MAKØK 3, BLOK 2, 2010/11
WRITTEN EXAM
- SUGGESTED ANSWERS -

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(Note that the answers are "suggested" in the sense that they include more than what can be realistically expected from even the best answer, taking the three-hour constraint into consideration.)

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In the simple New-Keynesian model with price rigidities only, optimal stabilization of cost-push shocks requires that the central bank can credibly commit to affect private-sector expectations.

A TRUE. By affecting inflation expectations, the inflationary consequences of the cost-push shock can be affected by the policymaker through two channels. Through the current output gap and through inflation expectations. Without a credible commitment, only a reduction in the current output gap can dampen the inflationary effects of the shock. If inflation expectations can be affected, the policymaker can get better current inflation stabilization by reducing the current output gap by less than in the absence of commitment.

(ii) In the New-Keynesian model with Calvo-style wage stickiness, a higher probability of not being able to change the wage (θ_w), leads to a lower welfare loss of nominal wage inflation.

A FALSE. More nominal rigidity under a Calvo mechanism, means that more prices dispersion will result from aggregate inflation. Translated into the model with wage rigidity, this means that any rate of nominal wage inflation will cause more dispersion in relative wages—and thus relative labor usage across labor types. Hence, nominal wage inflation is more welfare costly when θ_w is higher. (Indeed, in the New-Keynesian model with flexible wages, $\theta_w = 0$, there are no welfare loss from nominal wage changes.)

(iii) In the simple New-Keynesian model with price rigidities only, absence of any exogenous fluctuations in firms' desired markup, implies that the central bank can achieve the efficient allocation when an appropriate labor subsidy is in place.

A TRUE. With an appropriate labor subsidy, the average loss from monopoly power is neutralized. Hence, monetary policy can be addressed at fulfilling two objectives: Stabilizing the markup gap and avoiding price dispersion. In the simple model, these objectives do not conflict. A zero-inflation policy implies that the actual markup always equals the (constant) desired markup, and that all firms will set the same price resulting in no price dispersion. Hence, the Calvo mechanism is not a constraint on firms. The efficient allocation can therefore be obtained. (In terms of the conventional loss function, this is one of zero inflation and zero output gap.)

QUESTION 2:

Consider the following log-linear model of a closed economy:

$$
\begin{aligned}
y_t &= \mathrm{E}_t\{y_{t+1}\} - \sigma^{-1}\left(i_t - \mathrm{E}_t\{\pi_{t+1}\} - \rho\right), \quad \sigma > 0, & (1)\\
\pi_t &= \beta \mathrm{E}_t\{\pi_{t+1}\} + \kappa\left(y_t - y_t^n\right), \quad 0 < \beta < 1, \quad \kappa > 0, & (2)\\
i_t &= \rho + \phi_\pi \pi_t, \qquad \phi_\pi > 1, & (3)
\end{aligned}
$$

where y_t is output, i_t is the nominal interest rate, π_t is goods price inflation and y_t^n is the natural rate of output, which is assumed to be a mean-zero, serially uncorrelated shock. $\mathrm{E}_t\{.\}$ is the rational expectations operator conditional upon all information up to and including period t.

(i) Discuss (1) and (2) with focus on the underlying economic mechanisms. What does (3) represent? Explain.

A Here is should be mentioned that (1), the dynamic IS curve, is derived from a log-linearization of consumers' consumption-Euler equations: A higher real interest rate, i_t- $\mathrm{E}_t\left\{\pi_{t+1}\right\}$, make consumers increase future consumption relative to current. Equation (2), the New-Keynesian Phillips Curve, is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters (2) positively. The more price rigidity (e.g., the lower a probability of price adjustment under a Calvo price setting scheme), the smaller is κ. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be effective for some periods. Equation (3) is a simple specification for how monetary policy, in terms of nominal interest rate setting, is determined. It is a simple Taylor-type rule where the nominal interest rate in increased (more than one-for-one) when inflation increases.

(ii) Derive the solutions for y_t and π_t [Hint: Conjecture that the solutions are linear functions of the period's natural rate, y_t^n, and remember that $\mathrm{E}_t\left\{y_{t+1}^n\right\}=0$.]

A Use the hint and conjecture the following solutions (it is fine to note that the solution is unique in this model when indeed $\phi_\pi>1$):

$$\begin{aligned} y_t &= X_y y_t^n, \\ \pi_t &= X_\pi y_t^n, \end{aligned}$$

which under the assumptions about y_t^n implies

$$\begin{aligned} \mathrm{E}_t\left\{y_{t+1}\right\} &= 0, \\ \mathrm{E}_t\left\{\pi_{t+1}\right\} &= 0. \end{aligned}$$

Insert these conjectures into (1) and (2) [where (3) has already been substituted into (1)] to obtain

$$\begin{aligned} X_y y_t^n &= -\sigma^{-1}\phi_\pi X_\pi y_t^n, \\ X_\pi y_t^n &= \kappa\left(X_y y_t^n-y_t^n\right). \end{aligned}$$

Hence, the unknown coefficients are determined by

$$\begin{aligned} X_y &= -\sigma^{-1}\phi_\pi X_\pi, \\ X_\pi &= \kappa\left(X_y-1\right). \end{aligned}$$

We therefore get

$$
\begin{aligned}
X_y &= -\sigma^{-1}\kappa\phi_\pi\left(X_y - 1\right), \\
X_y\left(1+\sigma^{-1}\kappa\phi_\pi\right) &= \sigma^{-1}\kappa\phi_\pi, \\
X_y &= \frac{\sigma^{-1}\kappa\phi_\pi}{1+\sigma^{-1}\kappa\phi_\pi} > 0,
\end{aligned}
$$

and

$$
\begin{aligned}
X_\pi &= \kappa\left(\frac{\sigma^{-1}\kappa\phi_\pi}{1+\sigma^{-1}\kappa\phi_\pi} - 1\right), \\
X_\pi &= -\frac{\kappa}{1+\sigma^{-1}\kappa\phi_\pi} < 0.
\end{aligned}
$$

Output and inflation are therefore given by

$$
\begin{aligned}
y_t &= \frac{\sigma^{-1}\kappa\phi_\pi}{1+\sigma^{-1}\kappa\phi_\pi} y_t^n, \\
\pi_t &= -\frac{\kappa}{1+\sigma^{-1}\kappa\phi_\pi} y_t^n.
\end{aligned}
$$

(iii) What is the role of the parameter ϕ_π in terms of output and inflation's responses to shocks to the natural rate of output? Can ϕ_π be chosen such that the *output gap*, $\widetilde{y}_t \equiv y_t - y_t^n$, and inflation are stabilized completely? Why/why not?

A First remark that a positive shock to the natural rate decreases marginal costs, which reduces inflation. With the policy rule (3), this decreases the real interest rate, and thereby increases output. It is noted that the increase in output is smaller than the increase in the natural rate output; hence, the output gap $\widetilde{y}_t \equiv y_t - y_t^n$ decreases with a positive shock to the natural rate.

Then note that the policy rule parameter ϕ_π influences the way output and inflation respond to the shock. A higher value of ϕ_π, implies that increases in inflation is met by a more contractive monetary policy (stronger interest rate increase), which stabilizes inflation more. The coefficient X_π derived above indeed gets numerically smaller with a higher ϕ_π. As a consequence of this, output will respond more to shocks that affect inflation. Hence, a higher value of ϕ_π will make output more unstable in face of shocks to the natural rate; indeed the coefficient X_y derived above is increasing in ϕ_π.

The output gap, however, gets *more* stable as ϕ_π increases. With $\phi_\pi \longrightarrow \infty$ inflation is completely stabilized and output is "destabilized" such that it follows the movement in the natural rate perfectly. Hence, the output gap is fully stabilized. The policy rule *can* therefore be chosen such that the output *gap* and inflation are stabilized completely, as there is no trade off present when only shocks to the natural rate are a source of fluctuations (it can be mentioned that stabilizing inflation and the output gap is the welfare-optimal policy in this type of model).

QUESTION 3:

Assume a model of a closed economy under "small" permanent distortions, where the welfare-relevant loss function can be written as

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x \widehat{x}_t^2 \right) - \Lambda \widehat{x}_t \right], \quad 0 < \beta < 1, \quad \Lambda > 0, \quad \alpha_x > 0, \tag{1}$$

where π_t is goods price inflation and $\widehat{x}_t$ is the welfare-relevant output gap.

(i) Discuss the economic and model-consistent rationale for such a loss function.

A This type of loss function can be derived as the second-order Taylor approximation to (the negative of) the representative household's utility function in a model with monopolistic competition and Calvo-style price rigidities. In the economy, there are welfare losses from firms' monopoly power. Moreover, price rigidities cause losses from aggregate mark-ups being different from the desired markup, and under the Calvo-price structure, staggering cause inefficient dispersion of consumption of various goods. In absence of any fiscal measures to counteract the average monopoly distortion, the term $-\Lambda \widehat{x}_t$ captures that it would be desirable to have output above the (inefficient) natural rate on average. The quadratic terms reflect the costs from fluctuations. Inflation is proportional to the inefficient goods dispersion, and output gap fluctuations are proportional to the fluctuations in the markup gap (that causes inefficient fluctuations in consumption and labor).

(ii) Derive the optimal sequences of $(\widehat{x}_t, \pi_t)_{t=0}^{\infty}$ under discretionary policymaking when these have to obey

$$\pi_t = \beta \mathrm{E}_t \left\{ \pi_{t+1} \right\} + \kappa \widehat{x}_t + u_t, \quad \kappa > 0, \tag{2}$$

where u_t is a mean-zero, serially uncorrelated shock. For this purpose, treat $\widehat{x}_t$ as the policy instrument, and show that the relevant first-order condition for optimal policy together with (2) yield the difference equation

$$\pi_t = \frac{\alpha_x \beta}{\kappa^2 + \alpha_x} \mathrm{E}_t \{\pi_{t+1}\} + \frac{\alpha_x}{\kappa^2 + \alpha_x} u_t + \frac{\kappa}{\kappa^2 + \alpha_x} \Lambda. \tag{3}$$

A Under discretion, expectations are taken as given, so we have a sequence of one-period minimization problems:

$$\begin{aligned} \min_{\widehat{x}_t} L(\pi_t, \widehat{x}_t) &\equiv \frac{1}{2}\left(\pi_t^2 + \alpha_x \widehat{x}_t^2\right) - \Lambda \widehat{x}_t, \\ \text{s.t.} \quad \pi_t &= \kappa \widehat{x}_t + v_t, \\ v_t &\equiv \beta \mathrm{E}_t \{\pi_{t+1}\} + u_t. \end{aligned}$$

The relevant first-order condition is

$$\alpha_x \widehat{x}_t - \Lambda = -\kappa \pi_t.$$

Insert this back into (2) to eliminate $\widehat{x}_t$:

$$\pi_t = \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \left[\frac{\Lambda}{\alpha_x} - \frac{\kappa}{\alpha_x} \pi_t\right] + u_t,$$

which is rewritten as

$$\pi_t \left(1 + \kappa^2 / \alpha_x\right) = \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \frac{\Lambda}{\alpha_x} + u_t,$$

which can be written as (3). Since, $[\alpha_x \beta / (1 + \kappa^2 / \alpha_x)] < 1$ this has a unique stationary solution. We find this by conjecturing

$$\pi_t = A + B u_t,$$

which given the assumptions about u_t implies

$$\mathrm{E}_t \{\pi_{t+1}\} = A.$$

Inserted into difference equation:

$$A + B u_t = \frac{\alpha_x \beta}{\alpha_x + \kappa^2} A + \frac{\alpha_x}{\alpha_x + \kappa^2} u_t + \frac{\kappa}{\alpha_x + \kappa^2} \Lambda,$$

which gives two equations in the two undetermined coefficients:

$$A = \frac{\alpha_x \beta}{\alpha_x + \kappa^2} A + \frac{\kappa}{\alpha_x + \kappa^2} \Lambda,$$

$$B = \frac{\alpha_x}{\alpha_x + \kappa^2}.$$

The solution for A follows as:

$$\begin{aligned} A\left[\alpha_x + \kappa^2 - \alpha_x \beta\right] &= \kappa\Lambda, \\ A &= \frac{\kappa}{\kappa^2 + \alpha_x (1-\beta)}\Lambda. \end{aligned}$$

So, inflation is

$$\pi_t = \frac{\kappa}{\kappa^2 + \alpha_x (1-\beta)}\Lambda + \frac{\alpha_x}{\alpha_x + \kappa^2} u_t.$$

Output gap follows by using the first-order condition as

$$\begin{aligned} \widehat{x}_t &= \frac{\Lambda}{\alpha_x} - \frac{\kappa}{\alpha_x}\left[\frac{\kappa}{\kappa^2 + \alpha_x (1-\beta)}\Lambda + \frac{\alpha_x}{\alpha_x + \kappa^2} u_t\right], \\ &= \frac{\Lambda}{\alpha_x}\left[1 - \frac{\kappa^2}{\kappa^2 + \alpha_x (1-\beta)}\right] - \frac{\kappa}{\alpha_x + \kappa^2} u_t, \\ &= \frac{1-\beta}{\kappa^2 + \alpha_x (1-\beta)}\Lambda - \frac{\kappa}{\alpha_x + \kappa^2} u_t. \end{aligned}$$

(iii) Show that optimal inflation therefore is given by

$$\pi_t = \frac{\kappa}{\kappa^2 + \alpha_x (1-\beta)}\Lambda + \frac{\alpha_x}{\kappa^2 + \alpha_x} u_t.$$

Discuss the average properties and business cycle properties of this solution. Will inflation be fully stabilized under any circumstances?

A One sees that average inflation is positive as $\Lambda > 0$. This is the inflation bias of discretionary policymaking when there is an incentive to increase output above the natural rate (as the natural rate is inefficient). There will only be a slight effect on output, since inflation expectations also increase in recognition of this incentive. (As $\beta \to 1$, the NKPC is vertical in the long run, and there is no average effects on output, but an inflation bias remains.)

In terms of business cycle properties, one sees that an inflation shock raises inflation, but not by the full amount, as the loss function penalizes both inflation and output fluctuations ($\alpha_x > 0$). The policymaker optimally spreads out the impact on the shock on inflation and the output gap.

Inflation will only be fully stabilized in reponse to the u_t shock if either $\alpha_x \to 0$ or $\kappa \to \infty$. In the former case, there is no welfare rationale for stabilizing the output gap, so optimal policy can be directed towards full inflation stabilization (the trade-off disappears). In the latter case, there is approximately full price flexibility, and price stability can be attained by an infinitesimal change in the output gap

(iv) Discuss how commitment policies can improve on the solution under discretion.

A Commitment policies have the properties that the policymaker credible sticks to a policy plan made in period $t = 0$. In consequence, the policymaker will be able to affect expected future variables. In this context, it means that the policymaker will be able to affect inflation expectations. The policymaker can never be worse off by this additional option. In this model, an improvement in the inflation and output performance can be achieved. Firstly, the average inflation bias can be eliminated in the long run. Secondly, the stabilization properties can be improved. For example, if an inflationary shock occurs, it is optimal to perform a milder but prolonged contractionary policy. Through this, the policymaker affects inflation expectations in a downward direction, which helps stabilize current inflation (at a lower output gap cost). It can be mentioned that the first-order conditions for optimal policy under commitment indeed portrays a policy inertia, or, history dependence, which secure that a contractive policy today is continued into the future.