On stable and unstable roots in the simple New-Keynesian Model

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In the slides of November 26 (page 26), the dynamics of the simple two-equation New-Keynesian model (with a Taylor-type interest-rate rule) is presented in matrix form as follows

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} \mathrm{E}_t \{\widetilde{y}_{t+1}\} \\ \mathrm{E}_t \{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T (\widehat{r}_t - v_t), \tag{26}$$

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \phi_\pi \beta \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}, \qquad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}, \qquad \Omega \equiv \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}$$

It is then stated that uniqueness of a stationary equilibrium requires that the eigenvalues of $\mathbf{A}_T$ lies within the unit circle. This can create a bit of confusion, if one uses the standard reference for uniqueness in linear, rational expectations models, Blanchard and Kahn (1980). They state that uniqueness requires that the number of unstable eigenvalues should equal the number of "jump" variables", or, non-predetermined variables. So according to them, we should have two unstable eigenvalues, or, two unstable roots in the system, if we should have a unique stationary equilibrium. (See their Proposition 1, p. 1308).

But there is actually no inconsistency. It is only a matter of how one sets up the system of difference equations. Blanchard and Kahn set up the system with variables dated $t + 1$ on the left-hand side. Hence, the system depicted in (26) is rewritten as

$$\left[\begin{array}{c} \mathrm{E}_t\left\{\widetilde{y}_{t+1}\right\} \\ \mathrm{E}_t\left\{\pi_{t+1}\right\} \end{array}\right] = \mathbf{A}_T^{-1}\left[\begin{array}{c} \widetilde{y}_t \\ \pi_t \end{array}\right] - \mathbf{A}_T^{-1}\mathbf{B}_T\left(\widehat{r}_t - v_t\right).$$

So, in Blanchard and Kahn terms, the matrix $\mathbf{A}_T^{-1}$ should have both eigenvalues *outside* the unit circle in order for the model to deliver a unique equilibrium. This is the same as requiring that $\mathbf{A}_T$ should have both eigenvalues *inside* the unit circle.

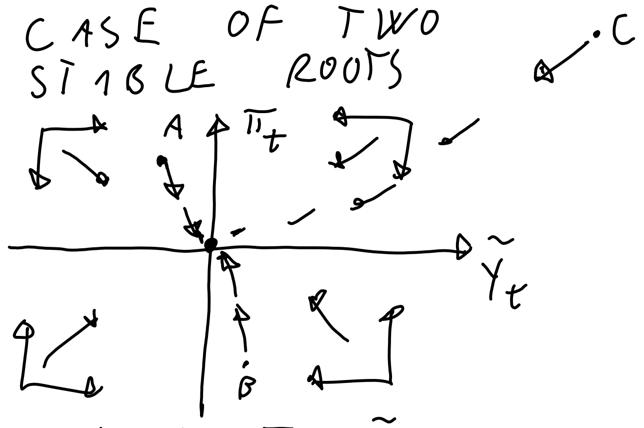
On the next two pages I attach the drawings from the lectures that in a rudimentary (and *very* heuristically) way hopefully demonstrate how unstable roots are necessary in the New-Keynesian model for uniqueness, or, determinacy.

References

[1] Blanchard, O. J. and C. M. Kahn, 1980, The Solution of Linear Difference Models under Rational Expectations, Econometrica 48, 1305-1311.

DYNAMICS OF $\tilde{Y}_t, \pi_t$ AND THE ROOTS OF $\mathbf{A}_T^{-1}$; – TENTATIVE ILLUSTRATIONS

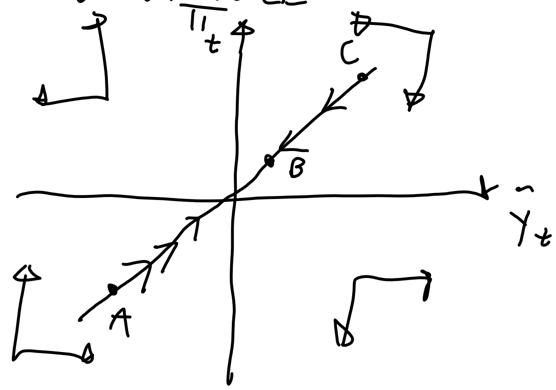
CASE OF TWO STABLE ROOTS



PROBLEM: $\pi_t, \tilde{Y}_t$ ARE NOT DETERMINED

$\approx$ <u>INDETERMINACY</u>: INFINITELY MANY STABLE SOLUTIONS. A, B, C ARE ALL NON-EXPLOSIVE SOLUTIONS

CASE OF ONE STABLE + ONE UNSTABLE



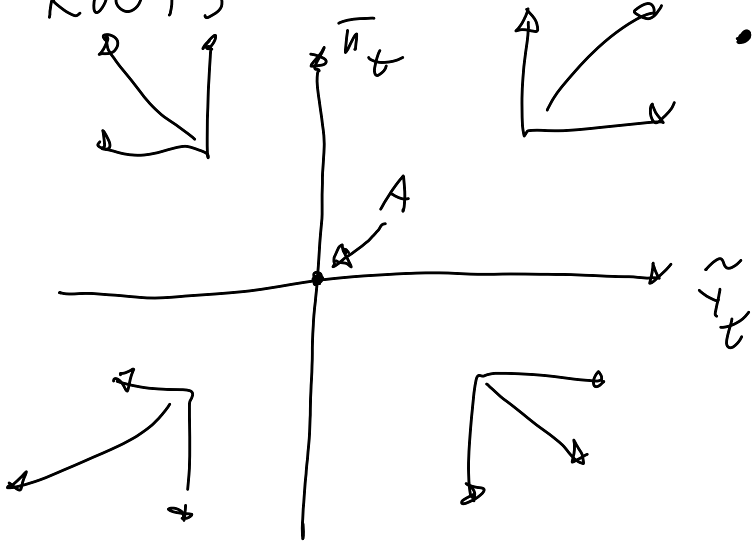
SADDLE PATH.

$\Rightarrow$ INDETERMINACY! BOTH $\pi_t, \tilde{Y}_t$ ARE ENDOGENOUS

$\phi_\pi = \phi_Y = 0$

* THIS IS THE DYN. OF NK MODEL WHEN i_t IS CONSTANT OR AN EXOGENOUS PROCESS

CASE OF TWO UNSTABLE ROOTS



$\Rightarrow$ ONE STATIONARY SOLUTION (I.E., NON-EXPLOSIVE)

$\boxed{\text{UNIQUENESS OF } \tilde{Y}_t, \pi_t \approx \text{DETERMINACY}}$

GENERAL REF.:

BLANCHARD & KAHN (1980, ECTRA)

- UNIQUENESS REQUIRES
  \# UNSTABLE ROOTS
  = \# ENDOGENOUS VARIABLES ("JUMP" VARIABLES; "NON-PREDETERMINED" VARIABLES)

"WITH SHOCKS)
"A" MOVES AS THE MODEL PREDICTS.

(IT DOES NOT CAUSE INSTABILITY)