MakØk3, Fall 2010 (blok 2)

"Business cycles and monetary stabilization policies"

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Lecture 3, November 30: **The Basic New Keynesian Model** (Galí, Chapter 3)

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Introduction

- The classical flex-price model(s) covered so far cannot account for realistic short-run dynamics following monetary shocks
  - In case there are any real effects, the magnitude appears modest
  - No liquidity effect present (long run = short run)

  Some "sand in the wheels" are necessary to explain the effects of money on output seen in data

- Obvious candidate is incomplete nominal adjustment. In flex-price models, money neutrality holds both in the short and long run
  - If money neutrality fails in the short run, the impact of monetary policy will clearly be stronger

- Classical model is therefore amended with nominal rigidities (and explicit determination of prices by firms)
- Result is Dynamic Stochastic General Equilibrium (DSGE) model of the New-Keynesian type

A basic New-Keynesian model

- Goods market
  - Demand side: Households consume a basket of goods (based on utility maximization)
  - Supply side: Firms produce *different* consumption goods (maximize profits under monopolistic competition)
- Labor market
  - Demand side: Firms hire labor (maximize profits under monopolistic competition)
  - Supply side: Households supply labor (based on utility maximization)
- Financial markets
  - Households optimally invest in a one-period risk-less bond
  - Households also hold money. We again just assume it (and generally do not make much use of it here, as focus will be on interest-rate policies)

- *Wages* are perfectly flexible (this assumption is relaxed in Chapter 6)

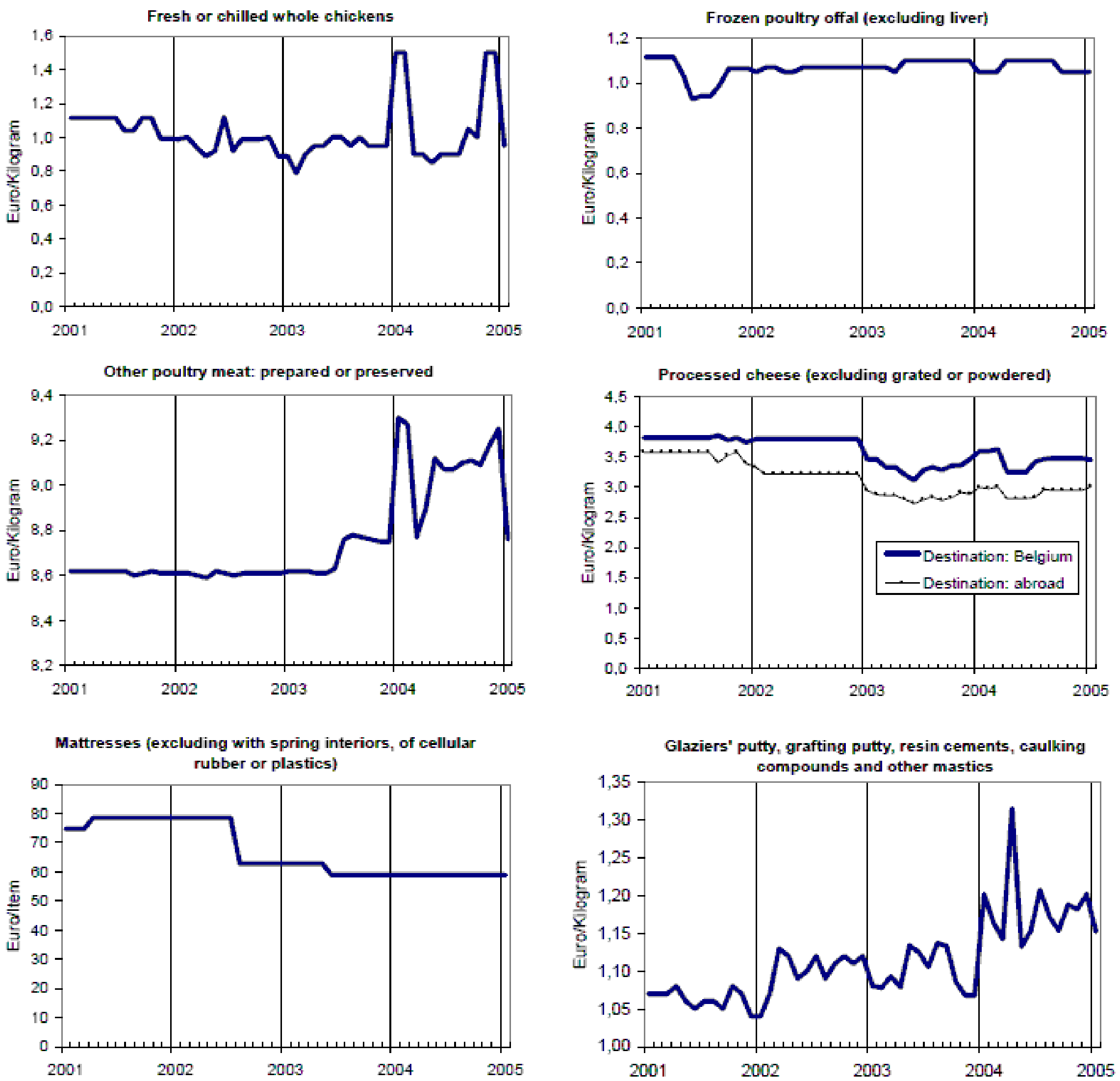
- *All goods prices* are subject to inflexibility, i.e., stickiness:
  - Every period, each firm faces a state-independent probability θ of "being stuck" with its price
  - So, $0 \leq \theta \leq 1$ is natural "index" for stickiness in the economy

- The probability is *independent* of when the firm last changed its price; i.e., time-dependent price stickiness (as opposed to state-dependent price stickiness)

- Stylistic representation of "staggering." The independence of history facilitates aggregation:
  - θ is the fraction of firms not adjusting prices in a period
  - $1 - \theta$ is the fraction of firms adjusting is a period
  - $1 + \theta + \theta^2 + \theta^3 + ... = 1/(1 - \theta)$ is average duration of a price contract

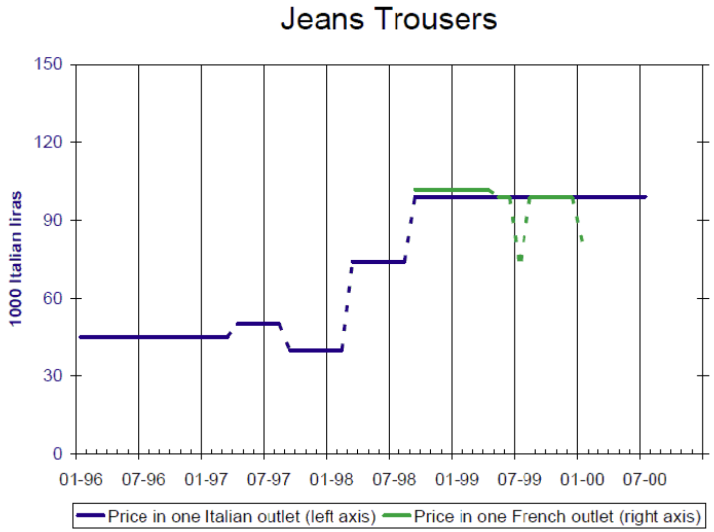
- When "allowed" to change the price, expectations about future prices become of importance

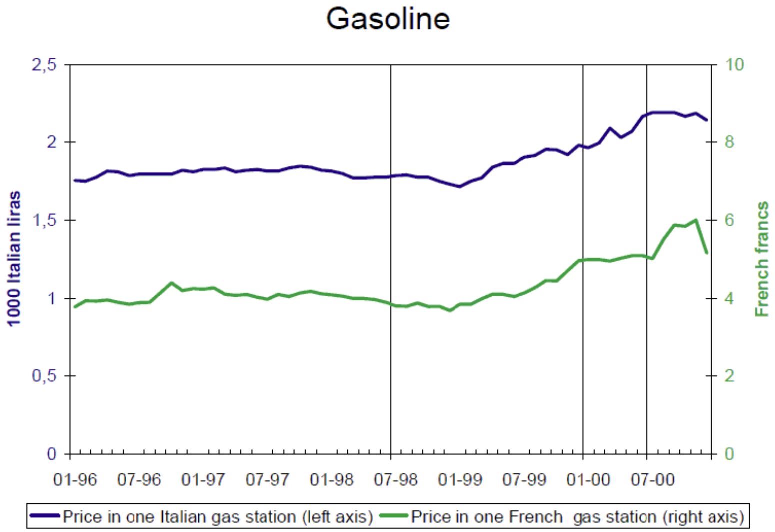
A view at prices changes: Belgium, 2001-2005

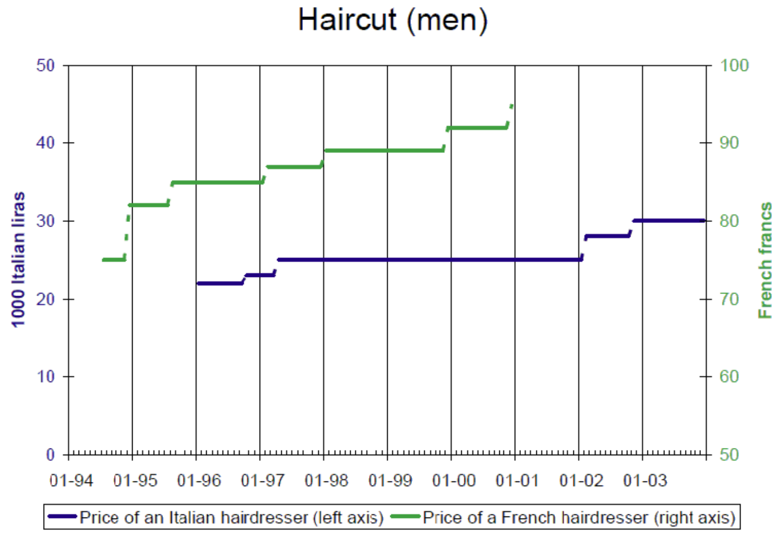


Source: Cornille and Dossche (2006)

A view at prices changes: France and Italy, 1996-2001







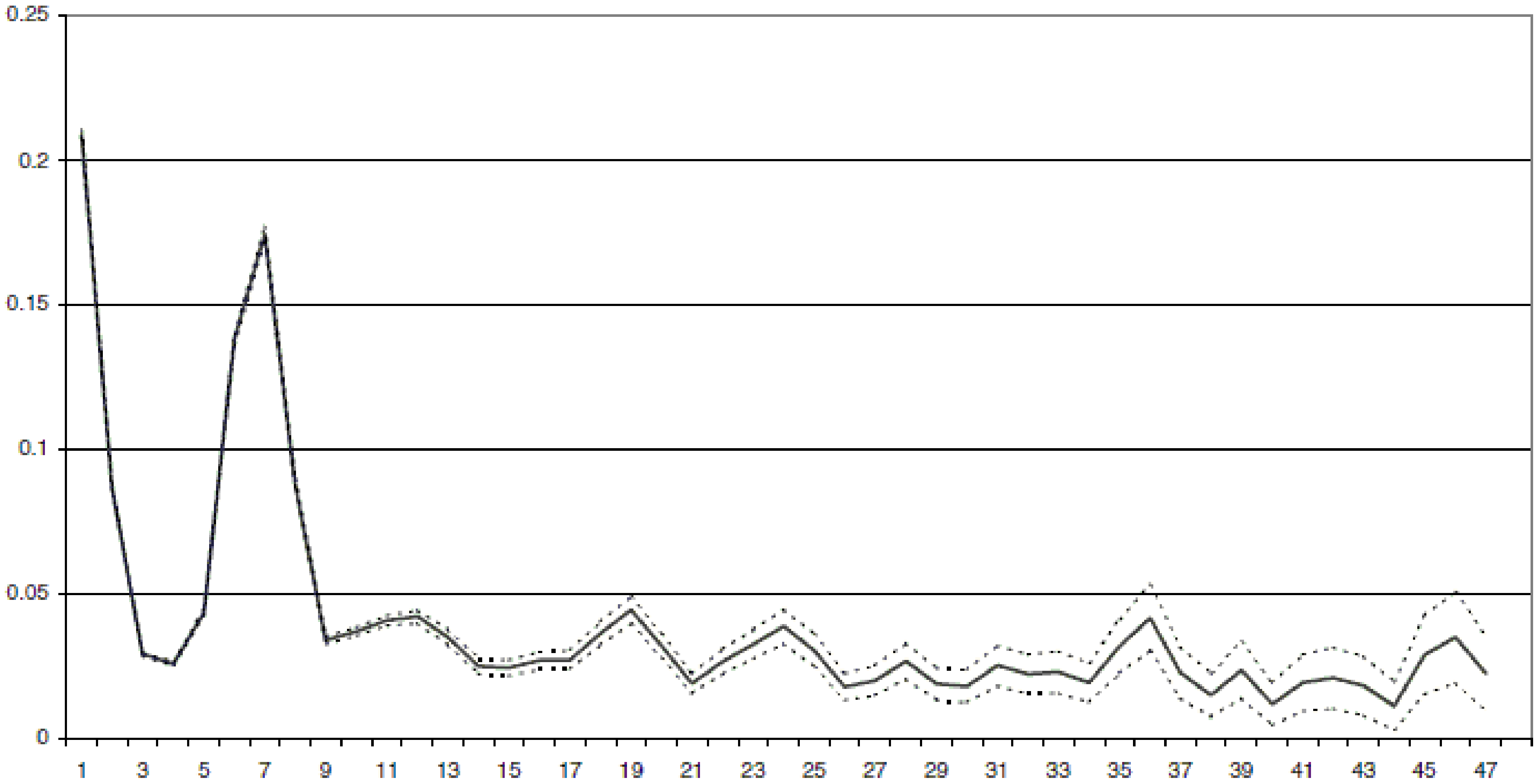
Source: Dhyne *et al.* (2005)

A view at prices changes: (Monthly) frequencies of price adjustments; Euro area vs. US

	Euro area	US
Frequency of price changes		
50 product sample	15.1 p.c.	24.8 p.c.
Larger sample	15.3 p.c.	26.1 p.c.
Trimmed mean	16.9 p.c.	
Average duration		
Based on frequencies at the euro area product category level	13.0 months	6.7 months
Based on frequencies at the country-product category level	15.1 months	
Inverting the aggregate frequency of price changes	6.6 months	3.8 months
Median price duration	10.6 months	4.6 months

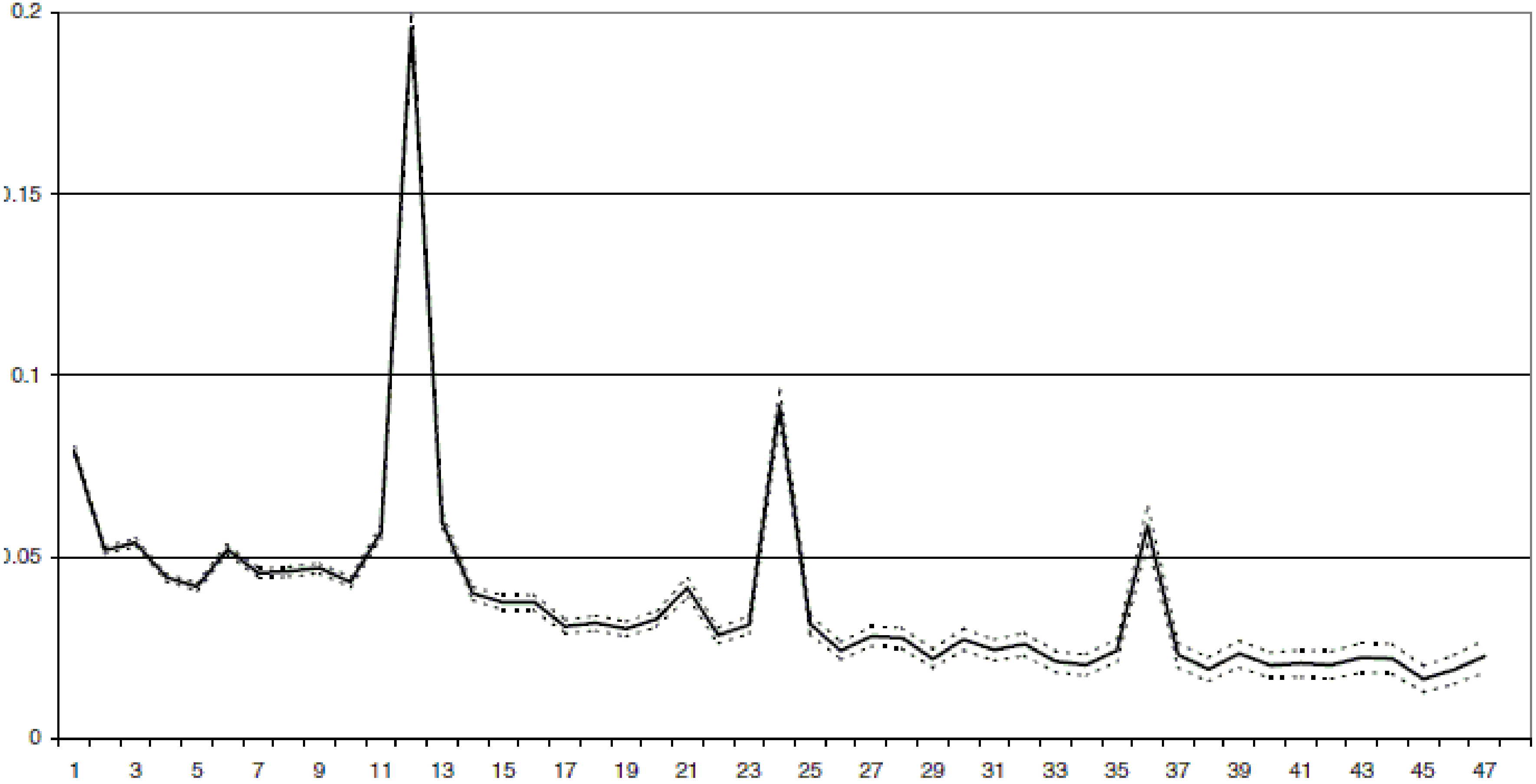
Source: Dhyne *et al.* (2005) (with US data based on Bils and Klenow, 2004)

A view at prices changes: France, hazard rates in clothing industry



Source: Baudry *et al.* (2004)

A view at prices changes: France: Hazard Rates in service industry



Source: Baudry *et al.* (2004)

Summary of findings from data:

- Aggregate prices are sticky (as also indicated by VAR analyses)
- Price setting in different sectors differs;
  - some sectors exhibit long periods of fixed prices (time dependent price setting)
  - some sectors exhibit continuous changing prices
- Price setting is asynchronized across sectors
- In some sectors, the probability of price adjustment is (nearly) independent of the life-span of the price

Relation to theoretical modelling of price rigidities:

- The staggered price setting scheme captures many of these features, and their aggregate implications, in a simple and handy way
- Modelling due to Calvo (1983); now just called "Calvo pricing"

Household behavior

- The essentials of household behavior are like in the basic classical model
- Household maximize expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \qquad 0 < \beta < 1.$$

- Main difference: C_t is not one good, but a basket of the different goods produced by the different producers (indexed by $i \in [0,1]$)

$$C_t \equiv \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right]^{\frac{\varepsilon}{\varepsilon-1}}, \qquad \varepsilon > 1$$

- Budget constraint therefore becomes:

$$\int_0^1 P_t(i)\, C_t(i)\, \mathrm{d}i + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

(and a No-Ponzi Game constraint ruling out explosive debt)

- Household chooses optimal paths of C_t, N_t and B_t (taking prices P_t, W_t and Q_t as given), but must now choose the composition of C_t as well

Finding the optimal composition of C_t:

- Assume that total nominal expenditures on consumption are Z_t

$$\int_0^1 P_t(i)\, C_t(i)\, \mathrm{d}i = Z_t$$

- Optimal demand for $C_t(i)$, is found by

$$\max_{C_t(i)} \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}} \qquad \text{s.t.} \int_0^1 P_t(i)\, C_t(i)\, \mathrm{d}i = Z_t$$

- The relevant first-order condition is

$$\begin{aligned} \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, \mathrm{d}i\right]^{\frac{1}{\varepsilon-1}} C_t(i)^{-\frac{1}{\varepsilon}} &= \lambda P_t(i), \\ \left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\, \mathrm{d}i\right]^{\frac{1}{\varepsilon-1}} C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} &= \lambda P_t(i)\, C_t(i) \end{aligned}$$

where λ is the Lagrange multiplier associated with the constraint

- Integrating over all goods:

$$\left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{1}{\varepsilon-1}} \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i = \lambda \int_0^1 P_t(i) C_t(i) \mathrm{d}i$$
$$C_t = \lambda Z_t$$

- Let P_t be the price index associated with C_t, then $P_t C_t = Z_t$ and

$$\lambda = \frac{1}{P_t}$$

- Inserted into first-order condition:

$$\left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i\right]^{\frac{1}{\varepsilon-1}} C_t(i)^{-\frac{1}{\varepsilon}} = \frac{P_t(i)}{P_t},$$
$$C_t^{\frac{1}{\varepsilon}} C_t(i)^{-\frac{1}{\varepsilon}} = \frac{P_t(i)}{P_t}$$

- The relative demand for good i is therefore

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} C_t \tag{1}$$

- Note that ε is the elasticity of substitution between goods. (Hence, the limiting case of perfect competition applies when $\varepsilon \to \infty$.)

The price index

- The precise form of the price index P_t follows from

$$P_t C_t = \int_0^1 P_t(i)\, C_t(i)\, \mathrm{d}i$$

and the expression for relative demand

- I.e., we get

$$\begin{aligned} P_t C_t &= \int_0^1 P_t(i) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t \; \mathrm{d}i \\ P_t &= \left[\int_0^1 P_t(i)^{1-\varepsilon} \, \mathrm{d}i \right]^{\frac{1}{1-\varepsilon}} \end{aligned}$$

Remaining household optimality conditions

- Precisely as in the simple classical model:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta \mathrm{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

- With $U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$, $\sigma, \varphi > 0$, we have the same log-linear expressions:
  - Labor supply schedule:

$$\sigma c_t + \varphi n_t = w_t - p_t \tag{2}$$

  - Consumption-Euler equation:

$$c_t = \mathrm{E}_t \{c_{t+1}\} - \sigma^{-1} (i_t - \mathrm{E}_t \{\pi_{t+1}\} - \rho) \tag{3}$$

    with $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$

Firms' behavior and aggregate inflation dynamics

- Generally, optimal price-setting of a firm that can change its price in period t:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k}\left(Y_{t+k|t}\right)\right)\right\}$$

$$\text{s.t. } Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon} C_{t+k} \tag{8}$$

- I.e., the maximal expected discounted stream of profits of choosing P_t^* taking into account the probabilities that it is fixed in the future

- Ψ_{t+k} is total costs of sales at price P_t^* in period $t+k$ (will depend on labor costs and productivity)

- $Q_{t,t+k}$ is the "stochastic discount factor" (or present-value operator), equal to the one facing consumers between periods t and $t+k$:

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+k}}\right)$$

Note

$$\begin{aligned}
Q_{t,t+1} &= \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \\
Q_{t,t+2} &= \beta^2 \left(\frac{C_{t+2}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+2}}\right) = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \beta \left(\frac{C_{t+2}}{C_{t+1}}\right)^{-\sigma} \left(\frac{P_{t+1}}{P_{t+2}}\right) = Q_{t,t+1} Q_{t+1,t+2} \\
Q_{t,t+k} &= Q_{t,t+1} Q_{t+1,t+2} \cdots\cdots Q_{t+k-2,t+k-1} Q_{t+k-1,t+k}
\end{aligned}$$

- First-order condition for P_t^*:

$$\sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} \left(Y_{t+k|t} + P_t^* \frac{\partial Y_{t+k|t}}{\partial P_t^*} - \psi_{t+k|t} \frac{\partial Y_{t+k|t}}{\partial P_t^*} \right) \right\} = 0,$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}\left(Y_{t+k|t}\right)$ is nominal marginal costs

$$\begin{aligned} \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(1 + \frac{P_t^*}{Y_{t+k|t}} \frac{\partial Y_{t+k|t}}{\partial P_t^*} - \psi_{t+k|t} \frac{1}{Y_{t+k|t}} \frac{\partial Y_{t+k|t}}{\partial P_t^*} \right) \right\} &= 0, \\ \sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(1 - \varepsilon + \psi_{t+k|t} \frac{\varepsilon}{P_t^*} \right) \right\} &= 0. \end{aligned}$$

- We then get the central pricing equation:

$$\sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M} \psi_{t+k|t} \right) \right\} = 0, \qquad (9)$$

with $\mathcal{M} \equiv \varepsilon / (\varepsilon - 1) > 1$ being the *mark-up*; a measure of the monopoly distortion

- This is rewritten as

$$\sum_{k=0}^{\infty} \theta^k \mathrm{E}_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0, \tag{10}$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ is real marginal costs and $\Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1}$ is gross inflation between $t-1$ and $t+k$

- Optimality condition is log-linearized around a zero-inflation steady state:

$$\frac{P_t^*}{P_{t-1}} = 1, \quad \Pi_{t-1,t+k} = 1, \quad Y_{t+k|t} = Y, \quad MC = \mathcal{M}^{-1}, \quad Q_{t,t+k} = \beta^k$$

- One gets

$$\sum_{k=0}^{\infty} (\beta\theta)^k \, \mathrm{E}_t \left\{ p_t^* - p_{t-1} - \widehat{mc}_{t+k|t} - p_{t+k} + p_{t-1} \right\} = 0$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc \; = \; mc_{t+k|t} + \log \mathcal{M} \; = \; mc_{t+k|t} + \log \frac{\varepsilon}{\varepsilon - 1} \; = \; mc_{t+k|t} + \mu$

- Hence,

$$\begin{aligned} \sum_{k=0}^{\infty} (\beta\theta)^k \, p_t^* \;&=\; \sum_{k=0}^{\infty} (\beta\theta)^k \, \mathrm{E}_t \left\{ \widehat{mc}_{t+k|t} + p_{t+k} \right\} \\ p_t^* \;&=\; (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \, \mathrm{E}_t \left\{ \widehat{mc}_{t+k|t} + p_{t+k} \right\} \end{aligned}$$

Optimal price is a function of expected current and future real marginal costs and aggregate prices

- Marginal costs depend on production function and factor payments
- Each firm has the following production function (i.e., identical technology):

$$Y_t(i) = A_t N_t(i)^{1-\alpha}, \qquad 0 \le \alpha < 1 \tag{5}$$

- Total costs

$$TC_t = W_t N_t(i)$$

Real marginal costs

$$MC_t(i) = \frac{W_t}{P_t A_t (1-\alpha) N_t(i)^{-\alpha}} \left(= \frac{W_t}{P_t \; MPN_t} = \frac{W_t}{P_t A_t (1-\alpha) \left(Y_t(i)/A_t\right)^{-\frac{\alpha}{1-\alpha}}} = \frac{W_t}{P_t (1-\alpha) A_t^{\frac{1}{1-\alpha}} Y_t(i)^{-\frac{\alpha}{1-\alpha}}} \right)$$

- Special case of constant returns to scale, $\alpha = 0$

$$MC_{t+k|t} = MC_{t+k} \left(= \frac{W_{t+k}}{P_{t+k} A_{t+k}} \right)$$

$$p_t^* = (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \, \mathrm{E}_t \left\{ \widehat{mc}_{t+k} + p_{t+k} \right\}$$

- This is the unique stationary solution to the first-order rational expectations difference equation:

$$p_t^* = \beta\theta \mathrm{E}_t \left\{ p_{t+1}^* \right\} + (1-\beta\theta)\left(\widehat{mc}_t + p_t\right)$$

Aggregate price dynamics

- Remember, θ is fraction of price setters that keep past period's price; θ is fraction of price setters that set new prices
- By definition of the price index:

$$P_t = \left[\theta \left(P_{t-1}\right)^{1-\varepsilon} + (1-\theta)\left(P_t^*\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \qquad \Longrightarrow \qquad \Pi_t^{1-\varepsilon} = \theta + (1-\theta)\left(\frac{P_t^*}{P_{t-1}}\right)^{1-\varepsilon}$$

- Log-linearized around the zero-inflation steady state:

$$(1-\varepsilon)\,\pi_t = (1-\varepsilon)(1-\theta)\left(p_t^* - p_{t-1}\right) \qquad \Longrightarrow \qquad \pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right) \tag{7}$$

- Rewrite the difference equation for optimal price setting:

$$p_t^* - p_{t-1} = \beta\theta \mathrm{E}_t\left\{p_{t+1}^* - p_t\right\} + (1-\beta\theta)\left(\widehat{mc}_t + p_t\right) - p_{t-1} + \beta\theta p_t$$

$$p_t^* - p_{t-1} = \beta\theta \mathrm{E}_t\left\{p_{t+1}^* - p_t\right\} + (1-\beta\theta)\,\widehat{mc}_t + \pi_t$$

- Use aggregate dynamics to obtain:

$$(1-\theta)^{-1}\pi_t = (1-\theta)^{-1}\beta\theta\mathrm{E}_t\left\{\pi_{t+1}\right\} + (1-\beta\theta)\,\widehat{mc}_t + \pi_t$$

$$\pi_t = \beta\mathrm{E}_t\left\{\pi_{t+1}\right\} + \lambda\widehat{mc}_t, \qquad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}, \qquad \lim_{\theta\to 0}\lambda = \infty.$$

This is the basic inflation-adjustment equation in New-Keynesian theory

General case of decreasing returns to scale, $0 < \alpha < 1$

- We define aggregate real marginal costs (in logs):

$$\begin{aligned} mc_t &\equiv w_t - p_t - mpn_t \\ &= w_t - p_t - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \end{aligned}$$

- We have for a price-changing firm:

$$\begin{aligned} mc_{t+k|t} &= w_{t+k} - p_{t+k} - \frac{1}{1-\alpha}\left(a_{t+k} - \alpha y_{t+k|t}\right) - \log(1-\alpha) \\ &= mc_{t+k} + \frac{\alpha}{1-\alpha}\left(y_{t+k|t} - y_{t+k}\right) \end{aligned}$$

- By the demand function in logs (here using equilibrium condition $y_{t+k} = c_{t+k}$):

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_{t+k}\right) \tag{14}$$

- Optimal price setting:

$$p_t^* = (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \, \mathrm{E}_t\left\{\widehat{mc}_{t+k} - \frac{\alpha\varepsilon}{1-\alpha}\left(p_t^* - p_{t+k}\right) + p_{t+k}\right\}$$

- Associated inflation dynamics

$$\pi_t = \beta \mathrm{E}_t\left\{\pi_{t+1}\right\} + \lambda\widehat{mc}_t, \qquad \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}, \qquad \lim_{\theta\to 0}\lambda = \infty. \tag{16}$$

Equilibrium outcomes

- Goods market clearing on each goods market

$$Y_t(i) = C_t(i), \qquad \text{all} \quad 0 \leq i \leq 1$$

- In the aggregate:

$$Y_t = C_t,$$

where

$$Y_t \equiv \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, \mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

- Asset market equilibrium (as in classical economy):

$$y_t = \mathrm{E}_t\{y_{t+1}\} - \sigma^{-1}\left(i_t - \mathrm{E}_t\{\pi_{t+1}\} - \rho\right) \tag{12}$$

- Aggregate employment:

$$N_t = \int_0^1 N_t(i)\, \mathrm{d}i = \int_0^1 \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \mathrm{d}i$$

$$\begin{aligned} N_t &= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} \mathrm{d}i \\ & \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \mathrm{d}i \end{aligned}$$

- In logs:

$$n_t = \frac{1}{1-\alpha}(y_t - a_t) + \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}},$$

$$(1-\alpha)\, n_t = y_t - a_t + d_t, \qquad d_t \equiv (1-\alpha) \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}}.$$

- The variable d_t is a natural measure of *price dispersion (* proportional to the variance of prices)
- When we consider log deviations from a symmetric zero-inflation steady state, d_t can be ignored (it is of *second order*—see Appendix 3.2 and 3.3)
- Hence, as a log-linear approximation:

$$y_t = a_t + (1-\alpha)\, n_t, \tag{13}$$

as in the classical case

Relationship between real marginal cost and output

- We want a dynamic system in output and inflation; hence, marginal cost is replaced appropriately by output
- We have

$$mc_t = w_t - p_t - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha)$$

- Using the labor supply schedule

$$mc_t = \sigma c_t + \varphi n_t - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha)$$

$$\begin{aligned} mc_t &= \sigma y_t + \frac{\varphi}{1-\alpha}(y_t - a_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \\ &= \frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha} y_t - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha) \end{aligned} \tag{17}$$

- Under flexible prices, $\theta = 0$, $\widehat{mc}_t = 0$

$$\begin{aligned} mc_t &= mc \\ &= -\mu \;\; (= -\ln \mathcal{M}) \end{aligned}$$

- Hence, denoting y_t^n the flexible-price output, or, the *natural rate of output*,

$$mc = -\mu = \frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha} y_t^n - \frac{1+\varphi}{1-\alpha} a_t - \log(1-\alpha)$$

- The natural rate of output is given as

$$\begin{aligned} y_t^n &= \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t - \frac{[\mu - \log(1-\alpha)](1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \\ &= \psi_{ya}^n a_t + \vartheta_y^n \end{aligned} \tag{19}$$

- Under perfect competition, $\mu = 0$, this is output as in the classical model

- We get the marginal cost in deviation from steady state:

$$\widehat{mc}_t = mc_t - mc = \frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha}(y_t - y_t^n) \tag{20}$$

- We denote $\widetilde{y}_t \equiv y_t - y_t^n$ the *output gap*

- The "New-Keynesian" Phillips Curve ("NKPC"):

$$\pi_t = \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \widetilde{y}_t, \qquad \kappa \equiv \lambda \frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha} \tag{21}$$

- Rephrase the Euler equation in terms of the output gap:

$$
\begin{aligned}
y_t &= \mathrm{E}_t\{y_{t+1}\} - \sigma^{-1}(i_t - \mathrm{E}_t\{\pi_{t+1}\} - \rho) \\
\widetilde{y}_t &= \mathrm{E}_t\{\widetilde{y}_{t+1}\} - \sigma^{-1}(i_t - \mathrm{E}_t\{\pi_{t+1}\} - \rho) - y_t^n + \mathrm{E}_t\{y_{t+1}^n\} \\
\widetilde{y}_t &= \mathrm{E}_t\{\widetilde{y}_{t+1}\} - \sigma^{-1}(i_t - \mathrm{E}_t\{\pi_{t+1}\} - r_t^n) \qquad (22)
\end{aligned}
$$

where

$$r_t^n \equiv \rho + \sigma \mathrm{E}_t\{\Delta y_{t+1}^n\}$$

is the *natural rate of interest*

  - A dynamic "IS" curve ("DIS")
  - Let $\widehat{r}_t^n \equiv r_t^n - \rho$

- New-Keynesian Phillips curve and dynamic IS curve determines output gap and inflation conditional on monetary policy (i_t).

- So, in contrast with the classical model, monetary policy matters for output determination!

Interest rate policy in the New Keynesian model

- The case of a simple Taylor-type interest rate rule:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + \upsilon_t, \qquad \phi_\pi, \phi_y \geq 0, \tag{25}$$

where υ_t is a policy shock, $\upsilon_t = \rho_\upsilon \upsilon_{t-1} + \varepsilon_t^\upsilon$

- The policy rule, NKPC and DIS give the dynamics

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} \mathrm{E}_t \{\widetilde{y}_{t+1}\} \\ \mathrm{E}_t \{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T (\widehat{r}_t - \upsilon_t), \tag{26}$$

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \phi_\pi \beta \\ \sigma\kappa & \kappa + \beta \left(\sigma + \phi_y\right) \end{bmatrix}, \qquad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}, \qquad \Omega \equiv \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}$$

- A unique non-explosive equilibrium requires the eigenvalues of $\mathbf{A}_T$ to be stable (within the unit circle)

- Uniqueness depends on policy-rule parameters:

$$0 < (\phi_\pi - 1)\kappa + \phi_y (1 - \beta) \tag{27}$$

is necessary and sufficient condition (the “Taylor principle”)

  – Note $\phi_\pi > 1$ is sufficient condition (an “active” Taylor rule)

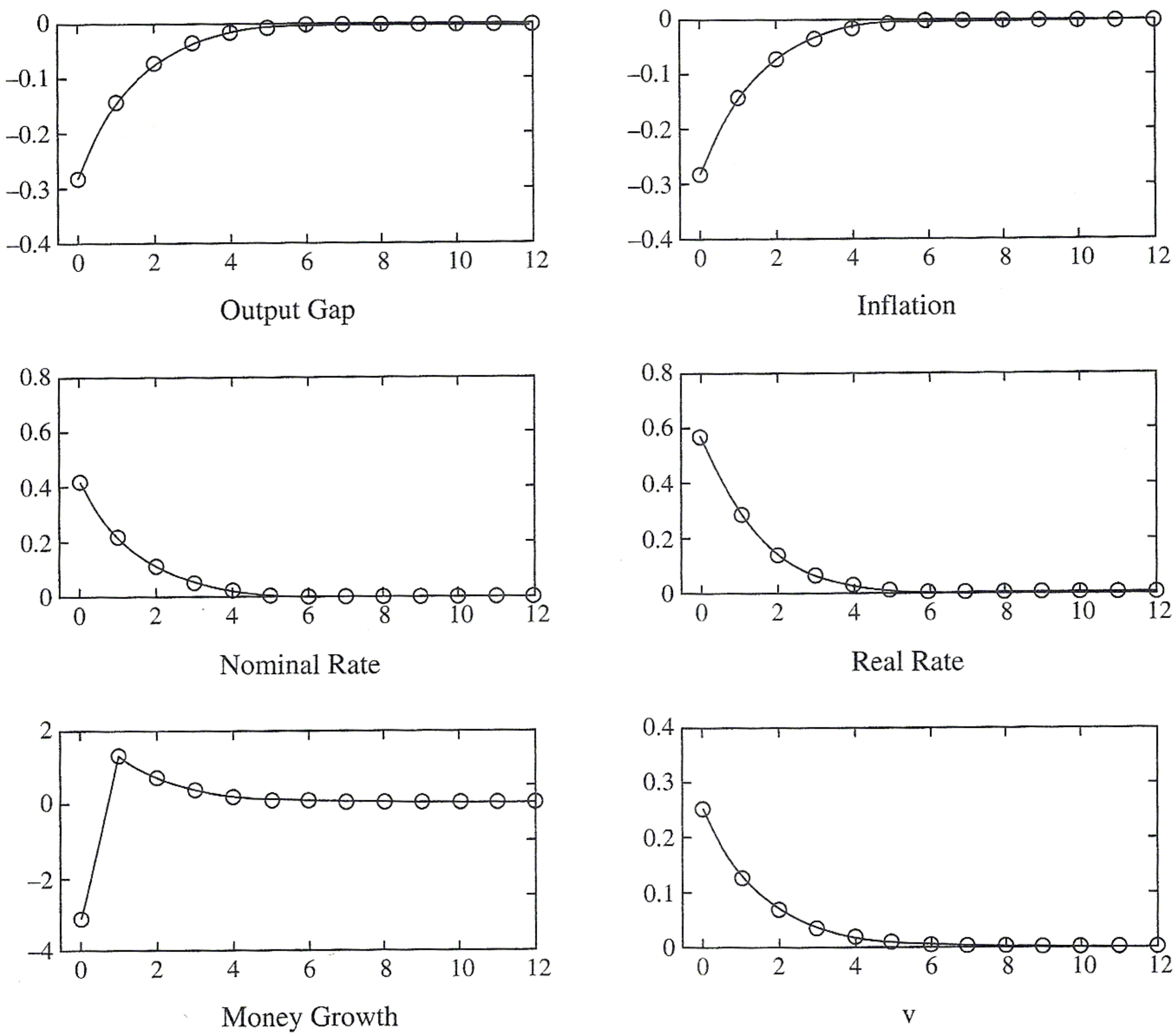


Figure 3.1 Effects of a Monetary Policy Shock (Interest Rate Rule)

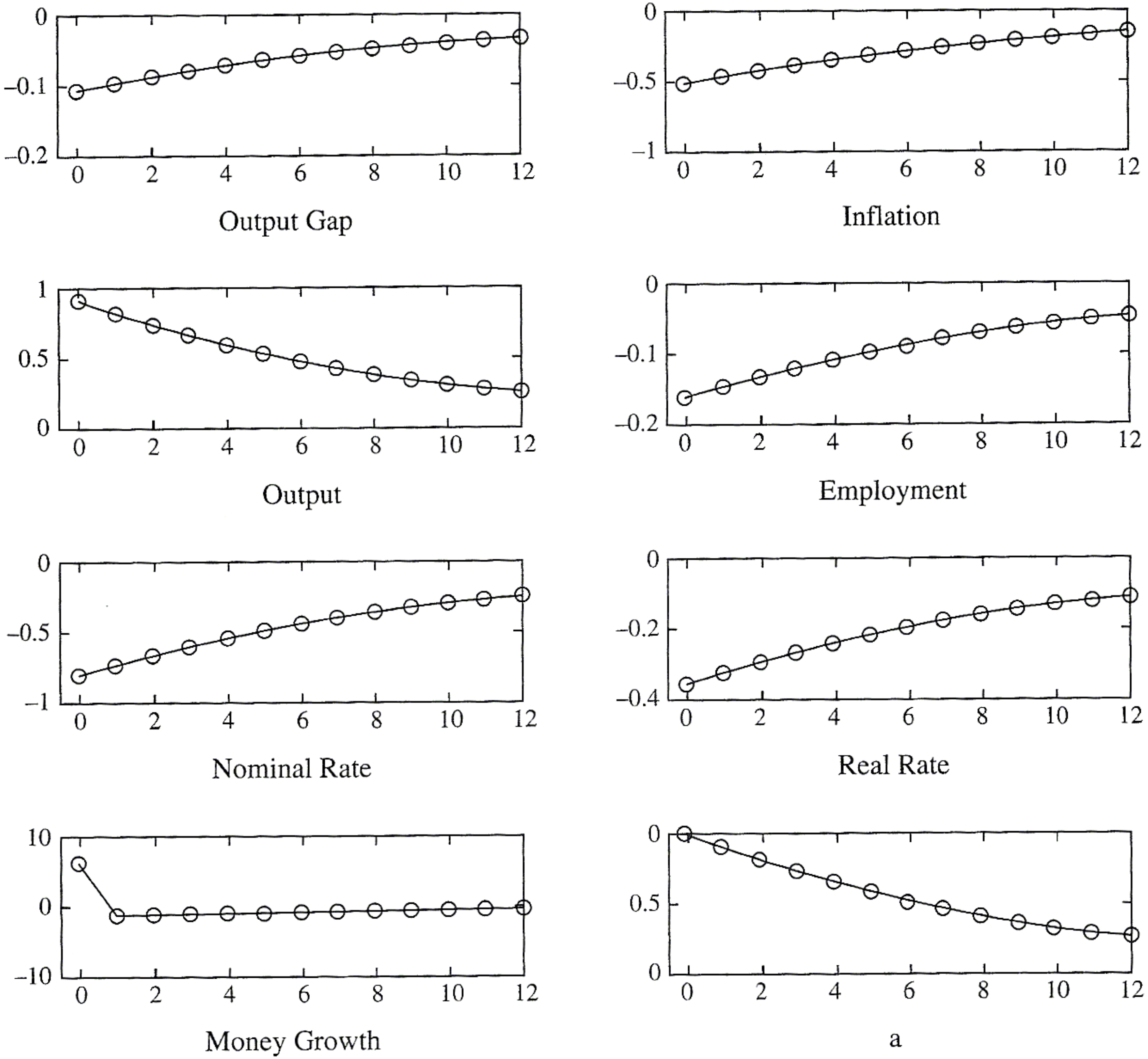


Figure 3.2 Effects of a Technology Shock (Interest Rate Rule)

Concluding remarks

- The New-Keynesian model is (compared to the classical model) a simple but powerful tool for analyzing monetary policy

- Given monetary policy works more in accordance with evidence, how can the model be used for policy evaluation?

- To come:
  - Welfare effects of different policies
  - Optimal monetary policy and credibility issues

Next time(s)

Monday, December 6: Exercises:

- Derive the dynamics of the New-Keynesian model on matrix form (or, state-space form), (26)
- Derive the condition for determinacy, (27) in Galí, Chapter 3. Hint: A 2×2 matrix has two stable roots, when the coefficients in the characteristic polynomial $\tau^2 + a_1\tau + a_0$ satisfy $|a_0| < 1$ and $|a_1| < 1 + a_0$
- Derive the explicit solutions for the output gap, inflation, and the nominal interest rate and output in New-Keynesian model of Galí, Chapter 3 (page 51) under an interest-rate rule (and in the absence of technology shocks).
  - Interpret the solutions economically

Tuesday, December 7:
Lecture: Monetary policy design and welfare in the simple NK Model (Galí, Chapter 4)