shocks are represented by a one percent increase of the interest rate. A technology shock will be a one percent positive technology with 0.9 persistence.\(^{29}\) House price shocks have a 0.8 persistence.\(^{30}\) I set the size of the shock to the housing demand parameter to 20% so that house prices

\(^{22}\)Lawrance (1991) estimates discount factors for poor consumers between 0.95 and 0.98 at quarterly frequency.

\(^{23}\)This value corresponds to the US. I assume here that the ratio is similar across most industrialized countries, given the lack of housing wealth data for European countries. See Aspachs and Rabanal (2008).

\(^{24}\)Microeconomic estimates usually suggest values in the range of 0 and 0.5 (for males). Domeij and Flodén (2006) show that in the presence of borrowing constraints this estimates could have a downward bias of 50%.

\(^{25}\)The countries that are included in the sample are Belgium, Germany, Greece, Spain, France, Italy, Hungary, Poland, Sweden and the United Kingdom.

\(^{26}\)This value is in the range of the estimates of Iacoviello (2005), Iacoviello and Neri (2008) and Campbell and Mankiw (1991) for the US, Canada, France and Sweden.

\(^{27}\)See McCallum (2001).

\(^{28}\)This value makes the model comparable with the standard models where fixed-rate mortgages are not considered.

\(^{29}\)This high persistence value for technology shocks is consistent with what is commonly used in the literature. Smets and Wouters (2002) estimate a value of 0.822 for this parameter in Europe, Iacoviello and Neri (2008) estimate is 0.93 for the US.

\(^{30}\)The persistence of the house price shock is consistent with the estimates in Iacoviello (2005) and Iacoviello and Neri (2008).
increase roughly by 1%.

3.2 Common Shock in a Monetary Union with Housing Market Heterogeneity

3.2.1 LTV Asymmetry

When countries in a monetary union are asymmetric in their housing markets, a common shock can affect them differently. The first source of asymmetry that I consider is differences in LTVs. The loan-to-value ratio is a crucial parameter because it implies the degree of credit accessibility for borrowers and therefore the strength of the financial accelerator. When LTVs are high, shocks that affect the value of the collateral are amplified due to the financial accelerator effect.

Figure 1 shows the effects of a monetary policy shock in a monetary union when countries differ in their LTVs. We consider Country B to be a small country with a low LTV of 0.2, as opposed to the rest of the union which has an LTV of 0.8. This theoretical experiment could illustrate the case of France in
An increase in the interest rate contracts the economy. Savers substitute intertemporally and prefer to save today to consume tomorrow. For borrowers, there is both a direct and an indirect effect that make their consumption decrease. First, their mortgage payments increase and therefore they consume less. The second effect comes through the collateral constraint. Since housing prices decrease following the interest rate increase, the value of their collateral decreases. Impatient agents are able to borrow less and hence consume less. This collateral effect, however, is stronger the higher the LTV parameter. We can see that the effects of this shock are amplified if the country has a high LTV, meaning that the financial accelerator is stronger there. Notice however that savers, who have access to international financial markets, are able to compensate the differences between the two scenarios.

The experiment for a common technology shock is not shown here because it is analogous. Also in this case total consumption would react more in the country that has a high LTV ratio. The interest rate would decrease and housing prices in both countries increase. The collateral effect is greater in that country with the higher LTV and therefore its consumption would increase by more.

3.2.2 Borrowers Proportion Asymmetry

The proportion of borrowers is also a source of cross-country asymmetry that matters for the transmission of shocks. The economic size of this group in Country A is captured by $1 - \gamma_A$ in the model. We consider Country B having a higher proportion of borrowers ($\gamma_B = 0.2$) than the rest of the union (Country A) where $\gamma_A = 0.7$. Similarly to the LTV heterogeneity case, we expect that when borrowers are very numerous, collateral constraints are a more pervasive feature of the economy. Figure 2 confirms this intuition. For borrowers, consumption decreases by more when they have a more important economic size. In this case, savers are able to offset only part of these differences through international financial markets. Housing also reacts more strongly in the country with more borrowers. In the aggregate we see that after a monetary policy shock consumption decreases by more where the proportion of borrowers is higher.

31 LTVs for first-time buyers reached a low 16% due to house price inflation and low interest rates.
32 Results are robust to the country size. Although the differences between the two settings are slightly amplified when country B is small, results mainly respond to the financial accelerator effect.
3.2.3 Mortgage Contract Asymmetry

Another source of heterogeneity in housing markets is the mortgage structure. Let us analyze now the case in which mortgage contracts in Country A are fixed rate and variable rate in Country B. This could be seen as Country B being for instance Spain and Country A the rest of EMU. Consider first an interest rate shock in a monetary union. For those consumers with variable-rate mortgages, after a positive interest-rate shock, interest rate payments increase by more than for the fixed-rate case. Also, the value of their collateral decreases by more. Then, the monetary policy shock hits strongly those individuals that are constrained. We can observe in figure 3 that consumption and housing demand for borrowers decrease slightly more persistently in the country in which consumers borrow at a variable rate. For housing demand the mortgage contract makes a difference, for both borrowers and savers housing demand reacts by more in the variable-rate scenario. For aggregate consumption differences between the two countries are quantitatively small. General equilibrium effects partially offset aggregate differences:
On the one hand, there is a redistribution between borrowers and savers. On the other hand, there are important wealth effects in the labor-supply decision, that is, variable-rate borrowers can simply decide to work more to compensate their consumption loss. These results are in line with Rubio (2009) that shows that in a closed economy, a larger proportion of borrowers or GHH preferences, which eliminate wealth effects in the labor supply, are able to generate larger aggregate differences between variable and fixed-rate scenarios.

Another issue which is crucial for the results in this particular case is the shock persistence. In figure 4 we see that a more persistent shock, such as a technology shock delivers larger aggregate differences between the two countries when the structure of mortgage contracts differs among them. In particular, we see strong differences in the behavior of housing demand and house prices across countries. Total consumption reacts by more in the fixed-rate case due to the procyclicality of the real interest rate. Variable-rate borrowers consume less because increase in real rate affects them negatively. However, fixed-rate consumers are better off in comparison and they can consume more. In Rubio (2009) it is also

Figure 3: Impulse Responses to a Monetary Policy Shock in a Monetary Union.
3.3 Asymmetric Shocks

3.3.1 House Price Shock

An important source of asymmetry within Europe is the house price evolution. In this framework I can study how asymmetric house price shocks are transmitted across countries. In a closed economy, a positive house price shock increases the value of the collateral, and total consumption increases, mainly due to the increase in consumption of borrowers. However, in an open economy, a country-specific house price shock can be transmitted internationally to other countries. If that were the case, the divergence caused by an asymmetric shock would be alleviated. Figure 5 shows the effects of a house price shock in Country A. Consumption in this country increases initially because of wealth effects. Housing demand by borrowers also increases. However, this asymmetric shock is slightly transmitted to Country B where consumption also increases because the countries are trading. Interest rates, especially in the case that aggregate differences increase with the persistence of the shock.
union, decrease and this makes house prices in Country B increase as well, giving an extra impulse to consumption in this country. Housing prices show a higher correlation across countries in the monetary union setting.

4 Welfare Analysis

We have seen that the transmission of shocks in a monetary union when there is housing market heterogeneity differs across countries. However, a remaining question is whether these countries should homogenize their structures or not. Maclellan et al. (1998) argue that countries in a monetary union should make an effort towards institutional homogenization in their housing markets. In this section I use welfare analysis to study whether this is always the case. In particular, I study if countries converging in their degree of credit accessibility and type of mortgage contracts would be beneficial. Here I focus on these two aspects because, although I consider them to be exogenous, they are the ones that are
most related to institutional features of the economy. Changes in regulation or recommendations of the central bank could make these parameters change.

To address these questions, I numerically evaluate how cross-country asymmetries affect welfare for a given policy rule and for common technology shocks. As discussed in Benigno and Woodford (2008), the two approaches that are recently used for welfare analysis in DSGE models include either characterizing the optimal Ramsey policy, or solving the model by using a second-order approximation to the structural equations for given policy and then evaluating welfare using this solution. As in Mendicino and Pescatori (2007) and Rubio (2009), I take this latter approach to be able to evaluate the welfare of the three types of agents separately. The individual welfare for savers and borrowers in Country A, respectively, is defined as follows:

\[
V_{u,t} = E_t \sum_{m=0}^{\infty} \beta^m \left( \ln C^u_{t+m} + j_t \ln H^u_{t+m} - \left( \frac{L^u_{t+m}}{\eta} \right)^{\eta} \right),
\]

\[
V_{cv,t} = E_t \sum_{m=0}^{\infty} \tilde{\beta}^m \left( \ln C^{cv}_{t+m} + j_t \ln H^{cv}_{t+m} - \left( \frac{L^{cv}_{t+m}}{\eta} \right)^{\eta} \right),
\]

\[
V_{cf,t} = E_t \sum_{m=0}^{\infty} \tilde{\beta}^m \left( \ln C^{cf}_{t+m} + j_t \ln H^{cf}_{t+m} - \left( \frac{L^{cf}_{t+m}}{\eta} \right)^{\eta} \right).
\]

Following Mendicino and Pescatori (2007), I define social welfare in Country A as a weighted sum of the individual welfare for the different types of households:

\[
V_t = (1 - \beta) V_{u,t} + \left( 1 - \tilde{\beta} \right) \left[ \alpha_A V_{cv,t} + (1 - \alpha_A) V_{cf,t} \right].
\]

Borrowers and savers’ welfare are weighted by \( 1 - \tilde{\beta} \) and \( (1 - \beta) \) respectively, so that the two groups receive the same level of utility from a constant consumption stream. Everything is symmetric for Country B.

Total welfare is defined as a weighted sum of the welfare in the two countries:

\[
W_t = nV_t + (1 - n) V_t^*.
\]

Welfare analysis in models with collateral constraints is a bit more complicated than in the standard

---

33See Monacelli (2006) for an example of the Ramsey approach in a model with heterogeneous consumers.
sticky-price model. First of all, in this kind of economy with borrowers and savers, there always appears a trade-off between the welfare of each agent. Total welfare is an aggregation of these two opposing forces that sometimes depends on parameterization. Furthermore, this economy has two distortions: the sticky price distortion and the collateral constraint distortion. Therefore, some features than in the standard model would deliver lower welfare, have the opposite outcome under this setting. For instance, Monacelli (2006), Mendicino and Pescatori (2007), and Rubio (2009) show that in a model with collateral constraints, the central bank should not find so aggressively against inflation because inflation helps relaxing the credit market inefficiency.

In order to gain some intuition on how welfare in this model depends on the parameters considered, I present figure 6.\textsuperscript{34} In this experiment I consider the two countries being identical and I evaluate welfare at the borrower, saver and country level. The monetary union welfare is the same as the country welfare because countries are symmetric in this case. The upper plot shows how welfare changes with the loan-to-value ratio while the lower plot displays welfare with respect to the degree of variability of the mortgage rate. In both subplots we can clearly see the trade-off between the welfare of borrowers and savers. In the first case, we see that the welfare of the borrowers decreases when the loan-to-value ratio increase, especially for high values of this parameter. On the contrary, savers’ welfare increase.

\textsuperscript{34}Notice that throughout all the graphs welfare has been rescaled for showing purposes.
This may seem counterintuitive because a high LTV relaxes the borrowing constraint. However, this is a result which has been already found in similar models. Campbell and Hercowitz (2009) perform a welfare analysis in a DSGE model with borrowers and savers and obtain that although high LTV ratios have a direct positive effect on welfare through the constraint relaxation, there may be other indirect effects that dominate. Notice that $k$ is a parameter that strongly affects the collateral constraint. A small change in this parameter can cause very large changes in borrowing that can be excessive. Higher LTVs lead to higher consumption levels because borrowing constraints are always binding; the more borrowers are offered, the more they take. But this, in turn, as shown in Campbell and Hercowitz (2009), changes relative prices. In particular, higher consumption levels imply higher interest rates. This could lead to a situation of overindebteness in the sense that high repayments could offset the positive effects on the constraint relaxation. Smith (2009) shows that these results do not rely on Campbell and Hercowitz (2009) specific assumptions, even in the simplest model with borrowers, savers and collateral constraints this effect takes place.\footnote{Smith (2009) shows that these results do not rely on Campbell and Hercowitz (2009) specific assumptions, even in the simplest model with borrowers, savers and collateral constraints this effect takes place.} For aggregate welfare, it is also the case that welfare decreases with higher LTVs.\footnote{For aggregate welfare, it is also the case that welfare decreases with higher LTVs.}

The lower plot in figure 6 displays how welfare evolves when we increase the proportion of variable rates in the economy. In this case, it is also clear how the welfare of the two agents move in different directions. Borrowers prefer fixed-rate contracts, as opposed to savers, who prefer variable rates. Even though under fixed interest rates the economy is losing a policy tool, this type of contracts reduce one of the distortions of the economy, and this enhances welfare for borrowers. Equation (66), the log-linearized collateral constraint in the appendix, shows how debt repayments in real terms decrease with the value of $\alpha$ and this relaxes the constraint. The difference with the previous case is that reducing $\alpha$ does not have such a strong impact on borrowing as increasing $k$. It helps borrowers lowering debt repayments in real terms but without causing overindebteness (notice that the steady-state value of borrowing does not depend on $\alpha$ but does depend on $k$). However, although fixed rates make the collateral constraint less tight for borrowers, this comes at the expense of savers, who have to bear all the risk associated with interest rate variability. For this parameterization, the welfare improvement for borrowers compensates it and also aggregate welfare increases with fixed-rate mortgages.\footnote{Rubio (2009) has a detailed discussion on this issue.}
geneity in the LTV ratio. In this experiment, Country A represents the majority of countries of the monetary union, whose LTV is 0.8. The upper plot shows how welfare changes when Country B, a small country in the union, deviates from the union LTV. Country B’s welfare is lower the higher the value of \( k \) is. Since the two countries are sharing financial and trade links, Country A can slightly benefit from the welfare loss. At the aggregate level, we see that the whole union is worse off when one of its components moves towards high LTVs. The lower plot compares welfare for the whole monetary union when countries are homogeneous in their LTVs with a situation in which Country B deviates from the common LTV (notice that, for convenience, the scale of the two plots is different). This graph permits us to assess whether countries in a monetary union should converge in their LTVs. Results show that heterogeneity in LTVs is not necessarily welfare worsening. I find that homogenization does not deliver the best outcome if both countries have value of \( k \) which is too high. Homogenization improves welfare only if countries move towards low LTVs.

In figure 8, I run a similar experiment but this time with respect to the variability of mortgage rates. Country A represents a monetary union in which consumers borrow at a fixed rate. I evaluate what happens in terms of welfare if Country B increases its proportion of variable-rate mortgages. When Country B deviates from having fixed-rate mortgages, as the rest of the union, its welfare decreases. Welfare in the other countries slightly increases but for the monetary union as a whole, welfare decreases.
when Country B increases its proportion of variable-rate mortgages. The question now is whether countries within the union should be homogeneous in their mortgage contracts. The lower plot shows that in this case, homogenization towards fixed-rate mortgages is welfare improving. For mortgage contracts, the heterogeneity by itself is not welfare worsening, it is having variable rates what makes the economy worse off. In fact, moving to a scenario in which both countries have variable-rate mortgages is worse than being heterogeneous. This result has important policy implications. It suggests that countries such as Spain or the United Kingdom (if it entered the EMU) should increase their proportion of fixed-rate contracts.

Table 1 presents a summary of welfare values for the same examples I used in simulations. The first column displays welfare values for the baseline case, a symmetric case in which the loan-to-value ratio is 0.8, the saver’s labor income share is 0.7 and mortgage rates are variable in the whole union. For the sake of comparison, I also consider other symmetric cases in which LTVs are low and mortgage rates are fixed. Finally, I consider two asymmetric cases in which first, the small country has a low LTV and second, it has variable-rate mortgages as opposed to the rest of the union.
### Table 1: Welfare values for symmetric and asymmetric cases

<table>
<thead>
<tr>
<th></th>
<th>Symmetric</th>
<th>Asymmetric</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare</td>
<td>Baseline $k_A = k_B = .2 \alpha_A = \alpha_B = 0$</td>
<td>$k_A = .8/k_B = .2 \alpha_A = 0/\alpha_B = 1$</td>
</tr>
<tr>
<td>Social A</td>
<td>$-3.486$  $-3.087$ $-1.476$</td>
<td>$-3.746$  $-1.067$</td>
</tr>
<tr>
<td>Savers A</td>
<td>$-116.53$ $-136.66$ $-469.86$</td>
<td>$-144.31$ $-447.89$</td>
</tr>
<tr>
<td>Borr A</td>
<td>$-116.05$ $-86.02$ $161.08$</td>
<td>$-115.14$ $170.57$</td>
</tr>
<tr>
<td>Social B</td>
<td>$-3.486$  $-3.087$ $-1.476$</td>
<td>$-0.817$  $-6.547$</td>
</tr>
<tr>
<td>Savers B</td>
<td>$-116.53$ $-136.66$ $-469.86$</td>
<td>$117.71$ $-392.47$</td>
</tr>
<tr>
<td>Borr B</td>
<td>$-116.05$ $-86.02$ $161.08$</td>
<td>$-99.73$ $-131.13$</td>
</tr>
<tr>
<td>Total</td>
<td>$-3.486$  $-3.087$ $-1.476$</td>
<td>$-3.453$  $-1.615$</td>
</tr>
</tbody>
</table>

### 5 Concluding Remarks

This paper explores how cross-country housing market heterogeneity affects the transmission of shocks and welfare in a monetary union. Since there is clear evidence of such heterogeneity across countries in Europe, it is relevant to evaluate to what extent this is important. Some normative conclusions regarding whether housing market homogenization is desirable are also presented.

For this purpose, I build a two-country DSGE model that features a housing market. A group of individuals in each country are credit constrained and need housing collateral to obtain loans. I consider two monetary regimes: the two countries conducting its own monetary policy under a flexible exchange rate system and a monetary union. I allow for countries to be heterogeneous in their LTVs, the labor-income share of each group and their mortgage contracts. House-price shocks can also be country specific.

I find that after a monetary policy shock, variables respond more strongly if the country has a high LTV, a high proportion of borrowers or mainly variable-rate mortgages. As for house price shocks, I find that the effects of a house price shock in one country are slightly transmitted to the other country, especially in the monetary union regime.

The recommendation that European countries should move towards institutional homogenization, particularly with respect to housing markets, is often heard. I perform welfare analysis to explore under which conditions this is the case. From a normative perspective, I find that housing market homogeniza-
tion *per se* is not necessarily beneficial. In line with recent studies, homogenization towards high LTVs decreases welfare, indirect effects dominate the direct effect of relaxing the borrowing constraints. As for mortgage contracts, results suggest that countries with predominantly variable-rate contracts should move towards fixed-rate contracts because they reduce the distorting effects of the collateral constraint without causing excessive borrowing.

This paper could serve as a basis for numerous extensions. One of the features of this kind of models is that borrowing constraints are always binding and the same agents are always constrained. An overlapping generations version of the model could deal with this issue and it would also allow to model mortgage contracts in a more realistic way. The introduction of an additional sector which produces houses or a rental market would also permit to study other relevant topics which were not the focus of the present paper. For future research, it would be also interesting to study what is the optimal monetary policy under the different sources of asymmetry or take a step towards estimation.

**References**


Appendix 1: Tables

<table>
<thead>
<tr>
<th>Loan-to-value ratios in European Countries (2006)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>France</td>
</tr>
<tr>
<td>Sweden</td>
</tr>
<tr>
<td>Denmark</td>
</tr>
<tr>
<td>Spain</td>
</tr>
<tr>
<td>Italy</td>
</tr>
<tr>
<td>United Kingdom</td>
</tr>
</tbody>
</table>

Source: European Mortgage Federation (Factsheets)

Table 2

<table>
<thead>
<tr>
<th>Predominant Type of Mortgage Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
</tr>
<tr>
<td>Austria</td>
</tr>
<tr>
<td>France</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>Greece</td>
</tr>
</tbody>
</table>


Table 3

38 Average LTV for all buyers. For France, first-time buyers in 2004.
### Residential Debt to GDP Ratio (2006)

<table>
<thead>
<tr>
<th>Country</th>
<th>Belgium</th>
<th>Italy</th>
<th>Spain</th>
<th>France</th>
<th>EU 27</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>36.3%</td>
<td>18.7%</td>
<td>58.6%</td>
<td>32.2%</td>
<td>49.0%</td>
</tr>
<tr>
<td>Denmark</td>
<td>100.8%</td>
<td>98.4%</td>
<td>23.5%</td>
<td>29.3%</td>
<td>81.1%</td>
</tr>
<tr>
<td>Germany</td>
<td>51.3%</td>
<td>23.5%</td>
<td>51.3%</td>
<td>32.2%</td>
<td>100.8%</td>
</tr>
<tr>
<td>Greece</td>
<td>29.3%</td>
<td>83.1%</td>
<td>29.3%</td>
<td>23.5%</td>
<td>100.8%</td>
</tr>
<tr>
<td>Spain</td>
<td>58.6%</td>
<td>56.7%</td>
<td>58.6%</td>
<td>58.6%</td>
<td>100.8%</td>
</tr>
<tr>
<td>Sweden</td>
<td>56.7%</td>
<td>56.7%</td>
<td>56.7%</td>
<td>56.7%</td>
<td>100.8%</td>
</tr>
</tbody>
</table>

Source: European Mortgage Federation

Table 4

### House Price % Change in European Countries

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2002</th>
<th>2003</th>
<th>2004</th>
<th>2005</th>
<th>2006</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>5.1</td>
<td>7.1</td>
<td>7.2</td>
<td>5.5</td>
<td>16.3</td>
<td>12.1</td>
</tr>
<tr>
<td>Denmark</td>
<td>7.9</td>
<td>5.3</td>
<td>5.2</td>
<td>11.7</td>
<td>17.0</td>
<td>16.2</td>
</tr>
<tr>
<td>Germany</td>
<td>0.0</td>
<td>-1.6</td>
<td>-0.8</td>
<td>-0.8</td>
<td>-1.7</td>
<td>-0.9</td>
</tr>
<tr>
<td>Greece</td>
<td>14.6</td>
<td>13.0</td>
<td>5.7</td>
<td>5.2</td>
<td>13.1</td>
<td>9.0</td>
</tr>
<tr>
<td>Spain</td>
<td>9.9</td>
<td>15.7</td>
<td>17.6</td>
<td>17.4</td>
<td>13.9</td>
<td>10.4</td>
</tr>
<tr>
<td>France</td>
<td>8.1</td>
<td>9.0</td>
<td>11.5</td>
<td>17.6</td>
<td>14.7</td>
<td>9.9</td>
</tr>
<tr>
<td>Ireland</td>
<td>8.0</td>
<td>3.6</td>
<td>14.1</td>
<td>11.2</td>
<td>10.6</td>
<td>13.6</td>
</tr>
<tr>
<td>Italy</td>
<td>7.9</td>
<td>10.0</td>
<td>10.7</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Latvia</td>
<td>n/a</td>
<td>14.0</td>
<td>17.5</td>
<td>4.9</td>
<td>48.6</td>
<td>n/a</td>
</tr>
<tr>
<td>Hungary</td>
<td>8.6</td>
<td>-1.1</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Poland</td>
<td>10.0</td>
<td>-4.2</td>
<td>-6.9</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>Sweden</td>
<td>8.0</td>
<td>6.3</td>
<td>6.6</td>
<td>9.6</td>
<td>9.6</td>
<td>11.4</td>
</tr>
<tr>
<td>UK</td>
<td>8.4</td>
<td>17.0</td>
<td>15.7</td>
<td>11.8</td>
<td>5.5</td>
<td>6.3</td>
</tr>
</tbody>
</table>

Source: European Mortgage Federation

Table 5
Appendix 2:

The Monetary Union Case

Unconstrained consumers in Country A:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln C_t^u + j_t \ln H_t^u - \frac{(L_t^u)^{\eta}}{\eta} \right),$$

subject to:

$$C_{At}^u + P_{Bt} C_{Bt}^u + q_t H_t^u + \frac{R_{At-1} b_{t-1}^u}{\pi_{At}} + \frac{R_{t-1} d_{t-1}}{\pi_{At}} + \frac{\psi}{2} d_t^2 \leq q_t H_t^u + w_t^u L_t^u + b_t^u + d_t + F_t + S_t,$$

where $R_t$ is an international interest rate. The non-arbitrage condition between home and foreign bonds implies now that

$$R_{At} = \frac{R_t}{(1 - \psi d_t)}.$$  \hspace{1cm} (39)

The equations for consumers in Country B are symmetric. The problem for the firms and the financial intermediary in each country is identical to the non-monetary union case.

The Taylor Rule becomes:

$$R_t = (R_{t-1})^\rho \left( \left[ (\pi_{At})^n (\pi_{Bt})^{1-n} \right]^{(1+\phi_y)} \left[ \left( \frac{Y_{At}}{Y_{At-1}} \right)^n \left( \frac{Y_{Bt}}{Y_{Bt-1}} \right)^{1-n} \right]^{\phi_y} R \right)^{1-\rho} \varepsilon_{R,t},$$ \hspace{1cm} (40)

Steady-State Relationships

Relative prices in the steady state are derived from equations (4), (15) and their counterparts for Country B:

$$\frac{n}{1-n} \frac{P_B}{P_A} = \frac{C_{A}^n}{C_{A}^c} = \frac{C_{A}^c}{C_{B}^c} = \frac{C_{A}^{uu*}}{C_{B}^{uu*}} = \frac{C_{A}^{cc*}}{C_{B}^{cc*}}$$ \hspace{1cm} (41)

Interest rates:

$$R_A = R = R_B = \bar{R} = \bar{R}^* = 1/\beta$$ \hspace{1cm} (42)

We can find the consumption to housing ratio for savers and borrowers in Country A by using the first order conditions for housing:
\[
\frac{C_A^u}{qH^u*} = \frac{n}{j} (1 - \beta) \tag{43}
\]

\[
\frac{C_A^c}{qH^c} = \frac{n}{j} \left[ \left(1 - \bar{\beta}\right) - k_A \left(\beta - \bar{\beta}\right) \right] = \frac{n}{j} \zeta \tag{44}
\]

Similarly, for Country B:

\[
\frac{C_B^u*}{q^*H^{u*}} = \frac{(1 - n)}{j^*} (1 - \beta) \tag{45}
\]

\[
\frac{C_B^c*}{q^*H^{c*}} = \frac{(1 - n)}{j^*} \left[ \left(1 - \bar{\beta}\right) - k_B \left(\beta - \bar{\beta}\right) \right] = \frac{(1 - n)}{j^*} \zeta^* \tag{46}
\]

Borrowing in the steady state:

\[
b^c = \beta k_AqH^c. \tag{47}
\]

\[
b^u + b^c = 0
\]

\[
b^c* = \beta k_Bq^*H^{c*}. \tag{48}
\]

\[
b^{u*} + b^{c*} = 0
\]

From the problem of the firm we have that in the steady state:

\[
w^u = \frac{1}{X} \frac{Y_A}{L^u} \tag{49}
\]

\[
w^c = \frac{1}{X} \left(1 - \gamma\right) \frac{Y_A}{L^c}, \tag{50}
\]

\[
w^{u*} = \frac{1}{X^*} \gamma \frac{Y_B}{L^{u*}}, \tag{51}
\]
\[ w^c = \frac{1}{X} (1 - \gamma) \frac{Y_B}{L^c}, \quad (52) \]

where \( X = X^* = \frac{\varepsilon - 1}{\varepsilon} \).

Combining the steady-state budget constraint for the unconstrained consumers in Country A with (43) and (49) we obtain:

\[ \frac{C^u_A}{Y_A} = \frac{n (\gamma + X - 1)}{X (1 - jk_A)} \quad (53) \]

Similarly, for constrained consumers:

\[ \frac{C^c_A}{Y_A} = \frac{1 - \gamma}{X} \frac{\zeta n}{\zeta + jk_A (1 - \beta)} \quad (54) \]

The market clearing condition for the good produced in Country A implies:

\[ \frac{C^*_A}{Y_A} = \frac{n}{1 - n} \left( 1 - \frac{C^u_A}{Y_A} - \frac{C^c_A}{Y_A} \right) \]

Using (43) and (53) we can find the housing to output ratio for the savers in Country A:

\[ \frac{H^u}{Y_A} = \frac{j (\gamma + X - 1)}{X q (1 - jk_A) (1 - \beta)} \quad (55) \]

Analogously, using (44) and (54) we can find the housing to output ratio for the constrained consumers in Country A:

\[ \frac{H^c}{Y_A} = \frac{(1 - \gamma) j}{X q} \frac{n}{\zeta + jk_A (1 - \beta)} \quad (56) \]

Similarly, for Country B:

\[ \frac{C^u_B}{Y_B} = \frac{(1 - n) (\gamma + X^* - 1)}{X^* (1 - j^*k_B)} \quad (57) \]

\[ \frac{C^c_B}{Y_B} = \frac{1 - \gamma}{X^*} \frac{\zeta (1 - n)}{\zeta + j^*k_B (1 - \beta)} \quad (58) \]

\[ \frac{H^uB}{Y_B} = \frac{j^* (\gamma + X^* - 1)}{X^* q^* (1 - j^*k_B) (1 - \beta)} \quad (59) \]
\[
\frac{H^c}{Y_B} = \frac{(1 - \gamma) j^*}{X^* q^*} \frac{(1 - n)}{\zeta + j^* k_B (1 - \beta)}
\]  

(60)

Log-linearized Equations

Variables in deviations from the steady state are expressed in lower-case and with a hat.

**Interest Rates**

\[
\hat{r}_{At} = \hat{r}_{Bt} + E_t (\hat{e}_{l+1} - \hat{e}_t) + \psi,
\]  

(61)

\[
\hat{r}_{At} = \hat{r}_{Bt} = 0.
\]  

(62)

**Aggregate Demand**

\[
\widehat{c}_{At} = E_t \widehat{c}_{At+1} - (\hat{r}_{At} - E_t \hat{\pi}_{At+1}),
\]  

(63)

\[
\widehat{c}_{Bt} = E_t \widehat{c}_{Bt+1} - (\hat{r}_{Bt} - E_t \hat{\pi}_{Bt+1}),
\]  

(64)

\[
\widehat{c}_{At} = \left(\frac{\zeta + j k_A (1 - \beta)}{\zeta}\right) \left(\hat{y}_{At} + \hat{\pi}_t - \hat{x}_t\right) - \frac{j}{\zeta} \left(\hat{h}_{t}^{\zeta} - \hat{h}_{t-1}^{\zeta}\right) + \frac{k A j}{\zeta} \left(\beta \hat{b}_{t}^{\gamma} - \hat{b}_{t-1}^{\gamma}\right) - k A j \left(\alpha A \hat{r}_{At-1} - \hat{\pi}_{At}\right),
\]  

(65)

\[
\hat{b}_{t}^{c} = E_t \hat{q}_{t+1} + \hat{h}_t^{c} - (\alpha A \hat{r}_{At} - E_t \hat{\pi}_{At+1}),
\]  

(66)

\[
\hat{c}_{Bt}^{*} = \left(\frac{\zeta^* + j^* k_B (1 - \beta)}{\zeta^*}\right) \left(\hat{y}_{Bt} + \hat{\pi}_t^* - \hat{x}_t\right) - \frac{j^*}{\zeta^*} \left(\hat{h}_{t}^{\zeta^*} - \hat{h}_{t-1}^{\zeta^*}\right) + \frac{k B j^*}{\zeta^*} \left(\beta \hat{b}_{t}^{\gamma^*} - \hat{b}_{t-1}^{\gamma^*}\right) - k B j^* \left(\alpha B \hat{r}_{Bt-1} - \hat{\pi}_{Bt}\right),
\]  

(67)

\[
\hat{b}_{t}^{c*} = E_t \hat{q}_{t+1}^* + \hat{h}_t^{c*} - (\alpha B \hat{r}_{Bt} - E_t \hat{\pi}_{Bt+1}),
\]  

(68)
\[ \hat{c}_{At} - \hat{c}_{Bt} = \hat{c}^*_A - \hat{c}^*_B \]  

(69)

**Housing Equations**

\[ \hat{h}^u_t = \frac{1}{1 - \beta} (\hat{c}^u_{At} - \hat{q}_t) - \frac{\beta}{1 - \beta} E_t (\hat{c}^u_{At+1} - \hat{q}_{t+1}) , \]  

(70)

\[ \hat{h}^c_t = \frac{1}{1 - \beta} (\hat{c}^c_{Bt} - \hat{q}_t) - \frac{\beta}{1 - \beta} E_t (\hat{c}^c_{Bt+1} - \hat{q}_{t+1}) , \]  

(71)

\[ \hat{h}^c_t = \frac{1 - k_A}{\zeta} \hat{c}^c_t - \frac{1}{\zeta} \hat{q}_t - \frac{k_A}{\zeta} (\alpha_A \hat{r}_{At} - E_t \hat{\pi}_{At+1}) + \frac{\beta}{\zeta} E_t \hat{q}_{t+1} - \frac{\beta (1 - k_A)}{\zeta} E_t \hat{c}^c_{t+1} . \]  

(72)

\[ \hat{h}^c_t = \frac{1 - k_B}{\zeta^*} \hat{c}^c_t - \frac{1}{\zeta^*} \hat{q}_t^* - \frac{k_B}{\zeta^*} (\alpha_B \hat{r}_{Bt} - E_t \hat{\pi}_{Bt+1}) + \frac{\beta}{\zeta^*} E_t \hat{q}_{t+1}^* - \frac{\beta (1 - k_B)}{\zeta^*} E_t \hat{c}^c_{t+1} . \]  

(73)

**Aggregate Supply**

\[ \hat{y}_{At} = \frac{\eta + 1}{\eta - 1} \hat{c}_t - \frac{1}{\eta - 1} (\gamma \hat{c}^u_{At} + (1 - \gamma) \hat{c}^c_{At} + \hat{x}_t) , \]  

(74)

\[ \hat{y}_{At} = \left( C^u_A + C^c_A \right) \hat{c}_t + \left( 1 - \frac{C^u_A}{Y_A} - \frac{C^c_A}{Y_A} \right) \hat{c}^c_{At} \]  

(75)

\[ \hat{y}_{Bt} = \frac{\eta + 1}{\eta - 1} \hat{c}_t^* - \frac{1}{\eta - 1} (\gamma \hat{c}^u_{Bt} + (1 - \gamma) \hat{c}^c_{Bt} + \hat{x}_t^*) , \]  

(76)

\[ \hat{y}_{Bt} = \left( C^u_B + C^c_B \right) \hat{c}_t^* + \left( 1 - \frac{C^u_B}{Y_B} - \frac{C^c_B}{Y_B} \right) \hat{c}^c_{Bt} , \]  

(77)

\[ \hat{\pi}_{At} = \beta \hat{\pi}_{At+1} - \bar{k} \hat{x}_t + u_{At} , \]  

(78)

\[ \hat{\pi}_{Bt} = \beta \hat{\pi}_{Bt+1} - \bar{k} \hat{x}_t^* + u_{Bt} , \]  

(79)

where \( \bar{k} = \frac{(1 - \theta)(1 - \beta)}{\theta} \) and \( u_{At} \) and \( u_{Bt} \) are cost-push shocks.
Monetary Policy

\[ \hat{r}_{At} = \rho A \hat{r}_{At-1+} (1 - \rho A) \left[ (1 + \phi_{At}) \hat{\pi}_{At} + \phi_{Ay} \hat{y}_{At} \right] + \hat{\epsilon}_{AR,t}, \]  

(80)

\[ \hat{r}_{Bt} = \rho B \hat{r}_{Bt-1+} (1 - \rho B) \left[ (1 + \phi_{Bt}) \hat{\pi}_{Bt} + \phi_{By} \hat{y}_{Bt} \right] + \hat{\epsilon}_{BR,t}, \]  

(81)

Notice that under the monetary union regime (80) and (81) become:

\[ \hat{r}_t = \rho \hat{r}_{t-1+} (1 - \rho) \left\{ (1 + \phi_{At}) \left[ n \hat{\pi}_{At} + (1 - n) \hat{\pi}_{Bt} \right] + \phi_{y} \left[ n \hat{y}_{At} + (1 - n) \hat{y}_{Bt} \right] \right\} + \hat{\epsilon}_{R,t} \]  

(82)