MakØk3, Fall 2012 (Blok 2)

"Business cycles and monetary stabilization policies"

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Lecture 2, November 20: **A Classical Model** (Galí, Chapter 2)

Introductory remarks

- The standard neoclassical model for (exogenous) economic growth is the simple Solow model featuring a fixed savings rate (no microfoundations)
- When extended with optimizing savings behavior, we get the Ramsey model
  - Heavily applied model in macroeconomics
- Models have *no* role for money and monetary policy

- Purpose of this lecture is to introduce money in such a classical model with microfoundations
- How will money and monetary policy affect the economy?
  - Does this type of “workhorse” model give implications and predictions that looks like the real world (so it could be a useful tool)?

- While the end result is not uplifting, we luckily learn a lot of material that are extremely useful in next lectures

A basic classical model

- Goods market
  - Demand side: Households consume (based on utility maximization)
  - Supply side: Firms produce consumption good (maximize profits under perfect competition)
- Labor market
  - Demand side: Firms hire labor (maximize profits under perfect competition)
  - Supply side: Households supply labor (based on utility maximization)
- Financial markets
  - Households optimally invest in a one-period risk-less bond
  - Households also hold money. Why? We just assume it for now

- *All* prices are perfectly flexible

Household behavior

- Household maximize expected utility

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right), \qquad 0 < \beta < 1. \tag{1}$$

- Budget constraint:

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t \tag{2}$$

(and a No-Ponzi Game constraint ruling out explosive debt)

- Household chooses optimal paths of C_t, N_t and B_t (taking prices P_t, W_t and Q_t as given)
- Let's solve its maximization problem using a Lagrangian method

$$\frac{\partial \mathcal{L}}{\partial C_t} = 0, \qquad U_{c,t} - \lambda_t P_t = 0, \qquad U_{c,t} = \lambda_t P_t \tag{*}$$

$$\frac{\partial \mathcal{L}}{\partial N_t} = 0, \qquad U_{n,t} + \lambda_t W_t = 0, \qquad U_{n,t} = -\lambda_t W_t \tag{**}$$

$$\frac{\partial \mathcal{L}}{\partial B_t} = 0, \qquad \lambda_t Q_t - \beta \mathrm{E}_t \left\{\lambda_{t+1}\right\} = 0, \qquad \lambda_t Q_t = \beta \mathrm{E}_t \left\{\lambda_{t+1}\right\} \tag{***}$$

- From (*) and (**):

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{4}$$

- From (*) and (***)

$$Q_t = \beta \mathrm{E}_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \tag{5}$$

- With $U\left(C_t, N_t\right) \equiv \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{\varphi}}{1+\varphi}$, $\sigma, \varphi > 0$,

$$C^{\sigma} N_t^{\varphi} = \frac{W_t}{P_t} \tag{6}$$

$$Q_t = \beta \mathrm{E}_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{7}$$

- Log-linear versions of households' optimality conditions (lower-case letters are logs or log-deviations from steady state)

- Labor supply:

$$\sigma c_t + \varphi n_t = w_t - p_t \tag{8}$$

- Bonds pay out 1 the period after the purchase at price Q_t. Bond yield is:

$$\begin{aligned} yield &= \frac{1 - Q_t}{Q_t} \\ 1 + yield &= Q_t^{-1} \\ \log(1 + yield) &= -\log Q_t \\ yield &\simeq -\log Q_t \\ &\equiv i_t \end{aligned}$$

  This is the nominal interest rate

- With this definition, consumption-Euler equation becomes:

$$c_t = \mathrm{E}_t\{c_{t+1}\} - \sigma^{-1}(i_t - \mathrm{E}_t\{\pi_{t+1}\} - \rho) \tag{9}$$

  where $\pi_{t+1} \equiv p_{t+1} - p_t$ and $\rho \equiv -\log \beta$

- Money demand is postulated (for now):

$$m_t - p_t = y_t - \eta i_t, \qquad \eta > 0 \tag{10}$$

Firms' behavior

- Production function for the representative firm

$$Y_t = A_t N_t^{1-\alpha}, \qquad 0 < \alpha < 1 \tag{11}$$

- Profits in each period t:

$$P_t Y_t - W_t N_t \tag{12}$$

- Firms choose N_t to maximize

$$P_t A_t N_t^{1-\alpha} - W_t N_t$$

- First-order condition

$$\begin{aligned} (1-\alpha) P_t A_t N_t^{-\alpha} &= W_t, \\ (1-\alpha) A_t N_t^{-\alpha} &= \frac{W_t}{P_t} \end{aligned} \tag{13}$$

- Labor demand in logs:

$$\log(1-\alpha) + a_t - \alpha n_t = w_t - p_t \tag{14}$$

Equilibrium outcomes

- Goods market clearing (or, resource constraint, or, national account):

$$y_t = c_t \tag{*}$$

- Labor market equilibrium (labor supply equals labor demand):

$$\sigma c_t + \varphi n_t = \log(1-\alpha) + a_t - \alpha n_t \tag{**}$$

- Bond market clearing (household's intertemporal consumption decision with goods market clearing)

$$y_t = \mathrm{E}_t\{y_{t+1}\} - \sigma^{-1}(i_t - \mathrm{E}_t\{\pi_{t+1}\} - \rho) \tag{***}$$

- Production function in the aggregate:

$$y_t = a_t + (1-\alpha)\, n_t \tag{****}$$

- (*), (**) and (****) gives c_t, n_t and y_t
- (***) gives subsequently the real interest rate $r_t \equiv i_t - \mathrm{E}_t\{\pi_{t+1}\}$

- Typical classical features: Real variables like consumption, output and employment (and thus the real wage) are determined from the economy's supply side

- No role for monetary policy here
  - We haven't used the money demand function at all
  - The real interest rate is determined independently of monetary factors
  - Any fluctuation in main macro variables will result from fluctuations in a_t

- We need not even say anything about the nominal interest rate or the price level
  - If we want to determine nominal variables, however, we must specify monetary policy

Monetary policy and prices

The nominal interest rate as policy instrument

- Most central bank uses the nominal interest rate as policy instrument
- Model Problem: Specifying some exogenous path for the nominal interest rate in the model will *not* determine the price level. Note the "Fisherian relationship":

$$\mathrm{E}_t\left\{\pi_{t+1}\right\} = i_t - r_t$$

  where r_t is independent of monetary policy

- Hence, (a rational expectations difference equation)

$$\mathrm{E}_t\left\{p_{t+1}\right\} = p_t + i_t - r_t \tag{*}$$

- We can now introduce a shock which has nothing to do with the economy, ξ_{t+1}: $\mathrm{E}_t\left\{\xi_{t+1}\right\} = 0$. Then any price level satisfying

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1}$$

  is consistent with (*).

- The price level is *indeterminate*
  - Likewise the money supply: $m_t = p_t + y_t - \eta i_t$
  - Likewise the nominal wage

- An exogenous path for the nominal interest rate does not help determine nominal variables in the classical model
- The problem is that the exogeneity leaves the economy without a "nominal anchor"
- A solution is to specify the nominal interest rate as a "feedback rule" where it is specified to be adjusted in response to some nominal variable

- One possibility (out of many!)

$$i_t = \rho + \phi_\pi \pi_t, \qquad \phi_\pi > 0$$

- From the Fisher equation:

$$\phi_\pi \pi_t = \mathrm{E}_t \{\pi_{t+1}\} + r_t - \rho = \mathrm{E}_t \{\pi_{t+1}\} + \widehat{r}_t \tag{22}$$

- If $\phi_\pi > 1$, we have a *unique* stationary (i.e., non-explosive) solution

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(1+k)} \mathrm{E}_t \{\widehat{r}_{t+k}\} \tag{23}$$

- If $\phi_\pi < 1$, we have that *any path* of π_t satisfying (24) is an equilibrium (so, we have indeterminacy)

$$\pi_{t+1} = \phi_\pi \pi_t - \widehat{r}_{t+k} + \xi_{t+1} \tag{24}$$

• Taylor (1993):

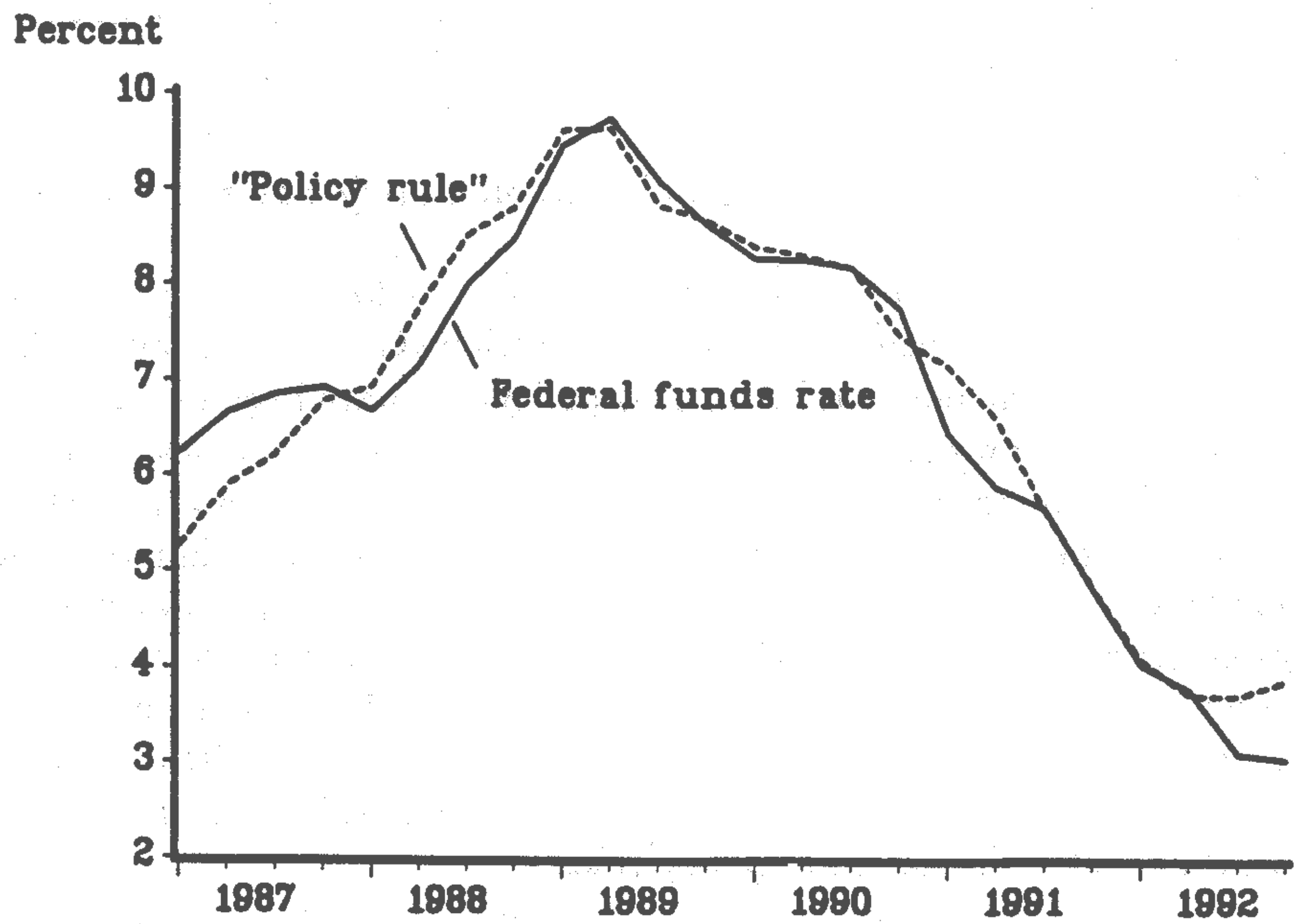

Figure 1. Federal funds rate and example policy rule.

The Taylor Rule:

$$i_t = \alpha + 1.5\,(\pi_t - \pi^*) + 0.5 y_t$$

Note the "$\phi_\pi > 1$" feature: This is the "Taylor principle"

The nominal money supply as policy instrument

- In this case the economy has a "nominal anchor": Consider an exogenous path for m_t
- Combine the money demand equation and the Fisher equation

$$\begin{aligned} m_t - p_t &= y_t - \eta i_t \\ i_t &= \mathrm{E}_t \{p_{t+1} - p_t\} + r_t \end{aligned}$$

$$p_t = -y_t + \eta \left(\mathrm{E}_t \{p_{t+1} - p_t\} + r_t\right) + m_t$$

Hence,

$$\begin{aligned} p_t &= \frac{\eta}{1+\eta} \mathrm{E}_t p_{t+1} + \frac{1}{1+\eta} m_t + u_t \\ u_t &\equiv \frac{1}{1+\eta} \left(\eta r_t - y_t\right) \ \text{(independent of monetary policy)} \end{aligned}$$

- Since $\eta > 0$ we have a *unique* stationary (i.e., non-explosive) solution

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t \{m_{t+k}\} + u_t'$$

$$u_t' \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \mathrm{E}_t \{u_{t+k}\} \ \text{(independent of monetary policy)}$$

- In terms of expected future money growth

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k \mathrm{E}_t \{ \Delta m_{t+k} \} + u_t' \qquad (25)$$

(where Δ is first difference operator)

- For given expected money growth, prices move one for one with the money supply

- The nominal interest rate is found by inserting (25) into the money demand function

$$\begin{aligned} i_t &= \eta^{-1} (y_t - m_t + p_t) \\ &= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k \mathrm{E}_t \{ \Delta m_{t+k} \} + u_t'' \\ u_t'' &\equiv \eta^{-1} (y_t + u_t') \text{ (independent of monetary policy)} \end{aligned}$$

- The nominal interest rate increases with expected future money growth

An example of a process for nominal money growth

- Let

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

- Assume no real shocks (hence y_t and r_t does not move; assume they are zero)
- Solution for prices and nominal interest rate

$$\begin{aligned} p_t &= m_t + \frac{\eta \rho_m}{1+\eta\left(1-\rho_m\right)} \Delta m_t, \\ i_t &= \frac{\rho_m}{1+\eta\left(1-\rho_m\right)} \Delta m_t \end{aligned}$$

- If money growth is persistent there are large effects on prices and positive effect on interest rates
  - No liquidity effect; i.e., an expansionary money growth shock increases the nominal interest rate (and real money falls equivalently)

Do all classical models leave monetary policy “useless”?

- No
- While *money neutrality prevails*, many models with flexible prices can have effects of monetary policy through variations in inflation and the nominal interest rate
  - There will not be monetary *superneutrality*

- Galí presents a model where money enters the utility function

$$U = U\left(C_t, M_t, N_t\right)$$

- Short cut for money’s liquidity services (e.g., saved leisure)
- The model features a micro-founded money demand relation just as postulated before
- Labor supply can now be affected by changes in the nominal interest rate:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

is affected if marginal rate of substitution is affected by money

Dynamic effects of money shocks in a classical money-in-the utility function (from Walsh, 2010)

- To assess the quantitative effects of money shocks, a money-in-the-utility function model is calibrated and simulated

  Calibration: Assign empirically plausible values to the parameters of the model

  Simulation:

  - Perform a linearization of the model's dynamic equations (everything is expressed as percentage deviations from steady state);
  - solve this system by numerical methods (various simulation programs are available on the internet);
  - create artificial time series data from the system

- From the artificial data, one evaluates the properties of the model in terms of:

  - Standard deviations of various relevant variables, and their s.d. relative to output
  - Correlation coefficients of various variables with output
  - Impulse response patterns of variables when shocks hit

- **Main results** (when $U_{cm} > 0$)
  - Steady-state non-superneutrality is of the form of: Higher money growth $\Longrightarrow$ higher inflation and nominal interest rate $\Longrightarrow$ lower money demand $\Longrightarrow$ lower marginal utility of consumption $\Longrightarrow$ lower labor supply $\Longrightarrow$ output
  - If money growth shocks, ε_t^m, shall pay a role, persistence in money growth is necessary ($\rho_m > 0$ is needed). Then, the shock will affects expected next-period inflation, and thus — through the Fisher equation — period t nominal interest rate.
  - The effects of money shocks on labor and output are stronger the more persistence in money growth, but the effects are quantitatively **very** small (nothing compared to what we saw from VARs)

  - Main effects of money shocks are on inflation and nominal interest rates
  - Positive money shocks lead to *higher* nominal interest rates. In contrast with liquidity effect seen in VARs (and in contrast with usual IS/LM story where nominal rates *fall* to increase money demand). Reason is flexible prices:
    * Prices adjust instantaneously so as to *reduce* real money supply, matching the fall in demand resulting from higher nominal interest rates.

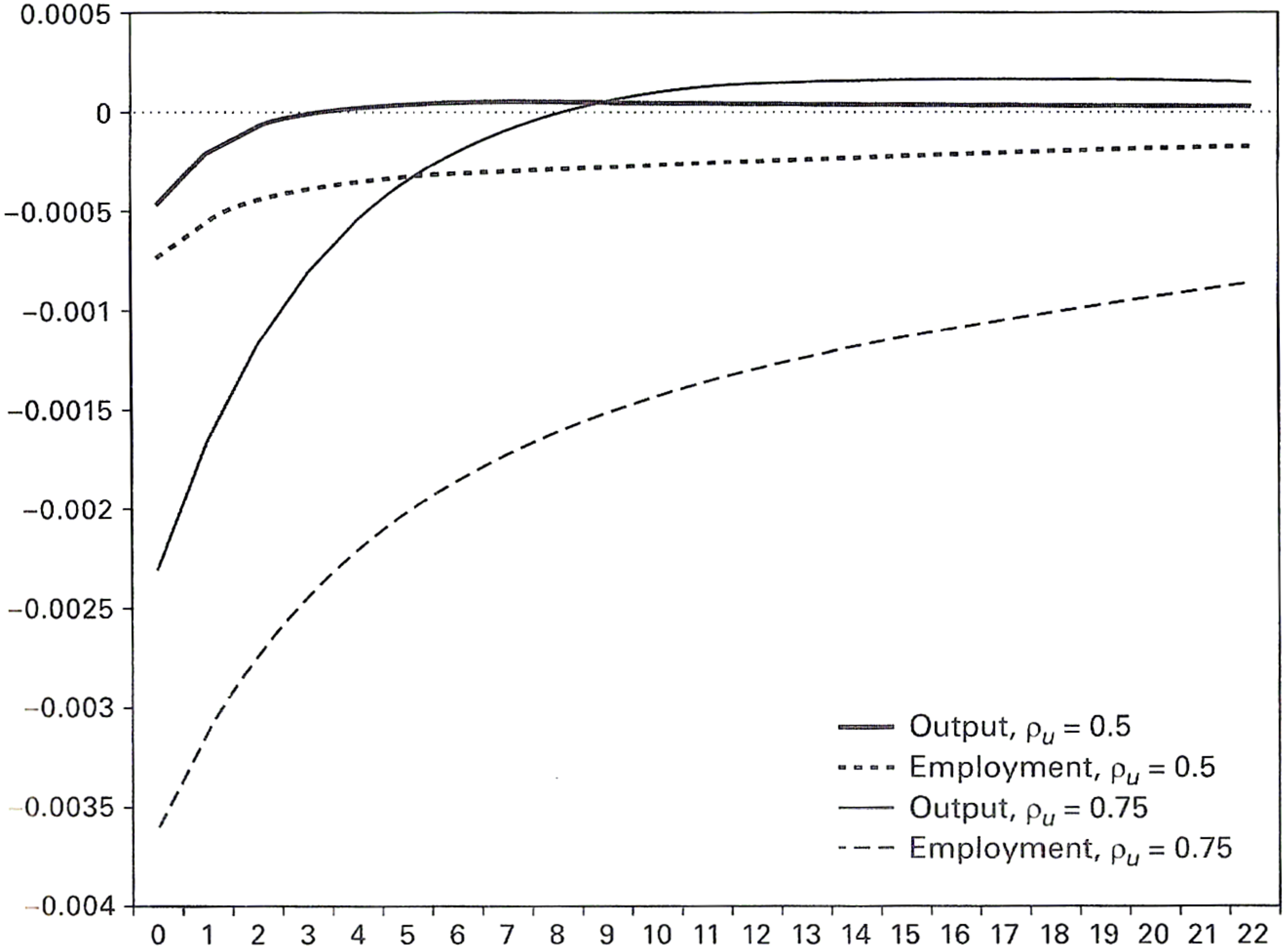

Figure 2.3
Response of output and employment to a positive money growth shock.

Source: Walsh, (2010): Monetary Theory and Policy (The MIT Press) ($\rho_u = \rho_m$)

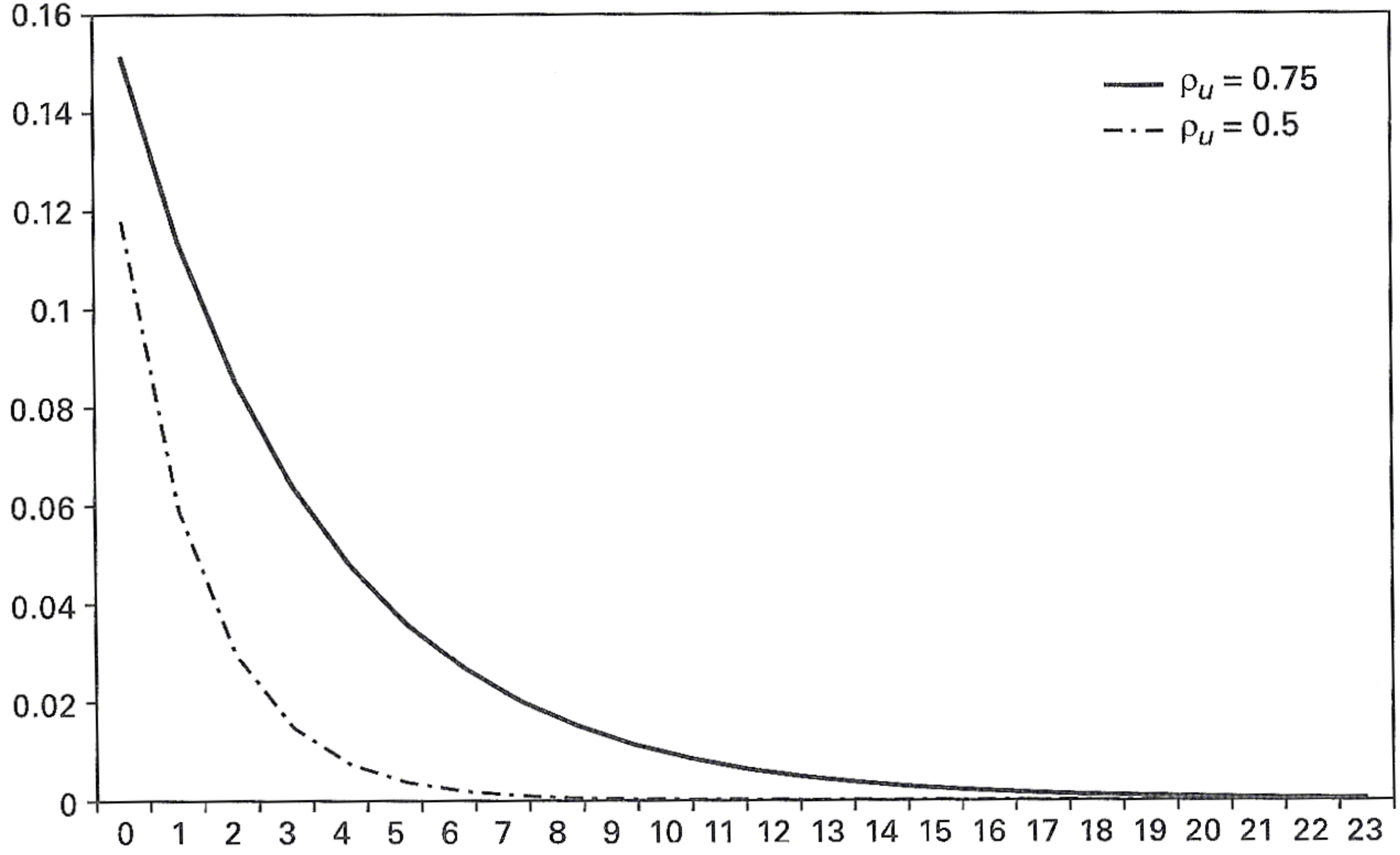

Figure 2.4
Response of the nominal interest rate to a positive money growth shock.

Source: Walsh, (2010): Monetary Theory and Policy (The MIT Press) ($\rho_u = \rho_m$)

Summary

- The classical model has little role for money in determination of real variables
- It should be seen as a model of the long run, or, the average hypothetical evolution of the economy
- I.e., it is important as a benchmark for showing the efficient allocation in a micro-founded economy (i.e., an ideal, but unrealistic, world)
- Some, like the money-in-the-utility function model can be used to assess the optimal long-run inflation rate
  - Optimum: $U_m = 0$. This requires $1 - \exp\{-i\} = 0$ [cf. eq. (28)], or simply $i = 0$ and thus $\pi = -r$
  - The Friedman rule; private opportunity cost of money is equal to social costs (zero)

- Model is not suitable for analyzing the short run implications of monetary shocks as the models, by nature, exhibits monetary neutrality (although not necessarily superneutrality). Impulse response patterns are unrealistic, even when there are effects on real variables
- Short and long run are virtually indistinguishable
- To remedy the short-run failure of such models, one *must* introduce nominal rigidities; This is the aim of New Keynesian Theory

Next time

Monday, November 26:

Lectures: The Basic New Keynesian Model (Galí, 2008, Chapter 3)