Exam set for International Monetary Economics

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WITH SUGGESTED ANSWERS[1]

[1]Remark that the word "suggested" is chosen with care. The answers do not protray the one and only way to answer the questions satisfactorily (and the answers given here are generally longer than what is expected in a two-hour exam).

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(1) In the simple New-Keynesian model with price rigidities only, optimal stabilization of cost-push shocks is the same under discretion and commitment.

A **FALSE**. By affecting inflation expectations, the inflationary consequences of the cost-push shock can be affected by the policymaker through two channels. Through the current output gap and through inflation expectations. Without a credible commitment, only a reduction in the current output gap can dampen the inflationary effects of the shock. If inflation expectations can be affected, which is the case under a policy commitment, the policymaker can get better current inflation stabilization by reducing the current output gap by less than in the absence of commitment.

(2) In a two-country New-Keynesian model where all goods are internationally traded, the welfare-relevant inflation variable to target for the Home country is its consumer-price inflation rate.

A **FALSE.** The welfare-relevant inflation variable is the producer-price inflation rate, as its variations distort the relative demand for Home goods, and thus Home production. Variations in prices of the Foreign country cause Foreign distortions, which are irrelevant from a Home perspective. (It is nice to mention that this will change under policy cooperation, where authorities should take into account fluctuations in all prices, Home as well as Foreign, in a collaborative fashion.)

(3) In the Calvo pricing model, expectations about future marginal costs are irrelevant for firms that by chance are able to reset their prices.

A **FALSE**. A firm that has the opportunity to change its price at a given date knows that it by some probability is going to we "stuck" with this price in the future. Therefore, expectations about future economic conditions, and thus the future output gap are essential, as the firm would like to set its price as a markup over marginal costs.

QUESTION 2:

Consider the following log-linear model of a closed economy:

$$\begin{aligned} \widetilde{y}_t &= \mathrm{E}_t\{\widetilde{y}_{t+1}\} - \sigma^{-1}(i_t - \mathrm{E}_t\{\pi_{t+1}\} - \rho), \qquad \sigma > 0, \quad \rho > 0 &(1)\\ \pi_t &= \beta \mathrm{E}_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t, \qquad 0 < \beta < 1, \quad \kappa > 0, &(2) \end{aligned}$$

where $\widetilde{y}_t$ is the output gap, i_t is the nominal interest rate (the monetary policy instrument), π_t is goods price inflation. $\mathrm{E}_t\{.\}$ is the rational expectations operator conditional upon all information up to and including period t. Inflation is assumed to be observed with some error such that

$$\pi_t^o = \pi_t + e_t, \tag{3}$$

where π_t^o denotes observed inflation, and e_t is a mean-zero, serially uncorrelated shock. It is assumed that the central bank sets the nominal interest rate according to a simple rule:

$$i_t = \rho + \phi \pi_t^o, \qquad \phi > 1. \tag{4}$$

(4) Solve for $\widetilde{y}_t$ and π_t [Hint: Conjecture that solutions are linear functions of e_t.], and explain how the policy-rule parameter ϕ affects output gap and inflation fluctuations.

A Use the hint, and conjecture

$$\begin{aligned} \widetilde{y}_t &= Ae_t, \\ \pi_t &= Be_t, \end{aligned}$$

where A, B are unknown coefficients to be determined. Substitute (3) into the policy rule (4), and insert the resulting nominal interest-rate equation $i_t = \rho + \phi(\pi_t + e_t)$ into (1):

$$\widetilde{y}_t = \mathrm{E}_t\{\widetilde{y}_{t+1}\} - \sigma^{-1}[\phi(\pi_t + e_t) - \mathrm{E}_t\{\pi_{t+1}\}].$$

Then insert the conjectures into this amended DIS equation and the NKPC to obtain:

$$\begin{aligned} Ae_t &= \mathrm{E}_t\{Ae_{t+1}\} - \sigma^{-1}[\phi(Be_t + e_t) - \mathrm{E}_t\{Be_{t+1}\}], \\ Be_t &= \beta \mathrm{E}_t\{Be_{t+1}\} + \kappa Ae_t. \end{aligned}$$

Then use that e_t is serially uncorrelated to reduce the expressions to

$$\begin{aligned} Ae_t &= -\sigma^{-1}\phi\left(Be_t + e_t\right), \\ Be_t &= \kappa A e_t. \end{aligned}$$

These must hold for any e_t, so therefore the coefficients are determined by

$$\begin{aligned} A &= -\sigma^{-1}\phi\left(B + 1\right), \\ B &= \kappa A. \end{aligned}$$

One finally obtains

$$\begin{aligned} B &= -\frac{\sigma^{-1}\phi}{\kappa^{-1} + \sigma^{-1}\phi} < 0, \\ A &= -\frac{\kappa^{-1}\sigma^{-1}\phi}{\kappa^{-1} + \sigma^{-1}\phi} < 0, \end{aligned}$$

and the solutions for the output gap and inflation as

$$\begin{aligned} \widetilde{y}_t &= -\frac{\phi}{\sigma + \kappa\phi} e_t, \\ \pi_t &= -\frac{\kappa\phi}{\sigma + \kappa\phi} e_t. \end{aligned}$$

It is seen that a higher value of ϕ results in a stronger impact of the "measurement error" e_t on both the output gap and inflation. Intuitively, a higher value of ϕ leads to a stronger policy response to a noisy variable, and this feeds into the economy. (It would be nice to note that $\phi = 0$ would be the best for output gap and inflation stability in terms of the effects of the e_t shock. This, however, would create the problem of indeterminacy, and thus endogenous fluctuations in the economy, as determinacy requires $\phi > 1$.)

(5) Does this model lend support to the view that a central bank should respond strongly towards inflation? Why/Why not?

A According to this model, the central bank should *not* respond strongly towards inflation. The reason being that a high value of ϕ makes the central bank "imports" the noise arising from not being able to observe inflation precisely.

QUESTION 3:

Consider the following log-linear model of a closed economy:

$$\widetilde{y}_t = \mathrm{E}_t\left\{\widetilde{y}_{t+1}\right\} - \sigma^{-1}\left(i_t - \mathrm{E}_t\left\{\pi_{t+1}\right\} - \rho - r_t^n\right), \qquad \sigma > 0, \quad \rho > 0 \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t\left\{\pi_{t+1}\right\} + \kappa \widetilde{y}_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{2}$$

where $\widetilde{y}_t$ is the output gap, i_t is the nominal interest rate (the monetary policy instrument), π_t is goods price inflation and r_t^n is the natural rate of interest, which is assumed to be a mean-zero, serially uncorrelated shock. $\mathrm{E}_t\{.\}$ is the rational expectations operator conditional on all information up to and including period t.

(6) Assume that the monetary authority wants to minimize the loss function

$$L = \frac{1}{2}\mathrm{E}_0\left\{\sum_{t=0}^{\infty}\beta^t\left[\lambda\widetilde{y}_t^2 + \pi_t^2\right]\right\}, \qquad \lambda > 0. \tag{3}$$

Discuss the micro-economic foundations for this loss function, and explain how κ affects λ.

A This type of loss function can be derived as the second-order Taylor approximation to (the negative of) the representative household's utility function in a model with monopolistic competition and Calvo-style price rigidities. In the economy, there are welfare losses from firms' monopoly power. Moreover, price rigidities cause losses from aggregate mark-ups being different from the desired markup, and under the Calvo-price structure, staggering cause inefficient dispersion of consumption of various goods. (It is excellent to mention that fiscal measures are assumed to counteract the average monopoly distortion, which would otherwise introduce a term like $-\Lambda\widehat{x}_t$ capturing that it would be desirable to have output above the—inefficient—natural rate.) The quadratic terms reflect the costs from fluctuations. Inflation is proportional to the inefficient goods dispersion, and output gap fluctuations are proportional to the fluctuations in the markup gap (that causes inefficient fluctuations in consumption and labor). The parameter κ is a function of the degree of price rigidity in the economy.

When prices are very sticky (low probability of resetting prices), κ is small and the costs of aggregate inflation fluctuations will be larger (as goods dispersion will be larger). This will be captured in the loss function by a smaller value of λ; inflation variability will be relatively more costly than output variability.

(7) Derive the optimal values of $\widetilde{y}_t$ and π_t under discretionary policymaking [Hint: Consider $\widetilde{y}_t$ the policy instrument, and acknowledge that under discretion the optimization problem becomes a sequence of static problems as expected values can be taken as given]. Discuss the solutions, and describe how the nominal interest rate will move with the natural rate of interest.

A To derive the solution under discretion one minimizes the per-period version of (3) w.r.t. $\widetilde{y}_t$ subject to (2). This results in the first-order condition

$$\lambda\widetilde{y}_t + \kappa\pi_t = 0.$$

Use this to eliminate $\widetilde{y}_t$ in (2) so as to get

$$\pi_t = \beta \mathrm{E}_t\left\{\pi_{t+1}\right\} - \kappa^2\lambda^{-1}\pi_t,$$

and thus

$$\pi_t = \frac{\beta}{1+\kappa^2\lambda^{-1}}\mathrm{E}_t\left\{\pi_{t+1}\right\}.$$

As the root of this difference equation is larger than one, it has a unique solution. This is readily seen to be $\pi_t = 0$. Combined with the first-order condition, the associated optimal value of the output gap is found, and the optimal solutions are

$$\begin{aligned} \widetilde{y}_t &= 0, \\ \pi_t &= 0. \end{aligned}$$

Hence, as there is no policy trade-offs the output gap and inflation can be fully stabilized period by period. By inspection of (1) it follows that the nominal interest rate must adjust one for one towards fluctuations in the natural rate of interest ($\mathrm{d}i_t/\mathrm{d}r_t^n = 1$ under optimal discretionary policymaking). This neutralizes any impact of the shock and is compatible with full stabilization of the output gap and inflation.

(8) Assume that the monetary policymaker instead follows a rule for nominal interest-rate setting given as

$$i_t = \rho + \phi\pi_t, \qquad \phi > 1. \tag{4}$$

Derive the solutions for $\widetilde{y}_t$ and π_t for the system (1), (2) and (4). [Hint: Conjecture that the solutions are linear functions of the period's natural rate of interest, r_t^n, and remember that $\mathrm{E}_t r_{t+1}^n = 0$.]. Discuss the differences between these solutions and the ones obtained under discretionary policymaking. Can the monetary policy rule (4) be parameterized such that it will "deliver" the outcomes under discretionary policymaking?

A Use the hint, and conjecture

$$\begin{aligned} \widetilde{y}_t &= Ar_t^n, \\ \pi_t &= Br_t^n, \end{aligned}$$

where A, B are unknown coefficients to be determined. Substitute (4) into (1):

$$\widetilde{y}_t = \mathrm{E}_t\left\{\widetilde{y}_{t+1}\right\} - \sigma^{-1}\left(\phi\pi_t - \mathrm{E}_t\left\{\pi_{t+1}\right\} - r_t^n\right).$$

Then insert the conjectures into this amended DIS equation and the NKPC to obtain:

$$\begin{aligned} Ar_t^n &= \mathrm{E}_t\left\{Ar_{t+1}^n\right\} - \sigma^{-1}\left(\phi Br_t^n - \mathrm{E}_t\left\{Br_{t+1}^n\right\} - r_t^n\right), \\ Br_t^n &= \beta\mathrm{E}_t\left\{Br_{t+1}^n\right\} + \kappa Ar_t^n. \end{aligned}$$

Then use that r_t^n is serially uncorrelated to reduce the expressions to

$$\begin{aligned} Ar_t^n &= -\sigma^{-1}\left(\phi Br_t^n - r_t^n\right), \\ Br_t^n &= \kappa Ar_t^n. \end{aligned}$$

These must hold for any e_t, so therefore the coefficients are determined by

$$\begin{aligned} A &= -\sigma^{-1}\left(\phi B - 1\right), \\ B &= \kappa A. \end{aligned}$$

One finally obtains

$$\begin{aligned} B &= \frac{\sigma^{-1}}{\kappa^{-1} + \sigma^{-1}\phi} > 0, \\ A &= \frac{\kappa\sigma^{-1}}{\kappa^{-1} + \sigma^{-1}\phi} > 0, \end{aligned}$$

and the solutions for the output gap and inflation as

$$\begin{aligned}\widetilde{y}_t &= \frac{\kappa\sigma^{-1}}{\kappa^{-1}+\sigma^{-1}\phi} r_t^n, \\ \pi_t &= \frac{\sigma^{-1}}{\kappa^{-1}+\sigma^{-1}\phi} r_t^n.\end{aligned}$$

In contrast with optimal policy, the shock to the natural rate of interest is now transmitted onto both the output gap and inflation. A positive realization increases demand and thus the output gap, which in turn increases inflation. The effect, however, is dampened by monetary policy, as $\phi > 1$ is seen to exert a reducing influence on the impact. This is because the nominal interest rate is increased by more than the increase in inflation thereby increasing the real interest rate, demand, output gap and inflation. It follows that if $\phi \to \infty$, output gap and inflation will be completely stabilized and the solutions under optimal discretionary policy can be attained.