Lecture 3, November 30: The Basic New Keynesian Model (Galí, Chapter 3)
Introduction

- The classical flex-price model(s) covered so far cannot account for realistic short-run dynamics following monetary shocks
  - In case there are any real effects, the magnitude appears modest
  - No liquidity effect present (long run = short run)

Some “sand in the wheels” are necessary to explain the effects of money on output seen in data

- Obvious candidate is incomplete nominal adjustment. In flex-price models, money neutrality holds both in the short and long run
  - If money neutrality fails in the short run, the impact of monetary policy will clearly be stronger

- Classical model is therefore amended with nominal rigidities (and explicit determination of prices by firms)

- Result is Dynamic Stochastic General Equilibrium (DSGE) model of the New-Keynesian type
A basic New-Keynesian model

• Goods market
  – Demand side: Households consume a basket of goods (based on utility maximization)
  – Supply side: Firms produce different consumption goods (maximize profits under monopolistic competition)

• Labor market
  – Demand side: Firms hire labor (maximize profits under monopolistic competition)
  – Supply side: Households supply labor (based on utility maximization)

• Financial markets
  – Households optimally invest in a one-period risk-less bond
  – Households also hold money. We again just assume it (and generally do not make much use of it here, as focus will be on interest-rate policies)
• *Wages* are perfectly flexible (this assumption is relaxed in Chapter 6)

• *All goods prices* are subject to inflexibility, i.e., stickiness:
  – Every period, each firm faces a state-independent probability $\theta$ of “being stuck” with its price
  – So, $0 \leq \theta \leq 1$ is natural “index” for stickiness in the economy

• The probability is *independent* of when the firm last changed its price; i.e., time-dependent price stickiness (as opposed to state-dependent price stickiness)

• Stylistic representation of “staggering.” The independence of history facilitates aggregation:
  – $\theta$ is the fraction of firms not adjusting prices in a period
  – $1 - \theta$ is the fraction of firms adjusting is a period
  – $1 + \theta + \theta^2 + \theta^3 + ... = 1 / (1 - \theta)$ is average duration of a price contract

• When “allowed” to change the price, expectations about future prices become of importance
A view at prices changes: Belgium, 2001-2005

Source: Cornille and Dossche (2006)
A view at prices changes: France and Italy, 1996-2001

Source: Dhyne et al. (2005)
A view at prices changes: (Monthly) frequencies of price adjustments; Euro area vs. US

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A view at prices changes: France, hazard rates in clothing industry

Source: Baudry et al. (2004)
A view at prices changes: France: Hazard Rates in service industry

Source: Baudry et al. (2004)
Summary of findings from data:

- Aggregate prices are sticky (as also indicated by VAR analyses)
- Price setting in different sectors differs;
  - some sectors exhibit long periods of fixed prices (time dependent price setting)
  - some sectors exhibit continuous changing prices
- Price setting is asynchronized across sectors
- In some sectors, the probability of price adjustment is (nearly) independent of the life-span of the price

Relation to theoretical modelling of price rigidities:

- The staggered price setting scheme captures many of these features, and their aggregate implications, in a simple and handy way
- Modelling due to Calvo (1983); now just called “Calvo pricing”
Household behavior

- The essentials of household behavior are like in the basic classical model

- Household maximize expected utility

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t), \quad 0 < \beta < 1.
\]

- Main difference: \( C_t \) is not one good, but a basket of the different goods produced by the different producers (indexed by \( i \in [0, 1] \))

\[
C_t \equiv \left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{1}{\varepsilon-1}}, \quad \varepsilon > 1
\]

- Budget constraint therefore becomes:

\[
\int_0^1 P_t(i) C_t(i) \, di + Q_t B_t \leq B_{t-1} + W_t N_t - T_t
\]

(and a No-Ponzi Game constraint ruling out explosive debt)

- Household chooses optimal paths of \( C_t, N_t \) and \( B_t \) (taking prices \( P_t, W_t \) and \( Q_t \) as given), but must now choose the composition of \( C_t \) as well
Finding the optimal composition of $C_t$:

- Assume that total nominal expenditures on consumption are $Z_t$

$$\int_0^1 P_t(i) C_t(i) \, di = Z_t$$

- Optimal demand for $C_t(i)$, is found by

$$\max_{C_t(i)} \left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}} \text{ s.t. } \int_0^1 P_t(i) C_t(i) \, di = Z_t$$

- The relevant first-order condition is

$$\left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{1}{\varepsilon-1}} \frac{1}{\varepsilon} C_t(i)^{\frac{1}{\varepsilon}} = \lambda P_t(i),$$

$$\left[ \int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{1}{\varepsilon-1}} \frac{1}{\varepsilon} C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} = \lambda P_t(i) C_t(i)$$

where $\lambda$ is the Lagrange multiplier associated with the constraint
• Integrating over all goods:

\[
\left[ \int_{0}^{1} C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{1}{\varepsilon-1}} \int_{0}^{1} C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di = \lambda \int_{0}^{1} P_t(i) C_t(i) \, di \\
C_t = \lambda Z_t
\]

• Let \( P_t \) be the price index associated with \( C_t \), then \( P_t C_t = Z_t \) and \( \lambda = \frac{1}{P_t} \)

• Inserted into first-order condition:

\[
\left[ \int_{0}^{1} C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right]^{\frac{1}{\varepsilon-1}} C_t(i)^{-\frac{1}{\varepsilon}} = \frac{P_t(i)}{P_t}, \\
C_t^{\frac{1}{\varepsilon}} C_t(i)^{-\frac{1}{\varepsilon}} = \frac{P_t(i)}{P_t}
\]

• The relative demand for good \( i \) is therefore

\[
C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t
\] (1)

• Note that \( \varepsilon \) is the elasticity of substitution between goods. (Hence, the limiting case of perfect competition applies when \( \varepsilon \to \infty \).)
The price index

- The precise form of the price index $P_t$ follows from

$$P_t C_t = \int_0^1 P_t(i) C_t(i) \, di$$

and the expression for relative demand

- I.e., we get

$$P_t C_t = \int_0^1 P_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-\varepsilon} C_t \, di$$

$$P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}}$$
Remaining household optimality conditions

- Precisely as in the simple classical model:

\[
\frac{-U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}
\]

\[
Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}
\]

- With \( U(C_t, N_t) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \), \( \sigma, \varphi > 0 \), we have the same log-linear expressions:

  - Labor supply schedule:

    \[
    \sigma c_t + \varphi n_t = w_t - p_t
    \]  \hspace{1cm} (2)

  - Consumption-Euler equation:

    \[
    c_t = E_t \{ c_{t+1} \} - \sigma^{-1} \left( i_t - E_t \{ \pi_{t+1} \} \right) - \rho
    \]  \hspace{1cm} (3)

    with \( i_t \equiv -\log Q_t \) and \( \rho \equiv -\log \beta \)
Firms’ behavior and aggregate inflation dynamics

- Generally, optimal price-setting of a firm that can change its price in period $t$:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( P_t^* Y_{t+k|t} - \Psi_{t+k} (Y_{t+k|t}) \right) \right\}$$

$$\text{s.t. } Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon} C_{t+k} \tag{8}$$

- I.e., the maximal expected discounted stream of profits of choosing $P_t^*$ taking into account the probabilities that it is fixed in the future

- $\Psi_{t+k}$ is total costs of sales at price $P_t^*$ in period $t + k$ (will depend on labor costs and productivity)

- $Q_{t,t+k}$ is the “stochastic discount factor” (or present-value operator), equal to the one facing consumers between periods $t$ and $t + k$:

$$Q_{t,t+k} \equiv \beta^k \left( \frac{C_{t+k}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+k}} \right)$$

Note

$$Q_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right)$$

$$Q_{t,t+2} = \beta^2 \left( \frac{C_{t+2}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+2}} \right) = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \beta \left( \frac{C_{t+2}}{C_{t+1}} \right)^{-\sigma} \left( \frac{P_{t+1}}{P_{t+2}} \right) = Q_{t,t+1} Q_{t+1,t+2}$$

$$Q_{t,k} = Q_{t,t+1} Q_{t+1,t+2} \cdots Q_{t+k-2,t+k-1} Q_{t+k-1,k}$$
• First-order condition for $P_t^*$:

$$
\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left( Y_{t+k|t} + P_t^* \frac{\partial Y_{t+k|t}}{\partial P_t^*} - \psi_{t+k|t} \frac{\partial Y_{t+k|t}}{\partial P_t^*} \right) \right\} = 0,
$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k} (Y_{t+k|t})$ is nominal marginal costs

$$
\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( 1 + \frac{P_t^*}{Y_{t+k|t}} \frac{\partial Y_{t+k|t}}{\partial P_t^*} - \psi_{t+k|t} \frac{1}{Y_{t+k|t}} \frac{\partial Y_{t+k|t}}{\partial P_t^*} \right) \right\} = 0,
$$

$$
\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( 1 - \varepsilon + \psi_{t+k|t} \frac{\varepsilon}{P_t^*} \right) \right\} = 0.
$$

• We then get the central pricing equation:

$$
\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left( P_t^* - \mathcal{M} \psi_{t+k|t} \right) \right\} = 0,
$$

(9)

with $\mathcal{M} \equiv \varepsilon / (\varepsilon - 1) > 1$ being the mark-up; a measure of the monopoly distortion
This is rewritten as
\[
\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,k|t} Y_{t+k|t} \left( \frac{P_t^*}{P_{t-1}} - M MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0, \tag{10}
\]
where \( MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k} \) is real marginal costs and \( \Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1} \) is gross inflation between \( t - 1 \) and \( t + k \).

Optimality condition is log-linearized around a zero-inflation steady state:
\[
\frac{P_t^*}{P_{t-1}} = 1, \quad \Pi_{t-1,t+k} = 1, \quad Y_{t+k|t} = Y, \quad MC = M^{-1}, \quad Q_{t,t+k} = \beta^k
\]

One gets
\[
\sum_{k=0}^{\infty} (\beta \theta)^k E_t \left\{ p_t^* - p_{t-1} - \hat{mc}_{t+k|t} - p_{t+k} + p_{t-1} \right\} = 0
\]
where \( \hat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc = mc_{t+k|t} + \log M = mc_{t+k|t} + \log \frac{\epsilon}{\epsilon - 1} = mc_{t+k|t} + \mu \)

Hence,
\[
\sum_{k=0}^{\infty} (\beta \theta)^k p_t^* = \sum_{k=0}^{\infty} (\beta \theta)^k E_t \left\{ \hat{mc}_{t+k|t} + p_{t+k} \right\}
\]
\[
p_t^* = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \left\{ \hat{mc}_{t+k|t} + p_{t+k} \right\}
\]

Optimal price is a function of expected current and future real marginal costs and aggregate prices.
• Marginal costs depend on production function and factor payments

• Each firm has the following production function (i.e., identical technology):

\[ Y_t(i) = A_t N_t(i)^{1-\alpha}, \quad 0 \leq \alpha < 1 \] (5)

• Total costs

\[ TC_t = W_t N_t(i) \]

Real marginal costs

\[ MC_t(i) = \frac{W_t}{P_t A_t (1 - \alpha) N_t(i)^{-\alpha}} \]

\[ = \frac{W_t}{P_t MPN_t} \]

\[ = \frac{W_t}{P_t A_t (1 - \alpha) (Y_t(i)/A_t)^{-\frac{\alpha}{1-\alpha}}} \]

\[ = \frac{W_t}{P_t (1 - \alpha) A_t^{\frac{1}{1-\alpha}} Y_t(i)^{-\frac{\alpha}{1-\alpha}}} \]

• Special case of constant returns to scale, \( \alpha = 0 \)

\[ MC_{t+k|t} = MC_{t+k} \left( = \frac{W_{t+k}}{P_{t+k} A_{t+k}} \right) \]

\[ p_t^* = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \{ \hat{mc}_{t+k} + p_{t+k} \} \]

• This is the unique stationary solution to the first-order rational expectations difference equation:

\[ p_t^* = \beta \theta E_t \{ p_{t+1}^* \} + (1 - \beta \theta) (\hat{mc}_t + p_t) \]
Aggregate price dynamics

- Remember, $\theta$ is fraction of price setters that keep past period’s price; $\bar{\theta}$ is fraction of price setters that set new prices.

- By definition of the price index:

$$P_t = \left[ \theta (P_{t-1})^{1-\varepsilon} + (1-\theta) (P_t^*)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \implies \Pi_t^{1-\varepsilon} = \theta + (1-\theta) \left( \frac{P_t^*}{P_{t-1}} \right)^{1-\varepsilon}$$

- Log-linearized around the zero-inflation steady state:

$$(1-\varepsilon) \pi_t = (1-\varepsilon) (1-\theta) (p_t^* - p_{t-1}) \implies \pi_t = (1-\theta) (p_t^* - p_{t-1}) \quad (7)$$

- Rewrite the difference equation for optimal price setting:

$$p_t^* - p_{t-1} = \beta \theta E_t \{p_{t+1}^* - p_t\} + (1-\beta \theta) (\hat{m}c_t + p_t) - p_{t-1} + \beta \theta p_t$$

$$p_t^* - p_{t-1} = \beta \theta E_t \{p_{t+1}^* - p_t\} + (1-\beta \theta) \hat{m}c_t + \pi_t$$

- Use aggregate dynamics to obtain:

$$(1-\theta)^{-1} \pi_t = (1-\theta)^{-1} \beta \theta E_t \{\pi_{t+1}\} + (1-\beta \theta) \hat{m}c_t + \pi_t$$

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \hat{m}c_t, \quad \lambda \equiv \frac{(1-\theta)(1-\beta \theta)}{\theta}, \quad \lim_{\theta \to 0} \lambda = \infty.$$ 

This is the basic inflation-adjustment equation in New-Keynesian theory.
General case of decreasing returns to scale, $0 < \alpha < 1$

- We define aggregate real marginal costs (in logs):
  \[
  mc_t \equiv w_t - p_t - mpn_t \\
  = w_t - p_t - \frac{1}{1-\alpha} (a_t - \alpha y_t) - \log (1-\alpha)
  \]

- We have for a price-changing firm:
  \[
  mc_{t+k|t} = w_{t+k} - p_{t+k} - \frac{1}{1-\alpha} (a_{t+k} - \alpha y_{t+k|t}) - \log (1-\alpha) \\
  = mc_{t+k} + \frac{\alpha}{1-\alpha} (y_{t+k|t} - y_{t+k})
  \]

- By the demand function in logs (here using equilibrium condition $y_{t+k} = c_{t+k}$):
  \[
  mc_{t+k|t} = mc_{t+k} - \frac{\alpha \varepsilon}{1-\alpha} (p^*_t - p_{t+k}) \tag{14}
  \]

- Optimal price setting:
  \[
  p^*_t = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \left\{ \widehat{mc}_{t+k} - \frac{\alpha \varepsilon}{1-\alpha} (p^*_t - p_{t+k}) + p_{t+k} \right\}
  \]

- Associated inflation dynamics
  \[
  \pi_t = \beta E_t \{\pi_{t+1}\} + \lambda \widehat{mc}_t, \quad \lambda \equiv \frac{(1-\theta)(1-\beta \theta)}{\theta} \frac{1-\alpha}{1-\alpha + \alpha \varepsilon}, \quad \lim_{\theta \to 0} \lambda = \infty. \tag{16}
  \]
Equilibrium outcomes

- Goods market clearing on each goods market
  \[ Y_t(i) = C_t(i), \quad \text{all } 0 \leq i \leq 1 \]

- In the aggregate:
  \[ Y_t = C_t, \]
  where
  \[ Y_t \equiv \left[ \int_0^1 Y_t(i) \frac{\frac{\varepsilon}{\varepsilon-1} - 1}{\varepsilon - 1} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}. \]

- Asset market equilibrium (as in classical economy):
  \[ y_t = E_t \{ y_{t+1} \} - \sigma^{-1} (i_t - E_t \{ \pi_{t+1} \} - \rho) \]  \hspace{1cm} (12)
• Aggregate employment:

\[ N_t = \int_0^1 N_t(i) \, di = \int_0^1 \left( \frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \, di \]

\[ N_t = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} \, di \]

\[ \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \, di \]

• In logs:

\[ n_t = \frac{1}{1 - \alpha} (y_t - a_t) + \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}}, \]

\[ (1 - \alpha) n_t = y_t - a_t + d_t, \quad d_t \equiv (1 - \alpha) \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}}. \]

• The variable \( d_t \) is a natural measure of price dispersion (proportional to the variance of prices)

• When we consider log deviations from a symmetric zero-inflation steady state, \( d_t \) can be ignored (it is of second order—see Appendix 3.2 and 3.3)

• Hence, as a log-linear approximation:

\[ y_t = a_t + (1 - \alpha) n_t, \quad (13) \]

as in the classical case
Relationship between real marginal cost and output

- We want a dynamic system in output and inflation; hence, marginal cost is replaced appropriately by output.

- We have

\[
mc_t = w_t - p_t - \frac{1}{1 - \alpha} (a_t - \alpha y_t) - \log (1 - \alpha)
\]

- Using the labor supply schedule

\[
mc_t = \sigma c_t + \varphi n_t - \frac{1}{1 - \alpha} (a_t - \alpha y_t) - \log (1 - \alpha)
\]

\[
m_{ct} = \sigma y_t + \frac{\varphi}{1 - \alpha} (y_t - a_t) - \frac{1}{1 - \alpha} (a_t - \alpha y_t) - \log (1 - \alpha)
\]

\[
= \frac{\sigma (1 - \alpha) + \varphi + \alpha}{1 - \alpha} y_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log (1 - \alpha) 
\]

(17)

- Under flexible prices, \( \theta = 0 \), \( \hat{mc}_t = 0 \)

\[
mc_t = mc
\]

\[
= -\mu \ (= -\ln M)
\]

- Hence, denoting \( y^n_t \) the flexible-price output, or, the natural rate of output,

\[
mc = -\mu = \frac{\sigma (1 - \alpha) + \varphi + \alpha}{1 - \alpha} y^n_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log (1 - \alpha)
\]
• The natural rate of output is given as
\[
y_t^n = \frac{1 + \varphi}{\sigma (1 - \alpha) + \varphi + \alpha} a_t - \frac{[\mu - \log (1 - \alpha)] (1 - \alpha)}{\sigma (1 - \alpha) + \varphi + \alpha} + \psi_{ya} a_t + \vartheta_y^n
\]
(19)

• Under perfect competition, \( \mu = 0 \), this is output as in the classical model

• We get the marginal cost in deviation from steady state:
\[
\widehat{mc}_t = mc_t - m = \sigma (1 - \alpha) + \varphi + \alpha \left( y_t - y_t^n \right) / (1 - \alpha)
\]
(20)

• We denote \( \tilde{y}_t \equiv y_t - y_t^n \) the output gap

• The “New-Keynesian” Phillips Curve (“NKPC”):
\[
\pi_t = \beta \mathbb{E}_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t, \quad \kappa \equiv \frac{\lambda \sigma (1 - \alpha) + \varphi + \alpha}{1 - \alpha}
\]
(21)
Rephrase the Euler equation in terms of the output gap:

\[
\begin{align*}
y_t &= E_t \{ y_{t+1} \} - \sigma^{-1} (i_t - E_t \{ \pi_{t+1} \} - \rho) \\
\tilde{y}_t &= E_t \{ \tilde{y}_{t+1} \} - \sigma^{-1} (i_t - E_t \{ \pi_{t+1} \} - \rho) - y_t^n + E_t \{ y_{t+1}^n \} \\
\tilde{y}_t &= E_t \{ \tilde{y}_{t+1} \} - \sigma^{-1} (i_t - E_t \{ \pi_{t+1} \} - r^n_t)
\end{align*}
\]

(22)

where

\[
r^n_t \equiv \rho + \sigma E_t \{ \Delta y^n_{t+1} \}
\]

is the \textit{natural rate of interest}

- A dynamic “IS” curve (“DIS”)
- Let \( \tilde{r}^n_t \equiv r^n_t - \rho \)

New-Keynesian Phillips curve and dynamic IS curve determines output gap and inflation conditional on monetary policy \( (i_t) \).

So, in contrast with the classical model, monetary policy matters for output determination!
Interest rate policy in the New Keynesian model

• The case of a simple Taylor-type interest rate rule:

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t, \quad \phi_\pi, \phi_y \geq 0, \]  

where \( v_t \) is a policy shock, \( v_t = \rho_v v_{t-1} + \varepsilon_t^v \)

\[ (25) \]

• The policy rule, NKPC and DIS give the dynamics

\[ \begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t \{ \tilde{y}_{t+1} \} \\ E_t \{ \pi_{t+1} \} \end{bmatrix} + B_T (\hat{r}_t - v_t), \]  

\[ (26) \]

\[ A_T \equiv \Omega \begin{bmatrix} \sigma \\ \sigma \kappa \\ \kappa + \beta (\sigma + \phi_y) \end{bmatrix}, \quad B_T \equiv \Omega \begin{bmatrix} 1 \\ 1 \\ \kappa \end{bmatrix}, \quad \Omega \equiv \frac{1}{\sigma + \phi_\pi \kappa + \phi_y} \]

• A unique non-explosive equilibrium requires the eigenvalues of \( A_T \) to be stable (within the unit circle)

• Uniqueness depends on policy-rule parameters:

\[ 0 < (\phi_\pi - 1) \kappa + \phi_y (1 - \beta) \]  

\[ (27) \]

is necessary and sufficient condition (the “Taylor principle”)

- Note \( \phi_\pi > 1 \) is sufficient condition (an “active” Taylor rule)
Figure 3.1 Effects of a Monetary Policy Shock (Interest Rate Rule)
Figure 3.2  Effects of a Technology Shock (Interest Rate Rule)
Concluding remarks

• The New-Keynesian model is (compared to the classical model) a simple but powerful tool for analyzing monetary policy

• Given monetary policy works more in accordance with evidence, how can the model be used for policy evaluation?

• To come:
  – Welfare effects of different policies
  – Optimal monetary policy and credibility issues
Next time(s)

Monday, December 6: Exercises:

• Derive the dynamics of the New-Keynesian model on matrix form (or, state-space form), (26)

• Derive the condition for determinacy, (27) in Galí, Chapter 3. Hint: A $2 \times 2$ matrix has two stable roots, when the coefficients in the characteristic polynomial $\tau^2 + a_1 \tau + a_0$ satisfy $|a_0| < 1$ and $|a_1| < 1 + a_0$

• Derive the explicit solutions for the output gap, inflation, and the nominal interest rate and output in New-Keynesian model of Galí, Chapter 3 (page 51) under an interest-rate rule (and in the absence of technology shocks).
  
  – Interpret the solutions economically

Tuesday, December 7:
Lecture: Monetary policy design and welfare in the simple NK Model (Galí, Chapter 4)