Lecture 4, December 7: Monetary Policy Design in the Basic New Keynesian Model (Galí, Chapter 4)
• We have now developed a simple model for business cycle and monetary policy analysis
  – E.g., we can examine the economy’s response to various shocks (including policy shocks)

• Next step is to examine the model’s normative implications: I.e., how should monetary policy be conducted?

• What should be the goals of monetary policy?

• What can and what cannot monetary policy achieve

• For this purpose we identify the inefficiencies of the New Keynesian economy, and evaluate whether and how policy can remedy these

• Importantly, a model consistent welfare criterion will be developed to assess various simple, suboptimal policy rules
Properties of the New-Keynesian model

Basic equations summarized:

- “NKPC”

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t, \quad \tilde{y}_t \equiv y_t - y^n_t \]

\[ \kappa \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \frac{\sigma (1 - \alpha) + \varphi + \alpha}{1 - \alpha} > 0 \]

- “DIS”

\[ \tilde{y}_t = E_t \{ \tilde{y}_{t+1} \} - \sigma^{-1} (i_t - E_t \{ \pi_{t+1} \} - r^n_t), \quad r^n_t \equiv \rho + \sigma E_t \{ \Delta y^n_{t+1} \} \]
Properties of a (friendly) “command economy”

- Relevant benchmark, or, ideal outcome, is the allocation chosen by a benevolent social planner (it identifies the efficient outcomes)

- The planner maximizes

\[
E_0 \sum_{t=0}^{\infty} \beta^t U (C_t, N_t) = E_0 \sum_{t=0}^{\infty} \beta^t U \left( \left[ \int_0^1 C_t(i) \frac{\varepsilon-1}{\varepsilon} \, di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \int_0^1 N_t(i) \, di \right),
\]

subject to

\[C_t(i) = A_t N_t(i)^{1-\alpha}\quad \text{all } i \in [0,1]\]

- By nature of the consumption basket, \(C_t(i) \neq C_t(j)\) is never optimal for a given \(C_t\). Therefore, optimality requires

\[C_t(i) = C_t, \quad N_t(i) = N_t, \quad \text{all } i \in [0,1].\]

- The problem then simplifies to

\[
\max U \left( A_t N_t^{1-\alpha}, N_t \right)
\]

Optimality condition:

\[(1 - \alpha) A_t N_t^{-\alpha} U_{c,t} + U_{n,t} = 0 \quad \frac{U_{n,t}}{U_{c,t}} = (1 - \alpha) A_t N_t^{-\alpha} \equiv MPN_t\]
Inefficiencies in the New Keynesian model

Monopolistic competition

- Monopolistic competition implies that prices are a markup over aggregate marginal costs; even under flexible prices:
  \[ P_t = \mathcal{M} \frac{W_t}{MPN_t}, \quad \mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1} > 1 \]

- The model's labor market equilibrium:
  \[ \frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} \leq MPN_t \]

Monopolistic competition results in too low employment and output

- Monetary policy is useless in addressing this market-structure inefficiency

- Fiscal (tax) policy can (in theory) solve the problem. Assume a labor cost subsidy \( \tau \) (financed lump sum from consumers):
  \[ P_t = \mathcal{M} \frac{(1 - \tau) W_t}{MPN_t} \]
  - Monopoly distortion is eliminated if \( \mathcal{M} (1 - \tau) = 1 \); this is assumed
  - Requires \( \tau = \varepsilon^{-1} \)
Nominal rigidities

- Price rigidities cause mark-up fluctuations with sticky prices:

\[ M_t = \frac{P_t}{(1 - \tau) W_t MP N_t} = \frac{P_t M}{W_t MP N_t} \]

\[ \frac{W_t}{P_t} = MP N_t \frac{M}{M_t} \neq MP N_t \]

- Staggered price setting causes price and thus output dispersion:

\[ C_t (i) \neq C_t (j) \quad \text{when} \quad P_t (i) \neq P_t (j) \]

- Inefficiencies due to nominal rigidities can be addressed by monetary policy (at least in part)
What should monetary policy ideally do?

- Assume the labor subsidy $\tau = \varepsilon^{-1}$ is in place; the natural rate of output is then efficient.

- Eliminate markup fluctuations, i.e., secure that $\widehat{mc}_t = 0$
  - Equivalent of securing:
    $$\tilde{y}_t = 0 \text{ all } t$$

- Avoid any price dispersion
  - Assuming no past relative price dispersion, $P_{t-1}(i) = P_{t-1}$, all $i \in [0, 1]$
  - No firms will change prices when $\widehat{mc}_t = 0$, $mc_t = mc$
  - Hence, $P_{t+j}(i) = P_{t+j} = P_{t-1+j}$, all $i \in [0, 1]$, $j = 0, 1, 2, \ldots$
  - Equivalent of fully stable aggregate prices:
    $$\pi_t = 0 \text{ all } t$$
How can this be done?

- Surprisingly there is no policy trade-offs—the ideal policy goals are attainable
  - (Special to the simple shock structure; by some denoted “a divine coincidence”.)

- Letting $i_t = r^n_t$, is compatible with attaining both $\tilde{y}_t = 0$ and $\pi_t = 0$

$$
\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa \tilde{y}_t,
\tilde{y}_t = E_t \{\tilde{y}_{t+1}\} - \sigma^{-1} (i_t - E_t \{\pi_{t+1}\} - r^n_t)
$$

- Problem. Setting $i_t = r^n_t$ leads to dynamic the system

$$
\begin{bmatrix}
\tilde{y}_t \\
\pi_t
\end{bmatrix} = A_O \begin{bmatrix}
E_t \{\tilde{y}_{t+1}\} \\
E_t \{\pi_{t+1}\}
\end{bmatrix}, \quad A_O \equiv \begin{bmatrix}
1 & \sigma^{-1} \\
\kappa & \beta + \kappa \sigma^{-1}
\end{bmatrix}
$$

- $A_O$ has one eigenvalue above one, and one below one. Indeterminacy; i.e., infinitely many stationary inflation and output gap paths
One could therefore follow the previously considered Taylor rule, amended with a response to the natural rate of interest:

\[ i_t = r^n_t + \phi_\pi \pi_t + \phi_y \tilde{y}_t, \quad \phi_\pi, \phi_y \geq 0, \]

This leads to the familiar dynamics

\[
\begin{bmatrix}
\tilde{y}_t \\
\pi_t
\end{bmatrix} = A_T
\begin{bmatrix}
E_t \{ \tilde{y}_{t+1} \} \\
E_t \{ \pi_{t+1} \}
\end{bmatrix}
\]

\[
A_T \equiv \Omega \begin{bmatrix}
\sigma & 1 - \phi_\pi \beta \\
\sigma \kappa & \kappa + \beta (\sigma + \phi_y)
\end{bmatrix}, \quad \Omega \equiv \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}
\]

Uniqueness requires:

\[ 0 < (\phi_\pi - 1) \kappa + \phi_y (1 - \beta) \]

(the “Taylor principle”)

The optimal allocation will be achieved in equilibrium
• Such a rule, however, poses a practical problem: $r^n_t$ is not observed in real time

• Therefore, more simple rules can be considered; i.e., rules depending on observable variables

• But how should one to assess their performance?

• I.e., how is it possible to compare one rule to another?

• By developing a welfare criterion!
Welfare criterion in the NK model

• The relevant welfare criterion in the NK model is

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\} \]

• How do we use this together with the log-linearized model?

• \( W \) is approximated by a second-order Taylor expansion (a first-order expansion would not rank different monetary policies, as these do not affect longs-run levels; i.e., steady states)

• Important second-order approximation for a variable \( Z \):

\[ \frac{Z_t - Z}{Z} \approx \hat{z}_t + \frac{1}{2} \hat{z}_t^2 \]

where \( \hat{z}_t \equiv \log (Z_t/Z) \)

• The approximation is performed around an efficient steady state—yields a simple expression

• If the approximation is around an inefficient steady state, one may get “spurious” welfare results by using a log-linear model (the ignored second-order terms may become important, i.e., policy dependent)
Initial Taylor expansion

\[ U_t - U \simeq U_c C \left( \frac{C_t - C}{C} \right) + U_n N \left( \frac{N_t - N}{N} \right) + \frac{1}{2} U_{cc} C^2 \left( \frac{C_t - C}{C} \right)^2 + \frac{1}{2} U_{nn} N^2 \left( \frac{N_t - N}{N} \right)^2 \]

(hence, separability, \( U_{cn} = 0 \), is assumed)

- In log-deviations

\[
U_t - U \simeq U_c C \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + U_n N \left( \hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) + \frac{1}{2} U_{cc} C^2 \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right)^2 + \frac{1}{2} U_{nn} N^2 \left( \hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right)^2
\]

\[
\simeq U_c C \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + U_n N \left( \hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) + \frac{1}{2} U_{cc} C^2 \hat{c}_t^2 + \frac{1}{2} U_{nn} N^2 \hat{n}_t^2
\]

as \( \hat{c}_t^3 \simeq \hat{c}_t^4 \simeq \hat{n}_t^3 \simeq \hat{n}_t^4 \simeq 0 \) in a second-order expansion

- Rearranging:

\[
U_t - U \simeq U_c C \left( \hat{c}_t + \frac{1}{2} \hat{c}_t^2 + \frac{1}{2} U_{cc} C \hat{c}_t^2 \right) + U_n N \left( \hat{n}_t + \frac{1}{2} \hat{n}_t^2 + \frac{1}{2} U_{nn} N \hat{n}_t^2 \right)
\]

- Simplifying:

\[
U_t - U \simeq U_c C \left( \hat{c}_t + \frac{1 - \sigma}{2} \hat{c}_t^2 \right) + U_n N \left( \hat{n}_t + \frac{1 + \varphi}{2} \hat{n}_t^2 \right)
\]

where

\[
\sigma \equiv -\frac{U_{cc} C}{U_c} > 0, \quad \varphi \equiv \frac{U_{nn} N}{U_n} > 0
\]
• Using the goods-market equilibrium condition \( \hat{c}_t = \hat{y}_t \):

\[
U_t - U \simeq U_c C \left( \hat{y}_t + \frac{1 - \sigma}{2} \hat{y}_t^2 \right) + U_n N \left( \hat{n}_t + \frac{1 + \varphi}{2} \hat{n}_t^2 \right)
\]

• Now comes a “tricky” part: Rewrite \( \hat{n}_t \) in terms of output

**Relationship between employment, output and relative prices**

• From last lecture:

\[
N_t = \int_0^1 N_t(i) \, di = \int_0^1 \left( \frac{Y_t(i)}{A_t} \right) \frac{1}{1 - \alpha} \, di
\]

\[
N_t = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1 - \alpha}} \int_0^1 \left( \frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1 - \alpha}} \, di = \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1 - \alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1 - \alpha}} \, di
\]

• In logs:

\[
n_t = \frac{1}{1 - \alpha} (y_t - a_t) + \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1 - \alpha}} \, di,
\]

\[
(1 - \alpha) n_t = y_t - a_t + d_t, \quad d_t \equiv (1 - \alpha) \log \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1 - \alpha}} \, di.
\]

• Around a zero inflation steady state (where \( d = 0 \)):

\[
(1 - \alpha) \hat{n}_t = \hat{y}_t - a_t + d_t
\]
• We need to find $d_t$, the measure of price dispersion, as it is second-order term that will have welfare effects.

• Start by the definition of the price index:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\varepsilon} \, di \right]^{1 \over 1-\varepsilon}$$

• Then,

$$1 = \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{1-\varepsilon} \, di$$

$$= \int_0^1 \exp [(1 - \varepsilon) (p_t(i) - p_t)] \, di$$

$$\simeq 1 + (1 - \varepsilon) \int_0^1 (p_t(i) - p_t) \, di + \frac{(1 - \varepsilon)^2}{2} \int_0^1 (p_t(i) - p_t)^2 \, di$$

(*)

in a second-order approximation around $p(i) = p$.

• Letting $E_i \{ p_t(i) \} \equiv \int_0^1 p_t(i) \, di$ be the mean of log prices across sectors,

$$p_t \simeq E_i \{ p_t(i) \} + \frac{1 - \varepsilon}{2} \int_0^1 (p_t(i) - p_t)^2 \, di$$

(**)
Then assess the specific relative price expression of \( d_t \):

\[
\int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \, di = \int_0^1 \exp \left[ -\frac{\varepsilon}{1-\alpha} (p_t(i) - p_t) \right] \, di \\
\approx 1 - \frac{\varepsilon}{1-\alpha} \int_0^1 (p_t(i) - p_t) \, di + \frac{1}{2} \left( \frac{\varepsilon}{1-\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 \, di
\]

From (*) we have that

\[
\int_0^1 (p_t(i) - p_t) \, di \approx -\frac{1 - \varepsilon}{2} \int_0^1 (p_t(i) - p_t)^2 \, di
\]

Hence,

\[
\int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \, di \approx 1 + \frac{\varepsilon (1-\varepsilon)}{2 (1-\alpha)} \int_0^1 (p_t(i) - p_t) \, di + \frac{1}{2} \left( \frac{\varepsilon}{1-\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 \, di \\
\approx 1 + \frac{1}{21 - \alpha \Theta} \int_0^1 (p_t(i) - p_t)^2 \, di \quad \Theta \equiv \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon}
\]

Using (**) we get

\[
\int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \, di \approx 1 + \frac{\varepsilon}{21 - \alpha \Theta} \int_0^1 (p_t(i) - \mathbb{E}_i \{p_t(i)\})^2 \, di = 1 + \frac{\varepsilon}{21 - \alpha \Theta} \text{var}_i \{p_t(i)\}
\]

where \( \text{var}_i \{p_t(i)\} \) is price variance across sectors
Since
\[ d_t \equiv (1 - \alpha) \log \int_0^1 \left( \frac{P_t(i)}{\bar{P}_t} \right)^{-\frac{\varepsilon}{1-\alpha}} \, di \]
we get
\[ d_t \approx \frac{1}{2} \frac{\varepsilon}{\Theta} \text{var}_i \{ p_t(i) \} \]
(which also proves that we rightfully ignored it when looking at the linear dynamics, as \( d_t \) is a second-order term)

We then substitute \( \hat{n}_t = (1 - \alpha)^{-1} \hat{y}_t - (1 - \alpha)^{-1} a_t + (1 - \alpha)^{-1} d_t \) into
\[ U_t - U \approx U_c C \left( \hat{y}_t + \frac{1 - \sigma}{2} \hat{y}_t^2 \right) + U_n N \left( \hat{n}_t + \frac{1 + \varphi}{2} \hat{n}_t^2 \right) \]
and get
\[ U_t - U \approx U_c C \left( \hat{y}_t + \frac{1 - \sigma}{2} \hat{y}_t^2 \right) + \frac{U_n N}{1 - \alpha} \left( \hat{y}_t + d_t + \frac{1}{2} \frac{\varphi}{(1 - \alpha)} (\hat{y}_t - a_t)^2 \right) + \text{t.i.p.} \]
where t.i.p. is “terms independent of policy” and the third-order effects and higher are ignored

Rewrite so we get utility change measured as percentage change in steady-state consumption:
\[ \frac{U_t - U}{U_c C} \approx \hat{y}_t + \frac{1 - \sigma}{2} \hat{y}_t^2 + \frac{U_n N}{U_c C (1 - \alpha)} \left( \hat{y}_t + d_t + \frac{1 + \varphi}{2} (\hat{y}_t - a_t)^2 \right) + \text{t.i.p.} \]
- Now remember that we are approximating around an efficient steady state. Hence,

\[-\frac{U_n}{U_c} = MPN = (1 - \alpha) AN^{-\alpha} \equiv (1 - \alpha) \frac{Y}{N}\]

- Therefore,

\[-\frac{U_n}{U_c} = (1 - \alpha) \frac{C}{N}\]

or,

\[-\frac{U_n}{U_C C (1 - \alpha)} = -1\]

- The utility approximation therefore simplifies to

\[
\frac{U_t - U}{U_C C} \simeq \frac{1 - \sigma}{2} \tilde{y}_t^2 - \frac{1}{2} \frac{\varepsilon}{\Theta} \text{var} \{ p_t(i) \} - \frac{1}{2 (1 - \alpha)} (\tilde{y}_t - a_t)^2 + \text{t.i.p.}
\]

\[
= -\frac{1}{2} \left[ \frac{\varepsilon}{\Theta} \text{var} \{ p_t(i) \} + (\sigma - 1) \tilde{y}_t^2 + \frac{1 + \varphi}{1 - \alpha} (\tilde{y}_t - a_t)^2 \right] + \text{t.i.p.}
\]

\[
= -\frac{1}{2} \left[ \frac{\varepsilon}{\Theta} \text{var} \{ p_t(i) \} + (\sigma + \frac{\alpha + \varphi}{1 - \alpha}) \tilde{y}_t^2 - 2 \frac{1 + \varphi}{1 - \alpha} \tilde{y}_t a_t \right] + \text{t.i.p.}
\]

\[
= -\frac{1}{2} \left[ \frac{\varepsilon}{\Theta} \text{var} \{ p_t(i) \} + (\sigma + \frac{\alpha + \varphi}{1 - \alpha}) \left( \tilde{y}_t^2 - 2 \frac{1 + \varphi}{1 - \alpha} \tilde{y}_t y_t^n \right) \right] + \text{t.i.p.}
\]

as

\[y_t^n = \frac{1 + \varphi}{\sigma (1 - \alpha) + \varphi + \alpha} a_t.\]
• The welfare measure is therefore approximately

\[ \mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U_{tc}}{U_{tc}} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\} + \left( \sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) \tilde{y}_t^2 \right] + \text{t.i.p.} \]

• We finally use Lemma 2 from Woodford (2003):

\[ \sum_{t=0}^{\infty} \beta^t \text{var}_i \{p_t(i)\} = \frac{\theta}{(1 - \theta)(1 - \beta \theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2 \]

• We then get

\[ \mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U_{tc}}{U_{tc}} = \\
-\frac{1}{2} \left( \sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{\varepsilon}{\kappa} \pi_t^2 + \tilde{y}_t^2 \right] \]

as

\[ \kappa \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon} \frac{\sigma (1 - \alpha) + \varphi + \alpha}{1 - \alpha} = \frac{(1 - \theta)(1 - \beta \theta)}{\Theta} \frac{\sigma (1 - \alpha) + \varphi + \alpha}{1 - \alpha} \]
The performance of various policy rules

- With this utility-based welfare loss one can assess the performance of various policy rules
- One can perform optimal policy exercises as linear-quadratic optimization problems (next time)

- Galí exemplifies the importance of price stability in the New-Keynesian model by assessing the performance of the policy rule

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \]

(note: a function of \( \hat{y}_t \), not \( \bar{y} \)) for various policy parameters:

<table>
<thead>
<tr>
<th></th>
<th>Taylor Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_\pi )</td>
<td>1.5 1.5 5 1.5</td>
</tr>
<tr>
<td>( \phi_y )</td>
<td>0.125 0 0 1</td>
</tr>
<tr>
<td>( (\sigma_\xi, \rho_\xi) )</td>
<td>— — — —</td>
</tr>
<tr>
<td>( \sigma (\hat{y}) )</td>
<td>0.55 0.28 0.04 1.40</td>
</tr>
<tr>
<td>( \sigma (\pi) )</td>
<td>2.60 1.33 0.21 6.55</td>
</tr>
<tr>
<td>welfare loss</td>
<td>0.30 0.08 0.002 1.92</td>
</tr>
</tbody>
</table>
Concluding remarks

- The New-Keynesian model offers a simple framework for welfare-based policy analysis.

- Models is (in principle) immune to the Lucas critique, and the welfare criterion is consistent with the one used to derive the economy’s behavioral equations.

- One can rank various policy rules as well as meaningfully compare their quantitative welfare differences.

- The simple model is obviously too simple to represent the real world, but its basic features “survive” in large-scale versions used in many inflation-targeting central banks.
Next time(s)

Monday, December 13: Exercises:

• Prove Lemma 2 on page 89 (this will yield a prize!)
• Exercise 4.1 in Galí (2008)
• Exercise 4.2 in Galí (2008)

Tuesday, December 14
Lecture: Monetary policy trade-offs, optimal policy and credibility issues (Galí, Chapter 5)