Capital, Endogenous Separations, and the Business Cycle*

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December 29, 2009

Abstract

We implement capital in an endogenous separations New Keynesian matching model. In contrast to the vintage capital theory, we suggest a more general approach, such that workers have unrestricted access to a proportional share of the capital stock. We find that the introduction of capital generates an important channel for the transmission of aggregate productivity shocks, using a capital-labor trade-off. The model generates higher volatilities of key variables and therefore enhances the performance of the matching model to generate stylized facts in response to an aggregate productivity shock. However, there is almost no difference for monetary policy shocks.

*Keywords:* Capital, Endogenous Separations, Search and Matching.

*JEL classification:* E22, E32, J64.

*We would like to thank Larry Christiano, Ester Faia, Christian Merkl and Céline Poilly for highly valuable comments. In addition, we thank seminar participants at the Kiel Institute for the World Economy and the University of Pavia.*
1. Introduction

The ability of search and matching models to replicate stylized facts has been assessed by various authors. Shimer (2005) in his seminal contribution finds that the standard search and matching model with exogenous separations within a RBC partial equilibrium context is not able to replicate the fluctuations of key labor market variables. A promising solution approach was introduced by Hall (2005) implementing a real wage rigidity, increasing the surpluses of the firm over the cycle and increasing the response of vacancies. However, it appears that real wage rigidity on the one hand is not able to significantly improve the model’s performance, as shown by Krause and Lubik (2007). Additionally, there is evidence that the wage of new entrants is not rigid as shown by Haeck et al. (2009). A different solution proposed by Hagedorn and Manovskii (2008) sets the value of leisure close to the wage and the worker’s bargaining power close to zero. Therefore, firms relative profits are heavily affected from productivity changes, hence increasing the volatility. However, their approach shows an unrealistic high sensitivity of the unemployment rate with respect to unemployment benefit changes. It appears that a satisfying solution has not been found yet.

As proposed by Hornstein et al. (2005), the introduction of capital might be an important feature that has been rather neglected in the search and matching RBC and NKM literature. Capital, and to be more precisely the cost of investment, add an additional variable to the firm’s decision problem. Fluctuations along this margin should also have an impact on the labor market decisions of the firm, creating a capital-labor trade-off for the firm. In contrast to the recent literature, we do not assume that there is heterogeneity in matches, since we allow unrestricted access to capital and technology to all matches at any time. Therefore, we deviate from the vintage capital literature.\footnote{See e.g. Boucekkine et al. (2006), Eyigungor (2006) or Hornstein et al. (2007).} The vintage approach is based upon the seminal contribution from Caballero and Hammour (1991), assuming that due to the process of innovation jobs that contain the newest technologies are created, whereas outdated jobs are destructed. Along this line, Michelacci and Lopez-Salido (2007) and Costain and Reiter (2008) introduce the vintage theory to matching models in a RBC context with exogenous separations. Our approach has to be understood as a more general case of modelling capital and consistently as a starting point in the process of understanding the relevance of capital in
matching models. In the following, we build a New Keynesian DSGE model with search frictions, capital and purely endogenous separations. Although there is no consensus in the literature on the proper determination of the separation margin, following Fujita and Ramey (2007, 2008) and Ramey (2008) empirical evidence seems to favor endogenous separations. Balleer (2009) shows that the separation rate increases after a positive technology shock and that the standard model generates the volatility of these variables conditional on technology shocks. In addition, Barnichon (2009) finds that around business cycle turning points the separation rate is causative for most of unemployment movements.

Firms rent any desired quantity of capital on a frictionless capital market as in Pissarides (2000). We evaluate the performance along the labor market dimension. For this purpose, and to have a transparent judgement basis for later purpose, we scrutinize U.S. data and perform a statistical analysis. Our findings are presented in Table 1.

- Table 1 about here -

The main findings can be summarized as follows

- **Unemployment and Vacancies**
  Unemployment is almost 9 times as volatile as productivity, while vacancies are even 10 times as volatile. Labor market tightness is even 19 times as large as labor productivity, while the correlation between unemployment and vacancies is strongly negative.

- **Job Finding Rates**
  The job finding rate is about twice as volatile as labor productivity, strongly autocorrelated and pro-cyclical.

- **Job Creation and Destruction Rates**
  The job creation and destruction rates are negatively correlated (-0.36) and show much smaller standard deviations than unemployment or vacancies.

We show that the introduction of capital adds an important channel for the transmission of aggregate productivity shocks. We create the empirical values of

\(^2\)To be consistent with the findings from Shimer (2005), we apply his methodology.

\(^3\)Values for the job creation and destruction rates are based on Krause and Lubik (2007), using HP filtered data from 1964:Q1 to 2002:Q3.
standard deviations for unemployment and vacancies in response to an aggregate productivity shock. However, we find that the transmission channel is less important for the propagation of monetary shocks.

The remainder is organized as follows. In the next section we derive the model for later analysis. In section 3 we calibrate and close the model while section 5 compares the performance of the capital model with a standard search and matching model. In section 6 we finally draw the conclusion.

2. A Matching Model with Capital

In this section, we present a New Keynesian model with endogenous separations, search frictions and capital. Households maximize consumption by choosing the optimal consumption path of a CES aggregate of differentiated products and make the investment decision. Firms, acting on a monopolistically competitive market, maximize profits by setting prices - time-dependent pricing à la Rotemberg (1982) - and choosing the optimal levels of employment and capital subject to price adjustment costs, hiring costs and capital adjustment costs. Separations are driven by job-specific productivity shocks affecting new and old jobs generating a flow of worker. In addition, the monetary authority targets the nominal interest rate by a standard Taylor-rule.

2.1. Consumer Preferences

We assume a discrete-time economy with an infinite living representative household who makes its investment decision and seeks to maximize its utility given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma} - 1}{1 - \sigma} \right]$$

(1)

where $E$ is the expectation operator, $C_t = \left[ \int_0^1 C_{it} \frac{d\epsilon}{\epsilon} \right]^{\frac{1}{1-\sigma}}$ is the Dixit-Stiglitz aggregator, $\beta$ is the discount factor, and the degree of risk aversion is given by $\sigma$. We assume that a household consists of a continuum of members, inelastically supplying one unit of labor and being represented by the unit interval. In addition, and following Merz (1995), Andolfatto (1996), and Poilly and Sahuc
(2008), we assume consumption pooling. The household maximizes consumption subject to the intertemporal budget constraint

\[ C_t + I_t + \frac{B_t}{P_t} = R_{t-1} \frac{B_{t-1}}{P_t} + W_t + bu_t + T_t + \Pi_t + r^K_t K_t, \]

(2)

where \( b \) is the value of home production, such that \( bu_t \) accordingly is the income of unemployed household members. \( W_t \) is labor income, \( B_t \) is bond holding which pays a gross interest rate \( R_t \). \( \Pi_t \) are aggregate profits and \( T_t \) are real lump sum transfers from the government. The household owns the capital stock and rents it to the firm, such that earnings from providing capital are given by \( r^K_t K_t \), where \( r^K_t \) is the rental rate. It is derived from the corresponding cost minimization problem of the firm. The capital stock accumulates according to

\[ K_{t+1} = (1 - \delta) K_t + \left( 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t, \]

(3)

where \( \delta \) is the depreciation rate, \( I_t \) corresponds to investment and \( S \left( \frac{b}{I_{t-1}} \right) \) is a function which describes investment adjustment costs as in Christiano et al. (2005).\(^4\) The first-order condition with respect to investment is given by

\[ 1 = Q_t \Phi_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right] + \beta E_t \left[ \Lambda_{t+1} Q_{t+1} \Phi_{t+1} S' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_{t+1}}{I_t} \right) \right], \]

(4)

where Tobin’s Q is the ratio of the Lagrangian multipliers attached to equations (2) and (3), i.e.

\[ Q_t = \beta E_t \left[ \Lambda_{t+1} r^K_{t+1} + Q_{t+1}(1 - \delta) \right], \]

(5)

where \( \Lambda_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \) is the stochastic discount factor. By minimizing total expenditures, we obtain the demand function \( C_{it} = \left( \frac{P_i}{P_t} \right)^{-\epsilon} C_i \) and by solving the households maximization problem, we obtain the standard Euler equation for intertemporal consumption flows, i.e.

\(^4\)In steady state, \( S(\cdot) \) satisfies \( S = 0, S' = 0 \) and \( S'' > 0 \).
\[ C^\tau_t = \beta R_t E_t \left[ \frac{P_t}{P_{t+1}} C_{t+1}^\tau \right]. \] (6)

### 2.2. Entry and Exit Site

The firm searches for workers on a discrete and closed market. Trade in the labor market is uncoordinated, costly and time-consuming. Therefore, labor market frictions are modelled via a Cobb-Douglas type matching function with constant returns to scale, i.e.\[ \Psi(u_t, v_t) = m(u_t)^\mu (v_t)^{1-\mu}. \] (7)

The function gives the number of new employment relationships at the beginning of the next period. Where \( u_t \) is the number of unemployed worker and \( v_t \) is the number of open vacancies, assumed to lie on the unit interval \( v_t = \int_0^1 v_t \, di \). Where \( \mu \in (0, 1) \) is the elasticity of the matching function with respect to unemployment and the matching efficiency is governed by \( m > 0 \). The matching function is homogenous of degree one, strictly increasing in each of its arguments, strictly concave and twice continuously differentiable. The homogeneity assumption leads to the probability of a vacancy being filled in the next period \( q(\theta_t) = m \theta_t^{-\mu} \), where \( \theta_t = v_t / u_t \) corresponds to labor market tightness.

The firm’s exit site is determined by endogenous separations only. Therefore, the total number of separations at firm \( i \) is given by \( \rho_{it} = F(\hat{a}_{it}) \), where \( \hat{a}_{it} \) is an endogenously determined critical threshold. If the specific productivity of a job is below this threshold, it is not profitable and separation takes place. \( F(a) \) is a time-invariant distribution with positive support \( f(a) \). Connecting the results for job creation and the job destruction enables us to determine the evolution of employment at firm \( i \) as

\[ n_{it+1} = (1 - \rho_{it+1})(n_{it} + v_{it} q(\theta_t)). \] (8)

The firm adjusts employment by posting vacancies and by setting the critical threshold, which then influences the separation rate.

\(^5\text{In their empirical analysis Petrongolo and Pissarides (2001) find that the Cobb-Douglas function with constant returns to scale is the most appropriate specification.}\)
2.3. Firm’s Maximization

If the matching process has been successful, production commences along the production function given by

$$y_t = A_t K_t^a [n_t \int a \frac{f(a)}{1-F(\tilde{a}_it)} da]^{1-\alpha} = A_t K_t^a [n_t H(\tilde{a}_it)]^{1-\alpha}, \quad (9)$$

where $\alpha < 1$, aggregate productivity $A_t$ is common to all firms, the specific productivity $a_{it}$ is idiosyncratic and every period it is drawn in advance of the production process from the corresponding distribution function. The worker specific production function can then be written as

$$y_{jt} = A_t k_{ijt}^{\alpha} \tilde{a}_{jt}^{1-\alpha}, \quad (10)$$

where worker $j$’s share of the overall capital in firm $i$ is $k_{ijt} = K_{it}/n_{it}$. Since we assume homogeneity in matches, every worker has a proportional access to the capital stock.

The firm solves the following maximization problem

$$\Pi_{i0} = E_{t0} \sum_{t=0}^{\infty} \beta^t \lambda_t \left[ \frac{P_{it}}{P_t} y_{it} - W_{it} - cv_{it} - \psi \left( \frac{P_{it}}{P_{it-1}} - \pi \right)^2 \right], \quad (11)$$

being real revenue depleted by total costs.\(^6\) Due to the introduction of nominal and real frictions, total costs are also determined by vacancy posting costs ($c > 0$), capital rental costs and price adjustment costs ($\psi \geq 0$).

The wage bill is given by the aggregate of individual wages

$$W_{it} = n_{it} \int \tilde{a}_{it} w_t(a) \frac{f(a)}{1-F(\tilde{a}_{it})} da. \quad (12)$$

The job creation condition is then given by

$$\frac{c}{q(\theta_t)} = E_t \beta_{t+1} (1 - \rho_{t+1}) \left[ (1 - \alpha) \phi_{t+1} A_{t+1} K_{t+1}^a n_{t+1}^{\alpha} H(\tilde{a}_{t+1})^{1-\alpha} - \frac{\partial W_{t+1}}{\partial n_{t+1}} + \frac{c}{q(\theta_{t+1})} \right]. \quad (13)$$

\(^6\)Perfect capital markets imply that the firm discounts with the households subjective discount factor.
where $\beta_{t+1} = \beta \frac{\lambda_{t+1}}{\lambda_t}$ is the stochastic discount factor. This condition reflects the hiring decision as a trade-off between the costs of a vacancy and the expected return. Where $1/q(\theta_t)$ is the duration of the relationship between firm and worker.

A key distinctiveness of New Keynesian models is their capability to elucidate the reciprocity of output and inflation. In these models inflation dynamics are defined by the New Keynesian Phillips curve (NKPC)\textsuperscript{7}

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{\phi}_t,$$

where $\kappa = (\epsilon - 1)/\psi$ and $\hat{\phi}_t$ reflects the marginal costs.

Subsequently, we will shed light on the wage setting process to derive an expression for the individual real wage which will allow us to study the firm’s separation decision more precisely and further determine the critical threshold.

### 2.4. Wage Setting

A successful match shares an economic rent which is split according to individual Nash bargaining. The firm-worker pair then solves the following problem

$$w_t = \arg\max_{w_t} \left\{ (W_t - U_t)\eta (J_t - V_t)^{1-\eta} \right\},$$

where the first term is the worker’s surplus, the latter term is the firm’s surplus and $0 \leq \eta \leq 1$ is the exogenously determined, constant relative bargaining power. It can be shown that the individual real wage satisfies the optimality condition

$$W_t(a_t) - U_t = \frac{\eta}{1-\eta} J_t(a_t).$$

To obtain an explicit expression for the individual real wage, we determine the asset value functions and substitute them into the Nash bargaining solution (16). For the firm the asset value of the job depends on the real revenue, the real wage and if the job is not destroyed, the discounted future value. Otherwise, the job is destroyed and hence has zero value. In terms of a Bellman equation the asset

\textsuperscript{7}For simplicity, we illustrate the log-linearized version of the New Keynesian Phillips Curve.
value is given by
\[ J_t(a_t) = \varphi_t A_t k_t^a a_t^{1-\alpha} - w_t(a_t) + E_t \beta_{t+1} \left(1 - \rho_{t+1}\right) \int_{a_{t+1}} J_{t+1}(a) \frac{f(a)}{1 - F(\tilde{a}_{t+1})} da \] (17)

The asset value of being employed for the worker consists of the real wage, the discounted continuation value and in case of separation the value of being unemployed
\[ W_t(a_t) = w_t(a_t) + E_t \beta_{t+1} (1 - \rho_{t+1}) \int_{\tilde{a}_{t+1}} W_{t+1}(a) \frac{f(a)}{1 - F(\tilde{a}_{t+1})} da \] (18)

Analogously, the asset value of a job seeker is given by
\[ U_t = b + E_t \beta_{t+1} \theta_t q(\theta_t) (1 - \rho_{t+1}) \int_{\tilde{a}_{t+1}} W_{t+1}(a) \frac{f(a)}{1 - F(\tilde{a}_{t+1})} da \] (19)

Unemployed worker receive the value of home production $b$, the discounted continuation value of being unemployed and if she is matched she receives the value of future employment. Inserting these value functions into the Nash bargaining solution yields the individual real wage
\[ w_t(a_t) = \eta(\varphi_t A_t k_t^a a_t^{1-\alpha} + c \theta_t) + (1 - \eta) b. \] (20)

The firm will endogenously separate from a worker, if and only if
\[ J_t(a_t) < 0, \] (21)

i.e. if the worker’s asset value is smaller than zero.

After some algebra, the threshold is given by
\[ \tilde{a}_t = \left\{ \frac{1}{(1 - \eta) \varphi_t A_t k_t^a} \left[ (1 - \eta)b + \eta c \theta_t - \frac{c}{q(\theta_t)} \right] \right\}^{\frac{1}{1-\alpha}}. \] (22)

\[ ^8 \text{Notice, that we drop subscripts } i \text{ due to symmetry.} \]
In the next section, we close our model by calibrating deep parameters and by defining monetary policy.

3. Model Solution

3.1. Closing the Model

The monetary authority targets the short-term nominal interest rate by following a standard Taylor rule, given by

$$\left( \frac{R_t}{\bar{R}} \right) = \left( \frac{\pi_t}{\bar{\pi}} \right) \phi_{\pi} \left( \frac{Y_t}{\bar{Y}} \right) \phi_{y} \rho_{t}$$

(23)

where $\phi_{\pi}$ and $\phi_{y}$ are the respective weights and the interest rate shock follows a standard AR(1), i.e. $\rho_{t} = \rho_{t-1} e^{\alpha_{\rho,t}}$. The aggregate productivity shock is formulated as

$$A_t = A_{t-1} e^{\alpha_{A,t}}.$$  

(24)

The i.i.d. error term is $\alpha_{A,t} \sim N(0, \sigma_A)$ with $\text{cov}(A_{t-1}, \alpha_{A,t}) = 0 \ \forall \ t$.

The resource constraint is given by

$$Y_t = C_t + I_t - cv_t.$$  

(25)

Then the model is linearized around its deterministic steady state and simulated with the software package Dynare.

For the given stochastic processes $\{A_t, \rho_t\}_{t=0}^\infty$ and the interest rate $\{r_t\}_{t=0}^\infty$ a determined equilibrium is a sequence of allocations and prices $\{a_t, C_t, I_t, jcr_t, jdr_t, K_t, m_t, n_t, \pi_t, \varphi_t, Q_t, \rho_t, r^K_t, \theta_t, u_t, v_t, w_t, y_t\}_{t=0}^\infty$, which for given initial conditions, satisfies equations (3), (4), (5), (6), (7), (8), (9), (13), (14), (20), (22), (23), (24), (25), the definitions for labor market tightness, job destruction and creation rate, the interest rate shock, the law of motion for unemployment, the separation rate, and the firm’s cost minimization solution.
3.2. Calibration

We calibrate the model on a quarterly basis for the United States and set parameter values according to stylized facts and the relevant literature. We set the Taylor Rule parameters to standard values of 1.5 with respect to inflation ($\phi_\pi$) and to 0.125 to output ($\phi_y$). The discount factor is set to 0.98 percent which results into a steady state interest rate of approximately 2 percent. For the risk aversion parameter $\sigma$ we choose to set the parameter to a value of 2. Following Krause and Lubik (2007), we set the price adjustment cost $\psi$ to a value of 105. With regard to the labor market variables, we choose a steady-state employment rate of 0.88 which results in an equilibrium unemployment rate of 12 percent analogously to Krause and Lubik (2007). The separation rate is set to 0.10 according to Hall (2005) which is due to endogenous separations only. The distribution parameters $\mu$ and $\sigma_{ln}$ of the log-normal distribution are chosen to match the observed volatility of job destruction. We set the parameters to 0 resp. 0.12. The job filling rate is assumed to equal 0.7 which corresponds to a monthly rate of 0.3 which is consistent with U.S. data\textsuperscript{9} whereas the probability for a worker of finding a job, $\theta_q(\theta)$ is equal to 0.6, which represents an unemployment average duration of 1.7 as in Cole and Rogerson (1999). In order to ensure a socially efficient outcome, we respect the Hosios (1990) rule, viz. the elasticity of matches with respect to unemployment $\mu$ is set equal to the workers bargaining power $\eta = 0.5$. Missing parameter values are computed from the steady state. The coefficient with respect to the capital share is set to the standard value of 1/3, inducing a labor share of roughly 60 percent. According to Smets and Wouters (2007), we set the second derivative of the investment adjustment cost function $S(\cdot)$ to 0.8. In line with Christiano et al. (2005) the depreciation rate is $\delta = 0.025$. The value of the autocorrelation of the aggregate productivity shock is set to 0.66, to balance the estimation results from Prescott (1986) and Altig et al. (2005). Prescott (1986) finds a value of 0.95, while Altig et al. (2005) find a value of 0.1 for a shock to capital embodied technology and 0.87 for a neutral technology shock. For the interest rate shock, we calibrate the persistence parameter to 0.49.

\textsuperscript{9}See Blanchard and Galí (2007).
4. Model Comparison

In this section, we analyze differences between a standard search and matching model with endogenous separations (baseline model, henceforth) and a model which additionally includes capital as developed above. At first, we consider an aggregate productivity shock. Subsequently, we scrutinize the dynamics to an expansionary monetary policy shock.

4.1. Productivity shock

Figure 1 shows the impulse responses to a positive aggregate productivity shock for both models.

- Figure 1 about here -

In the baseline model, GDP increases persistently due to the rise of aggregate productivity, which boosts production. Simultaneously, marginal costs decrease and consistently, - via the NKPC - inflation decreases. The monetary authority lowers its interest rates, since it puts more weight on inflation than on output. The expansionary behaviour of the central bank acts as a supplementary increase in aggregate demand such that firms increase their labor demand. As a consequence, the separation threshold decreases and firms aim to keep more workers in order to remedy increased demand. Analogously, job creation rises on impact, turns negative in the consecutive quarter, however. As characteristic for endogenous separation models, we identify a separation driven adjustment mechanism. In contrast to the baseline model, and as a consequence of the capital-labor trade-off, unemployment increases in the capital model, because firms substitute labor by capital. While in the baseline model the productivity of workers increases because aggregate productivity increases, the capital model suggests an additional channel, working along the per-worker capital stock, i.e. $k_{ijt}$. The direct effect works through the increase in the capital stock, viz. an increased capital stock makes workers more productive. The indirect effect is a consequence of the

10 If inflation would not react on impact, employment would fall, because the firm would be capable of satisfying the unchanged demand with less workers.

11 This follows straightforward from the job creation condition. Initially, expected profits rise - since productivity increases - but then hiring costs increase, driving the system back to the equilibrium.
substitution of labor. Since employment decreases, the capital share of worker $i$ clearly increases, additionally increasing her productivity. This, in turn, also rises the incentive to separate from less productive workers. Furthermore, the dynamic response of vacancies changes significantly in the model with capital. While in the baseline model vacancy postings drop on impact and converge from below to the steady state, in the capital model vacancies initially increase and converge from above. As before, the capital-labor trade-off causes this changed vacancy posting behavior. Consider the vacancy posting condition (13). Due to the increase in the capital stock and aggregate productivity, the expected profit from a vacancy increases. This is further enhanced by the same mechanism that we already identified. Because firms decrease employment, the per-worker capital stock significantly increases such that there is "capital left" for new workers, i.e. the expected profit from a posted vacancy is large, since the worker's productivity will be large due to (i) the large aggregate productivity and (ii) due to the large worker's capital share (especially, in relation to the steady state).

Our findings are mirrored in the second moments of our simulation (see Table 2).

- Table 2 about here -

With respect to our empirical analysis in the introduction, we show that the model with capital performs outstandingly. In particular, the standard deviations of vacancies and unemployment are in line with the evidence, such that there is no Shimer-puzzle. However, the volatility of labor market tightness is four times below the target value. In addition, except from the standard deviation of separations, all values are close to their empirical pendants. We admit that the challenge of the endogenous separation model, the separation driven adjustment mechanism, is still present in the capital model and causes the relatively bad performance of second moments. For instance, the fact that there is no Beveridge curve and the second moments with respect to the separation rate are not in line with empirical observations. In addition, in the capital model - and in contrast to the baseline model - the standard deviations of the job creation and destruction rates are too high compared to empirical estimates and there correlation is close to (plus) one (being around 0.5 in the baseline model). However, we find that the capital model produces higher persistence values. Unfortunately, this persistence is not endogenous in the sense that a one-off shock creates al-
most no persistence. Therefore, the model only generates higher persistence to an autocorrelated shock.

### 4.2. Interest Rate Shock

Figure 2 presents the reactions of the aggregates to a reduction of the monetary authority’s policy rate.

The transmission of monetary policy shocks is not - heavily - affected by the introduction of capital. Our qualitative and quantitative results are quite similar. An expansionary measure of the monetary authority leads to a boost in aggregate demand and in inflation, due to the pressure on marginal costs. The significant decrease of unemployment is due to the sharp reduction of the job destruction rate which results from a strong decline in separations, i.e. the separation driven adjustment mechanism. Initially, job creation increases; however it already turns negative in the second quarter. Capital accumulation makes almost no difference with respect to monetary policy interventions. The reason is, that the monetary policy shock acts as a demand shock, hence increasing firms output. Because the capital-labor trade-off is less heavily activated - as it was compared to the consideration of the productivity shock -, the dynamic responses of both systems look identical. Although, the transmission channel is less present in this case, it still creates additional variability of labor market variables. Our simulations of the business cycle statistics in Table 3 confirm these results.

As we have seen, the capital model creates significantly higher values of standard deviations for unemployment and vacancies. However, with respect to the remaining values of standard deviation, we infer almost no change. In addition, this holds for the correlations between the labor market variables. The relationship between the volatilities of the job creation and destruction rate between the two models is of the same size as we have discussed in the precedent section.
5. Robustness Issues

In what follows, we perform a robustness analysis of our results for the capital model and the productivity shock only. Setting the value of the autocorrelation of the shock to 0.89 (the empirical autocorrelation) significantly increases the fluctuations of all variables. However, our qualitative results remain unaffected and the nature of the shock allows us to choose a smaller value of the shock than normally employed in RBC models. Increasing the unemployment rate (0.3 in this example) leaves our quantitative results unaffected while it significantly decreases the volatility of aggregate fluctuations (e.g. std(v)=0.08). In particular, the standard deviations of the job creation and destruction rate are significantly reduced. Along this line, decreasing the unemployment rate (5% in this example) increases the fluctuations of labor market variables. Since this is a well-known effect of matching models, we continue with the discussion of the steady state separation rate. Setting this value to 0.15 as in Andolfatto (1996) leaves our results almost unaffected. A value of $\psi = 40$ increases the volatility of key variables such as unemployment or vacancies.

The value of capital adjustment costs plays a major role for the dynamics of the capital model. Increasing this value (doubled, in this example) increases the standard deviations of labor market variables, because adjustments along the capital margin are more expensive. In addition, we simulated the model with a money growth rule (results are available upon request) and find the same internal propagation mechanism for the volatility of labor market variables as described in the precedent section.\footnote{Our simulation shows no Shimer puzzle.} While we observe a significant improvement in the second moments of the model, the dynamic saddle path is almost identical to the model without capital.

6. Conclusion

In this paper we have shown that the introduction of capital adds an important channel for the propagation of productivity shocks. We develop a New Keynesian model with search frictions, purely endogenous separations and capital. In deviating from the vintage capital theory, we consider a more general case of capital
in DSGE models. In our model, workers have an unrestricted and proportional
access to the firm’s capital stock, i.e. we do not assume heterogeneity in matches.
Within this framework we show that the dynamic adjustment path of the capital
model deviates significantly from the baseline model to an aggregate productivity
shock. The reason for the differences between these two models - and in fact the
reason why there is no difference in response to the monetary policy shock - is
the fact that the firm substitutes labor by capital. In addition, the main channel
causing the discussed differences is the worker’s capital share being affected (i) di-
rectly, by the increase of the capital stock, and (ii) indirectly, by the decrease of
employment. While these channels are important for the transmission of the pro-
ductivity shock, they only play a minor role for the propagation of the monetary
policy shock. Consistently, in both cases the capital model creates larger - and
for the productivity shock even fits - the empirical fluctuations of unemployment
and vacancies. We admit that the underlying challenge of endogenous separation
models - the separation driven adjustment mechanism - is not resolved, such that
the second moments are still not entirely in line with the evidence.
With our more general approach of implementing capital, and by proposing an
additional transmission channel, we justify to further consider the role of capital,
capital adjustment costs and the capital-labor trade-off in sticky price matching
models. We believe that the consideration of specific shocks, e.g. match-specific
or worker-specific, should be a fruitful area for future research.
A. References


Tables and Figures

Figure 1: Impulse Responses to a 1% Productivity Shock.

Figure 2: Impulse Responses to a 1% Interest Rate Shock.
Table 1: Business Cycle Properties of the U.S. Economy.

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Notes: We use quarterly, seasonally adjusted, HP filtered ($\lambda = 10^5$) data from 1955:Q1 to 2009:Q2 provided by the OECD and the BLS. All variables respond to log deviations.

Table 2: Business Cycle Properties of the Matching Models - Productivity Shock.

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Notes: Theoretical Moments.
Table 3: Business Cycle Properties of the Matching Models - Interest Rate Shock.

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Notes: Statistics for the U.S. economy are computed using quarterly HP-filtered data from 1964:1 to 2002:3. Statistics for the model economies are computed by simulating the model 100 times for 200 periods. The statistics are averages over the HP-filtered simulations. The standard deviations of all variables are relative to output.