MakØk3, Fall 2010 (blok 2)

"Business cycles and monetary stabilization policies"

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Lecture 4, December 7: **Monetary Policy Design in the Basic New Keynesian Model** (Galí, Chapter 4)

- We have now developed a simple model for business cycle and monetary policy analysis
  - E.g., we can examine the economy's response to various shocks (including policy shocks)

- Next step is to examine the model's normative implications: I.e., how should monetary policy be conducted?
- What should be the goals of monetary policy?
- What can and what cannot monetary policy achieve

- For this purpose we identify the inefficiencies of the New Keynesian economy, and evaluate whether and how policy can remedy these
- Importantly, a model consistent welfare criterion will be developed to assess various simple, suboptimal policy rules

Properties of the New-Keynesian model

Basic equations summarized:

- "NKPC"

$$\pi_t = \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \widetilde{y}_t, \qquad \widetilde{y}_t \equiv y_t - y_t^n$$

$$\kappa \ \equiv \ \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha} > 0$$

- "DIS"

$$\widetilde{y}_t = \mathrm{E}_t \{\widetilde{y}_{t+1}\} - \sigma^{-1} (i_t - \mathrm{E}_t \{\pi_{t+1}\} - r_t^n), \qquad r_t^n \equiv \rho + \sigma \mathrm{E}_t \{\Delta y_{t+1}^n\}$$

Properties of a (friendly) "command economy"

- Relevant benchmark, or, ideal outcome, is the allocation chosen by a benevolent social planner (it identifies the efficient outcomes)

- The planner maximizes

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(\left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \,\mathrm{d}i\right]^{\frac{\varepsilon}{\varepsilon-1}}, \int_0^1 N_t(i)\,\mathrm{d}i\right),$$

  subject to

$$C_t(i) = A_t N_t(i)^{1-\alpha} \quad \text{all } i \in [0,1]$$

- By nature of the consumption basket, $C_t(i) \neq C_t(j)$ is never optimal for a given C_t. Therefore, optimality requires

$$C_t(i) = C_t, \qquad N_t(i) = N_t, \qquad \text{all } i \in [0,1].$$

- The problem then simplifies to

$$\max U\left(A_t N_t^{1-a}, N_t\right)$$

  Optimality condition:

$$(1-\alpha) A_t N_t^{-\alpha} U_{c.t} + U_{n,t} = 0 \qquad -\frac{U_{n,t}}{U_{c.t}} = (1-\alpha) A_t N_t^{-\alpha} \equiv MPN_t$$

Inefficiencies in the New Keynesian model

Monopolistic competition

- Monopolistic competition implies that prices are a markup over aggregate marginal costs; even under flexible prices:

$$P_t = \mathcal{M}\frac{W_t}{MPN_t}, \qquad \mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1} > 1$$

- The model's labor market equilibrium:

$$-\frac{U_{n,t}}{U_{c.t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Monopolistic competition results in too low employment and output

- Monetary policy is useless in addressing this market-structure inefficiency

- Fiscal (tax) policy can (in theory) solve the problem. Assume a labor cost subsidy τ (financed lump sum from consumers):

$$P_t = \mathcal{M}\frac{(1-\tau)\,W_t}{MPN_t}$$

  - Monopoly distortion is eliminated if $\mathcal{M}\,(1-\tau) = 1$; this is assumed
  - Requires $\tau = \varepsilon^{-1}$

Nominal rigidities

- Price rigidities cause mark-up fluctuations with sticky prices:

$$\mathcal{M}_t = \frac{P_t}{(1-\tau)\, W_t MPN_t} = \frac{P_t \mathcal{M}}{W_t MPN_t}$$

$$\frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

- Staggered price setting causes price and thus output dispersion:

$$C_t(i) \neq C_t(j) \quad \text{when} \quad P_t(i) \neq P_t(j)$$

- Inefficiencies due to nominal rigidities can be addressed by monetary policy (at least in part)

What should monetary policy ideally do?

- Assume the labor subsidy $\tau = \varepsilon^{-1}$ is in place; the natural rate of output is then efficient

- Eliminate markup fluctuations, i.e., secure that $\widehat{mc}_t = 0$
  - Equivalent of securing:

$$\widetilde{y}_t = 0 \text{ all } t$$

- Avoid any price dispersion
  - Assuming no past relative price dispersion, $P_{t-1}(i) = P_{t-1}$, all $i \in [0,1]$
  - No firms will change prices when $\widehat{mc}_t = 0$, $mc_t = mc$
  - Hence, $P_{t+j}(i) = P_{t+j} = P_{t-1+j}$, all $i \in [0,1]$, $j = 0, 1, 2, , ,$
  - Equivalent of fully stable aggregate prices:

$$\pi_t = 0 \text{ all } t$$

How can this be done?

- Surprisingly there is no policy trade-offs—the ideal policy goals are attainable
  - (Special to the simple shock structure; by some denoted "a divine coincidence".)

- Letting $i_t = r_t^n$, is compatible with attaining both $\widetilde{y}_t = 0$ and $\pi_t = 0$

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \widetilde{y}_t, \\ \widetilde{y}_t &= \mathrm{E}_t \{\widetilde{y}_{t+1}\} - \sigma^{-1} (i_t - \mathrm{E}_t \{\pi_{t+1}\} - r_t^n) \end{aligned}$$

- Problem. Setting $i_t = r_t^n$ leads to dynamic the system

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} \mathrm{E}_t \{\widetilde{y}_{t+1}\} \\ \mathrm{E}_t \{\pi_{t+1}\} \end{bmatrix}, \qquad \mathbf{A_O} \equiv \begin{bmatrix} 1 & \sigma^{-1} \\ \kappa & \beta + \kappa\sigma^{-1} \end{bmatrix}$$

- $\mathbf{A_O}$ has one eigenvalue above one, and one below one. Indeterminacy; i.e., infinitely many stationary inflation and output gap paths

- One could therefore follow the previously considered Taylor rule, amended with a response to the natural rate of interest:

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t, \qquad \phi_\pi, \phi_y \geq 0,$$

- This leads to the familiar dynamics

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} \mathrm{E}_t \{\widetilde{y}_{t+1}\} \\ \mathrm{E}_t \{\pi_{t+1}\} \end{bmatrix}$$

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \phi_\pi \beta \\ \sigma\kappa & \kappa + \beta\left(\sigma + \phi_y\right) \end{bmatrix}, \qquad \Omega \equiv \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}$$

- Uniqueness requires:

$$0 < (\phi_\pi - 1)\,\kappa + \phi_y\,(1 - \beta)$$

(the "Taylor principle")

- The optimal allocation will be achieved in equilibrium

- Such a rule, however, poses a practical problem: r_t^n is not observed in real time
- Therefore, more simple rules can be considered; i.e., rules depending on observable variables

- But how should one to assess their performance?
- I.e., how is it possible to compare one rule to another?

- By developing a welfare criterion!

Welfare criterion in the NK model

- The relevant welfare criterion in the NK model is

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\}$$

- How do we use this together with the log-linearized model?
- W is approximated by a second-order Taylor expansion (a first-order expansion would not rank different monetary policies, as these do not affect longs-run levels; i.e., steady states)

- Important second-order approximation for a variable Z:

$$\frac{Z_t - Z}{Z} \simeq \widehat{z}_t + \frac{1}{2} \widehat{z}_t^2$$

  where $\widehat{z}_t \equiv \log(Z_t / Z)$

- The approximation is performed around an efficient steady state—yields a simple expression
- If the approximation is around an inefficient steady state, one may get "spurious" welfare results by using a log-linear model (the ignored second-order terms may become important, i.e., policy dependent)

Initial Taylor expansion

$$U_t - U \simeq U_c C \left(\frac{C_t - C}{C}\right) + U_n N \left(\frac{N_t - N}{N}\right) + \frac{1}{2} U_{cc} C^2 \left(\frac{C_t - C}{C}\right)^2 + \frac{1}{2} U_{nn} N^2 \left(\frac{N_t - N}{N}\right)^2$$

(hence, separability, $U_{cn} = 0$, is assumed)

- In log-deviations

$$\begin{aligned} U_t - U &\simeq U_c C \left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) + U_n N \left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2\right) + \frac{1}{2} U_{cc} C^2 \left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right)^2 + \frac{1}{2} U_{nn} N^2 \left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2\right)^2 \\ &\simeq U_c C \left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2\right) + U_n N \left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2\right) + \frac{1}{2} U_{cc} C^2 \widehat{c}_t^2 + \frac{1}{2} U_{nn} N^2 \widehat{n}_t^2 \end{aligned}$$

as $\widehat{c}_t^3 \simeq \widehat{c}_t^4 \simeq \widehat{n}_t^3 \simeq \widehat{n}_t^4 \simeq 0$ in a second-order expansion

- Rearranging:

$$U_t - U \simeq U_c C \left(\widehat{c}_t + \frac{1}{2}\widehat{c}_t^2 + \frac{1}{2}\frac{U_{cc} C}{U_c}\widehat{c}_t^2\right) + U_n N \left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2 + \frac{1}{2}\frac{U_{nn} N}{U_n}\widehat{n}_t^2\right)$$

- Simplifying:

$$U_t - U \simeq U_c C \left(\widehat{c}_t + \frac{1-\sigma}{2}\widehat{c}_t^2\right) + U_n N \left(\widehat{n}_t + \frac{1+\varphi}{2}\widehat{n}_t^2\right)$$

where

$$\sigma \equiv -\frac{U_{cc} C}{U_c} > 0, \qquad \varphi \equiv \frac{U_{nn} N}{U_n} > 0$$

- Using the goods-market equilibrium condition $\widehat{c}_t = \widehat{y}_t$:

$$U_t - U \simeq U_c C \left(\widehat{y}_t + \frac{1-\sigma}{2} \widehat{y}_t^2 \right) + U_n N \left(\widehat{n}_t + \frac{1+\varphi}{2} \widehat{n}_t^2 \right)$$

- Now comes a "tricky" part: Rewrite $\widehat{n}_t$ in terms of output

Relationship between employment, output and relative prices

- From last lecture:

$$N_t = \int_0^1 N_t(i)\, \mathrm{d}i = \int_0^1 \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \mathrm{d}i$$

$$N_t = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} \mathrm{d}i = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \mathrm{d}i$$

- In logs:

$$n_t = \frac{1}{1-\alpha} (y_t - a_t) + \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \mathrm{d}i,$$

$$(1-\alpha)\, n_t = y_t - a_t + d_t, \qquad d_t \equiv (1-\alpha) \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \mathrm{d}i.$$

- Around a zero inflation steady state (where $d = 0$):

$$(1-\alpha)\, \widehat{n}_t = \widehat{y}_t - a_t + d_t$$

- We need to find d_t, the measure of price dispersion, as it is second-order term that will have welfare effects.

- Start by the definition of the price index:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} \, \mathrm{d}i \right]^{\frac{1}{1-\varepsilon}}$$

- Then,

$$\begin{aligned} 1 &= \int_0^1 \left(\frac{P_t(i)}{P_t} \mathrm{d}i \right)^{1-\varepsilon} \\ &= \int_0^1 \exp\left[(1-\varepsilon)\left(p_t(i) - p_t \right) \right] \mathrm{d}i \\ &\simeq 1 + (1-\varepsilon) \int_0^1 \left(p_t(i) - p_t \right) \mathrm{d}i + \frac{(1-\varepsilon)^2}{2} \int_0^1 \left(p_t(i) - p_t \right)^2 \mathrm{d}i \qquad (*) \end{aligned}$$

in a second-order approximation around $p(i) = p$.

- Letting $\mathrm{E}_i \{p_t(i)\} \equiv \int_0^1 p_t(i) \mathrm{d}i$ be the mean of log prices across sectors,

$$p_t \simeq \mathrm{E}_i \{p_t(i)\} + \frac{1-\varepsilon}{2} \int_0^1 \left(p_t(i) - p_t \right)^2 \mathrm{d}i \qquad (**)$$

- Then assess the specific relative price expression of d_t:

$$\begin{aligned}\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} \mathrm{d}i &= \int_0^1 \exp\left[-\frac{\varepsilon}{1-\alpha}(p_t(i)-p_t)\right] \mathrm{d}i \\ &\simeq 1-\frac{\varepsilon}{1-\alpha}\int_0^1 (p_t(i)-p_t)\,\mathrm{d}i+\frac{1}{2}\left(\frac{\varepsilon}{1-\alpha}\right)^2\int_0^1 (p_t(i)-p_t)^2\,\mathrm{d}i\end{aligned}$$

- From (*) we have that

$$\int_0^1 (p_t(i)-p_t)\,\mathrm{d}i \simeq -\frac{1-\varepsilon}{2}\int_0^1 (p_t(i)-p_t)^2\,\mathrm{d}i$$

- Hence,

$$\begin{aligned}\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} \mathrm{d}i &\simeq 1+\frac{\varepsilon(1-\varepsilon)}{2(1-\alpha)}\int_0^1 (p_t(i)-p_t)\,\mathrm{d}i+\frac{1}{2}\left(\frac{\varepsilon}{1-\alpha}\right)^2\int_0^1 (p_t(i)-p_t)^2\,\mathrm{d}i \\ &\simeq 1+\frac{1}{2}\frac{\varepsilon}{1-\alpha}\frac{1}{\Theta}\int_0^1 (p_t(i)-p_t)^2\,\mathrm{d}i \qquad \Theta\equiv\frac{1-\alpha}{1-\alpha+\alpha\varepsilon}\end{aligned}$$

- Using (**) we get

$$\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\varepsilon}{1-\alpha}} \mathrm{d}i \simeq 1+\frac{1}{2}\frac{\varepsilon}{1-\alpha}\frac{1}{\Theta}\int_0^1 (p_t(i)-\mathrm{E}_i\{p_t(i)\})^2\,\mathrm{d}i = 1+\frac{1}{2}\frac{\varepsilon}{1-\alpha}\frac{1}{\Theta}\mathrm{var}_i\{p_t(i)\}$$

where $\mathrm{var}_i\{p_t(i)\}$ is price variance across sectors

- Since
$$d_t \equiv (1-\alpha) \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \mathrm{d}i$$
we get
$$d_t \simeq \frac{1}{2} \frac{\varepsilon}{\Theta} \mathrm{var}_i \left\{ p_t(i) \right\}$$
(which also proves that we rightfully ignored it when looking at the linear dynamics, as d_t is a second-order term)

- We then substitute $\widehat{n}_t = (1-\alpha)^{-1} \widehat{y}_t - (1-\alpha)^{-1} a_t + (1-\alpha)^{-1} d_t$ into
$$U_t - U \simeq U_c C \left(\widehat{y}_t + \frac{1-\sigma}{2} \widehat{y}_t^2 \right) + U_n N \left(\widehat{n}_t + \frac{1+\varphi}{2} \widehat{n}_t^2 \right)$$
and get
$$\begin{aligned} U_t - U \; &\simeq \; U_c C \left(\widehat{y}_t + \frac{1-\sigma}{2} \widehat{y}_t^2 \right) \\ &\quad + \frac{U_n N}{1-\alpha} \left(\widehat{y}_t + d_t + \frac{1+\varphi}{2(1-\alpha)} (\widehat{y}_t - a_t)^2 \right) + \text{t.i.p.} \end{aligned}$$
where t.i.p. is "terms independent of policy" and the third-order effects and higher are ignored

- Rewrite so we get utility change measured as percentage change in steady-state consumption:
$$\frac{U_t - U}{U_c C} \simeq \widehat{y}_t + \frac{1-\sigma}{2} \widehat{y}_t^2 + \frac{U_n N}{U_c C (1-\alpha)} \left(\widehat{y}_t + d_t + \frac{1+\varphi}{2(1-\alpha)} (\widehat{y}_t - a_t)^2 \right) + \text{t.i.p.}$$

- Now remember that we are approximating around an efficient steady state. Hence,

$$-\frac{U_n}{U_c} = MPN = (1-\alpha)\, AN^{-\alpha} \equiv (1-\alpha)\,\frac{Y}{N}$$

- Therefore,

$$-\frac{U_n}{U_c} = (1-\alpha)\,\frac{C}{N}$$

or,

$$-\frac{U_n}{U_c}\frac{N}{C\,(1-\alpha)} = -1$$

- The utility approximation therefore simplifies to

$$\begin{aligned}
\frac{U_t - U}{U_c C} &\simeq \frac{1-\sigma}{2}\widehat{y}_t^2 - \frac{1}{2}\frac{\varepsilon}{\Theta}\mathrm{var}_i\,\{p_t\,(i)\} - \frac{1+\varphi}{2\,(1-\alpha)}\,(\widehat{y}_t - a_t)^2 + \text{t.i.p.} \\
&= -\frac{1}{2}\left[\frac{\varepsilon}{\Theta}\mathrm{var}_i\,\{p_t\,(i)\} + (\sigma-1)\,\widehat{y}_t^2 + \frac{1+\varphi}{1-\alpha}\,(\widehat{y}_t - a_t)^2\right] + \text{t.i.p.} \\
&= -\frac{1}{2}\left[\frac{\varepsilon}{\Theta}\mathrm{var}_i\,\{p_t\,(i)\} + \left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right)\widehat{y}_t^2 - 2\frac{1+\varphi}{1-\alpha}\widehat{y}_t a_t\right] + \text{t.i.p.} \\
&= -\frac{1}{2}\left[\frac{\varepsilon}{\Theta}\mathrm{var}_i\,\{p_t\,(i)\} + \left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right)\left(\widehat{y}_t^2 - 2\frac{1+\varphi}{1-\alpha}\widehat{y}_t y_t^n\right)\right] + \text{t.i.p.}
\end{aligned}$$

as

$$y_t^n = \frac{1+\varphi}{\sigma\,(1-\alpha)+\varphi+\alpha}a_t.$$

- The welfare measure is therefore approximately

$$\mathbb{W} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c C} = -\frac{1}{2} \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\Theta} \mathrm{var}_i \left\{ p_t(i) \right\} + \left(\sigma + \frac{\alpha + \varphi}{1-\alpha} \right) \widetilde{y}_t^2 \right] + \text{t.i.p.}$$

- We finally use Lemma 2 from Woodford (2003):

$$\sum_{t=0}^{\infty} \beta^t \mathrm{var}_i \left\{ p_t(i) \right\} = \frac{\theta}{(1-\theta)(1-\beta\theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$$

- We then get

$$\begin{aligned} \mathbb{W} &= \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c C} = \\ & -\frac{1}{2} \left(\sigma + \frac{\alpha + \varphi}{1-\alpha} \right) \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\kappa} \pi_t^2 + \widetilde{y}_t^2 \right] \end{aligned}$$

as

$$\begin{aligned} \kappa &\equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha} \\ &= \frac{(1-\theta)(1-\beta\theta)}{\theta} \Theta \frac{\sigma(1-\alpha)+\varphi+\alpha}{1-\alpha} \end{aligned}$$

The performance of various policy rules

- With this utility-based welfare loss one can assess the performance of various policy rules
- One can perform optimal policy exercises as linear-quadratic optimization problems (next time)

- Galí exemplifies the importance of price stability in the New-Keynesian model by assessing the performance of the policy rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t$$

(note: a function of $\widehat{y}_t$, not $\widetilde{y}$) for various policy parameters:

	Taylor Rule			
ϕ_π	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1
$(\sigma_\zeta, \rho_\zeta)$	—	—	—	—
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40
$\sigma(\pi)$	2.60	1.33	0.21	6.55
welfare loss	0.30	0.08	0.002	1.92

Concluding remarks

- The New-Keynesian model offers a simple framework for welfare-based policy analysis

- Models is (in principle) immune to the Lucas critique, and the welfare criterion is consistent with the one used to derive the economy's behavioral equations

- One can rank various policy rules as well as meaningfully compare their quantitative welfare differences

- The simple model is obviously too simple to represent the real world, but its basic features "survive" in large-scale versions used in many inflation-targeting central banks

Next time(s)

Monday, December 13: Exercises:

- Prove *Lemma 2* on page 89 (this will yield a prize!)
- Exercise 4.1 in Galí (2008)
- Exercise 4.2 in Galí (2008)

Tuesday, December 14
Lecture: Monetary policy trade-offs, optimal policy and credibility issues (Galí, Chapter 5)