

MakØk3, Fall 2012 (Blok 2)

“Business cycles and monetary stabilization policies”

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Lectures, December 3 and 10: **Monetary Policy Design in the Basic New Keynesian Model**
(Galí, Chapter 4)

- We have now developed a simple model for business cycle and monetary policy analysis
 - E.g., we can examine the economy's response to various shocks (including policy shocks)
- Next step is to examine the model's normative implications: I.e., how should monetary policy be conducted?
- What should be the goals of monetary policy?
- What can and what cannot monetary policy achieve
- For this purpose we identify the inefficiencies of the New Keynesian economy, and evaluate whether and how policy can remedy these
- Importantly, a model-consistent welfare criterion will be developed to assess various simple, suboptimal policy rules

Properties of the New-Keynesian model

Basic equations summarized:

- “NKPC”

$$\pi_t = \beta \mathbf{E}_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t, \quad \tilde{y}_t \equiv y_t - y_t^n$$

$$\kappa \equiv \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon} \frac{\sigma(1 - \alpha) + \varphi + \alpha}{1 - \alpha} > 0$$

- “DIS”

$$\tilde{y}_t = \mathbf{E}_t \{ \tilde{y}_{t+1} \} - \sigma^{-1} (i_t - \mathbf{E}_t \{ \pi_{t+1} \} - r_t^n), \quad r_t^n \equiv \rho + \sigma \mathbf{E}_t \{ \Delta y_{t+1}^n \}$$

Properties of a (friendly) “command economy”

- Relevant benchmark, or, ideal outcome, is the allocation chosen by a benevolent social planner (it identifies the efficient outcomes)
- The planner maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U \left(\left[\int_0^1 C_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}, \int_0^1 N_t(i) di \right),$$

subject to

$$C_t(i) = A_t N_t(i)^{1-\alpha} \quad \text{all } i \in [0, 1]$$

- By nature of the consumption basket, $C_t(i) \neq C_t(j)$ is never optimal for a given C_t . Therefore, optimality requires

$$C_t(i) = C_t, \quad N_t(i) = N_t, \quad \text{all } i \in [0, 1].$$

- The problem then simplifies to

$$\max U(A_t N_t^{1-\alpha}, N_t)$$

Optimality condition:

$$(1 - \alpha) A_t N_t^{-\alpha} U_{c,t} + U_{n,t} = 0 \quad - \frac{U_{n,t}}{U_{c,t}} = (1 - \alpha) A_t N_t^{-\alpha} \equiv MPN_t$$

Inefficiencies in the New Keynesian model

Monopolistic competition

- Monopolistic competition implies that prices are a markup over aggregate marginal costs; even under flexible prices:

$$P_t = \mathcal{M} \frac{W_t}{MPN_t}, \quad \mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1} > 1$$

- The model's labor market equilibrium:

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Monopolistic competition results in too low employment and output

- Monetary policy is useless in addressing this market-structure inefficiency
- Fiscal (tax) policy can (in theory) solve the problem. Assume a labor cost subsidy τ (financed lump sum from consumers):

$$P_t = \mathcal{M} \frac{(1 - \tau) W_t}{MPN_t}$$

- Monopoly distortion is eliminated if $\mathcal{M}(1 - \tau) = 1$ (requires $\tau = \varepsilon^{-1}$); this is assumed for technical reasons (to be clarified)

Nominal rigidities

- Price rigidities result in mark-up fluctuations:

$$\mathcal{M}_t = \frac{P_t}{(1 - \tau) W_t MPN_t} = \frac{P_t \mathcal{M}}{W_t MPN_t}$$
$$\frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

- Staggered price setting causes price and thus output dispersion:

$$C_t(i) \neq C_t(j) \quad \text{when} \quad P_t(i) \neq P_t(j)$$

- Inefficiencies due to nominal rigidities can be addressed by monetary policy (at least in part)

What should monetary policy ideally do?

- Assume the labor subsidy $\tau = \varepsilon^{-1}$ is in place; the natural rate of output is then efficient

- Eliminate markup fluctuations, i.e., secure that $\widehat{mc}_t = 0$

– Equivalent of securing:

$$\tilde{y}_t = 0 \text{ all } t$$

- Avoid any price dispersion

– Assuming no past relative price dispersion, $P_{t-1}(i) = P_{t-1}$, all $i \in [0, 1]$

– No firms will change prices when $\widehat{mc}_t = 0$, $mc_t = mc$

– Hence, $P_{t+j}(i) = P_{t+j} = P_{t-1+j}$, all $i \in [0, 1]$, $j = 0, 1, 2, \dots$

– Equivalent of fully stable aggregate prices:

$$\pi_t = 0 \text{ all } t$$

How can this be done?

- Surprisingly there is no policy trade-offs—the ideal policy goals are attainable
 - (Attributable to the simple shock structure; by some denoted “a divine coincidence”.)

- Letting $i_t = r_t^n$, is compatible with attaining both $\tilde{y}_t = 0$ and $\pi_t = 0$

$$\begin{aligned}\pi_t &= \beta \mathbf{E}_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t, \\ \tilde{y}_t &= \mathbf{E}_t \{ \tilde{y}_{t+1} \} - \sigma^{-1} (i_t - \mathbf{E}_t \{ \pi_{t+1} \} - r_t^n)\end{aligned}$$

- Problem. Setting $i_t = r_t^n$ leads to dynamic the system

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_O \begin{bmatrix} \mathbf{E}_t \{ \tilde{y}_{t+1} \} \\ \mathbf{E}_t \{ \pi_{t+1} \} \end{bmatrix}, \quad \mathbf{A}_O \equiv \begin{bmatrix} 1 & \sigma^{-1} \\ \kappa & \beta + \kappa \sigma^{-1} \end{bmatrix}$$

- \mathbf{A}_O has one eigenvalue above one, and one below one. Indeterminacy; i.e., infinitely many stationary inflation and output gap paths

- One could therefore follow the previously considered Taylor rule, amended with a response to the natural rate of interest:

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \tilde{y}_t, \quad \phi_\pi, \phi_y \geq 0,$$

- This leads to the familiar dynamics

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} \mathbf{E}_t \{ \tilde{y}_{t+1} \} \\ \mathbf{E}_t \{ \pi_{t+1} \} \end{bmatrix}$$

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \phi_\pi \beta \\ \sigma \kappa & \kappa + \beta (\sigma + \phi_y) \end{bmatrix}, \quad \Omega \equiv \frac{1}{\sigma + \phi_\pi \kappa + \phi_y}$$

- Uniqueness requires:

$$0 < (\phi_\pi - 1) \kappa + \phi_y (1 - \beta)$$

(the “Taylor principle”)

- The optimal allocation will be achieved in equilibrium

- Such a rule, however, poses a practical problem: r_t^n is not observed in real time
- Therefore, more simple rules can be considered; i.e., rules depending on observable variables

- But how should one to assess their performance?
- I.e., how is it possible to compare one rule to another?

- By developing a welfare criterion!

Welfare criterion in the NK model

- The relevant welfare criterion in the NK model is

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \right\}$$

- How do we use this together with the log-linearized model?
- W is approximated by a second-order Taylor expansion (a first-order expansion would not rank different monetary policies, as these do not affect long-run levels; i.e., steady states)
- Important second-order approximation for any variable Z :

$$\frac{Z_t - Z}{Z} \simeq \hat{z}_t + \frac{1}{2} \hat{z}_t^2$$

where $\hat{z}_t \equiv \log(Z_t/Z)$

- The approximation is performed around an efficient steady state—yields a simple expression, and secures focus on fluctuations only
- If the approximation is around an inefficient steady state, one may get “spurious” welfare results by using a log-linear model (the ignored second-order terms may become important, i.e., policy dependent; Kim and Kim, 2003, *JIE*)

Initial Taylor expansion

$$U_t - U \simeq U_c C \left(\frac{C_t - C}{C} \right) + U_n N \left(\frac{N_t - N}{N} \right) + \frac{1}{2} U_{cc} C^2 \left(\frac{C_t - C}{C} \right)^2 + \frac{1}{2} U_{nn} N^2 \left(\frac{N_t - N}{N} \right)^2$$

(hence, separability, $U_{cn} = 0$, is assumed as in the utility function often applied)

- In log-deviations

$$\begin{aligned} U_t - U &\simeq U_c C \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + U_n N \left(\hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) + \frac{1}{2} U_{cc} C^2 \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right)^2 + \frac{1}{2} U_{nn} N^2 \left(\hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right)^2 \\ &\simeq U_c C \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + U_n N \left(\hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) + \frac{1}{2} U_{cc} C^2 \hat{c}_t^2 + \frac{1}{2} U_{nn} N^2 \hat{n}_t^2 \end{aligned}$$

as $\hat{c}_t^3 \simeq \hat{c}_t^4 \simeq \hat{n}_t^3 \simeq \hat{n}_t^4 \simeq 0$ in a second-order expansion

- Rearranging:

$$U_t - U \simeq U_c C \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 + \frac{1}{2} \frac{U_{cc} C}{U_c} \hat{c}_t^2 \right) + U_n N \left(\hat{n}_t + \frac{1}{2} \hat{n}_t^2 + \frac{1}{2} \frac{U_{nn} N}{U_n} \hat{n}_t^2 \right)$$

- Simplifying:

$$U_t - U \simeq U_c C \left(\hat{c}_t + \frac{1 - \sigma}{2} \hat{c}_t^2 \right) + U_n N \left(\hat{n}_t + \frac{1 + \varphi}{2} \hat{n}_t^2 \right)$$

where

$$\sigma \equiv -\frac{U_{cc} C}{U_c} > 0, \quad \varphi \equiv \frac{U_{nn} N}{U_n} > 0$$

- Using the goods-market equilibrium condition $\widehat{c}_t = \widehat{y}_t$:

$$U_t - U \simeq U_c C \left(\widehat{y}_t + \frac{1 - \sigma}{2} \widehat{y}_t^2 \right) + U_n N \left(\widehat{n}_t + \frac{1 + \varphi}{2} \widehat{n}_t^2 \right)$$

- Now comes a “tricky” part: Rewrite \widehat{n}_t in terms of output

Relationship between employment, output and relative prices

- From last lecture:

$$N_t = \int_0^1 N_t(i) di = \int_0^1 \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} di$$

$$N_t = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} di = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} di$$

- In logs:

$$n_t = \frac{1}{1-\alpha} (y_t - a_t) + \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} di,$$

$$(1-\alpha) n_t = y_t - a_t + d_t, \quad d_t \equiv (1-\alpha) \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} di.$$

- Around a zero inflation steady state (where $d = 0$):

$$(1-\alpha) \widehat{n}_t = \widehat{y}_t - a_t + d_t$$

- We need to find d_t , the measure of price dispersion, as it is a second-order term, which will have welfare effects.

- Start by the definition of the price index:

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}$$

- Then,

$$\begin{aligned} 1 &= \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{1-\varepsilon} di \\ &= \int_0^1 \exp [(1-\varepsilon)(p_t(i) - p_t)] di \\ &\simeq 1 + (1-\varepsilon) \int_0^1 (p_t(i) - p_t) di + \frac{(1-\varepsilon)^2}{2} \int_0^1 (p_t(i) - p_t)^2 di \end{aligned} \quad (*)$$

in a second-order approximation around $p(i) = p$.

- Letting $E_i \{p_t(i)\} \equiv \int_0^1 p_t(i) di$ denote the mean of log prices across sectors, (*) becomes

$$p_t \simeq E_i \{p_t(i)\} + \frac{1-\varepsilon}{2} \int_0^1 (p_t(i) - p_t)^2 di \quad (**)$$

- Then consider the integral in the definition of d_t :

$$\begin{aligned} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} di &= \int_0^1 \exp \left[-\frac{\varepsilon}{1-\alpha} (p_t(i) - p_t) \right] di \\ &\simeq 1 - \frac{\varepsilon}{1-\alpha} \int_0^1 (p_t(i) - p_t) di + \frac{1}{2} \left(\frac{\varepsilon}{1-\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 di \end{aligned}$$

- From (*) we have that

$$\int_0^1 (p_t(i) - p_t) di \simeq -\frac{1-\varepsilon}{2} \int_0^1 (p_t(i) - p_t)^2 di$$

- Hence,

$$\begin{aligned} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} di &\simeq 1 + \frac{\varepsilon(1-\varepsilon)}{2(1-\alpha)} \int_0^1 (p_t(i) - p_t)^2 di + \frac{1}{2} \left(\frac{\varepsilon}{1-\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 di \\ &\simeq 1 + \frac{1}{2} \frac{\varepsilon}{1-\alpha} \frac{1}{\Theta} \int_0^1 (p_t(i) - p_t)^2 di \quad \Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\varepsilon} \end{aligned}$$

- Using (**) we get

$$\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} di \simeq 1 + \frac{1}{2} \frac{\varepsilon}{1-\alpha} \frac{1}{\Theta} \int_0^1 (p_t(i) - \mathbf{E}_i \{p_t(i)\})^2 di = 1 + \frac{1}{2} \frac{\varepsilon}{1-\alpha} \frac{1}{\Theta} \text{var}_i \{p_t(i)\}$$

where $\text{var}_i \{p_t(i)\}$ is price variance across sectors

- Since

$$d_t \equiv (1 - \alpha) \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\varepsilon}{1-\alpha}} di = (1 - \alpha) \log \left(1 + \frac{1}{2} \frac{\varepsilon}{1-\alpha} \frac{1}{\Theta} \text{var}_i \{p_t(i)\} \right)$$

we get

$$d_t \simeq \frac{1}{2} \frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\}$$

(which also proves that we rightfully ignored it when looking at the linear dynamics; d_t is a second-order term)

- We then substitute $\hat{n}_t = (1 - \alpha)^{-1} \hat{y}_t - (1 - \alpha)^{-1} a_t + (1 - \alpha)^{-1} d_t$ into

$$U_t - U \simeq U_c C \left(\hat{y}_t + \frac{1 - \sigma}{2} \hat{y}_t^2 \right) + U_n N \left(\hat{n}_t + \frac{1 + \varphi}{2} \hat{n}_t^2 \right)$$

and get

$$U_t - U \simeq U_c C \left(\hat{y}_t + \frac{1 - \sigma}{2} \hat{y}_t^2 \right) + \frac{U_n N}{1 - \alpha} \left(\hat{y}_t + d_t + \frac{1 + \varphi}{2(1 - \alpha)} (\hat{y}_t - a_t)^2 \right) + \text{t.i.p.}$$

where t.i.p. is “terms independent of policy” and the third-order effects and higher are ignored

- Rewrite, so we get utility change measured as percentage change in steady-state consumption:

$$\frac{U_t - U}{U_c C} \simeq \hat{y}_t + \frac{1 - \sigma}{2} \hat{y}_t^2 + \frac{U_n N}{U_c C (1 - \alpha)} \left(\hat{y}_t + d_t + \frac{1 + \varphi}{2(1 - \alpha)} (\hat{y}_t - a_t)^2 \right) + \text{t.i.p.}$$

- Now remember that we are approximating around an efficient steady state. Hence,

$$-\frac{U_n}{U_c} = MPN = (1 - \alpha) AN^{-\alpha} \equiv (1 - \alpha) \frac{Y}{N}$$

- Therefore,

$$-\frac{U_n}{U_c} = (1 - \alpha) \frac{C}{N}$$

or,

$$-\frac{U_n}{U_c} \frac{N}{C(1 - \alpha)} = 1$$

- The utility approximation therefore simplifies to

$$\begin{aligned} \frac{U_t - U}{U_c C} &\simeq \frac{1 - \sigma}{2} \widehat{y}_t^2 - \frac{1}{2} \frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\} - \frac{1 + \varphi}{2(1 - \alpha)} (\widehat{y}_t - a_t)^2 + \text{t.i.p.} \\ &= -\frac{1}{2} \left[\frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\} + (\sigma - 1) \widehat{y}_t^2 + \frac{1 + \varphi}{1 - \alpha} (\widehat{y}_t - a_t)^2 \right] + \text{t.i.p.} \\ &= -\frac{1}{2} \left[\frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\} + \left(\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) \widehat{y}_t^2 - 2 \frac{1 + \varphi}{1 - \alpha} \widehat{y}_t a_t \right] + \text{t.i.p.} \\ &= -\frac{1}{2} \left[\frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\} + \left(\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) (\widehat{y}_t^2 - 2 \widehat{y}_t y_t^n) \right] + \text{t.i.p.} \\ &= -\frac{1}{2} \left[\frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\} + \left(\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) (\widehat{y}_t^2 - 2 \widehat{y}_t y_t^n + (y_t^n)^2) \right] + \text{t.i.p.} \end{aligned}$$

$$y_t^n \equiv \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} a_t.$$

- The welfare measure is therefore approximately

$$\mathbb{W} = \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c C} = -\frac{1}{2} \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\Theta} \text{var}_i \{p_t(i)\} + \left(\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) \tilde{y}_t^2 \right] + \text{t.i.p.}$$

as $\tilde{y}_t = \hat{y}_t - y_t^n$.

- We finally use *Lemma 2* from Chapter 4's appendix:

$$\sum_{t=0}^{\infty} \beta^t \text{var}_i \{p_t(i)\} = \frac{\theta}{(1 - \theta)(1 - \beta\theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$$

and get

$$\begin{aligned} \mathbb{W} &= \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{U_t - U}{U_c C} = \\ &\quad -\frac{1}{2} \left(\sigma + \frac{\alpha + \varphi}{1 - \alpha} \right) \mathbf{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\kappa} \pi_t^2 + \tilde{y}_t^2 \right] \end{aligned}$$

with

$$\begin{aligned} \kappa &\equiv \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon} \frac{\sigma(1 - \alpha) + \varphi + \alpha}{1 - \alpha} \\ &= \frac{(1 - \theta)(1 - \beta\theta)}{\theta} \Theta \frac{\sigma(1 - \alpha) + \varphi + \alpha}{1 - \alpha} \end{aligned}$$

The performance of various policy rules

- With this utility-based welfare loss one can assess the performance of various policy rules
- One can perform optimal policy exercises as linear-quadratic optimization problems (next time)
- Galí exemplifies the importance of price stability in the New-Keynesian model by assessing the performance of the policy rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t$$

(note: a function of \hat{y}_t , not \tilde{y}) for various policy parameters:

	Taylor Rule			
ϕ_π	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1
$(\sigma_\zeta, \rho_\zeta)$	—	—	—	—
$\sigma(\tilde{y})$	0.55	0.28	0.04	1.40
$\sigma(\pi)$	2.60	1.33	0.21	6.55
<i>welfare loss</i>	0.30	0.08	0.002	1.92

Concluding remarks

- The New-Keynesian model offers a simple framework for welfare-based policy analysis
- Models is (in principle) immune to the Lucas critique, and the welfare criterion is consistent with the one used to derive the economy's behavioral equations
- One can rank various policy rules as well as meaningfully compare their quantitative welfare differences
- The simple model is obviously too simple to represent the real world, but its basic features “survive” in large-scale versions used in many inflation-targeting central banks