MakØk3, Fall 2012 (Blok 2)

"Business cycles and monetary stabilization policies"

Henrik Jensen
Department of Economics
University of Copenhagen

Lecture 5, November 17: **Monetary Policy Tradeoffs** (Galí, 2008, Chapter 5)

Introductory remarks

- In the simple New-Keynesian model, appropriate monetary policy can achieve the efficient allocation:
  - Conditional on a fiscal policy that neutralizes average monopolistic distortions, an appropriate monetary policy
    - eliminates fluctuations in the average mark-up
    - eliminates price (and output) dispersion
- Implications of appropriate policy, $\pi_t = 0$, $\widetilde{y}_t = 0$ all t
- The utility-based welfare measure

$$\mathbb{W} = -\frac{1}{2}\left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right) \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\kappa}\pi_t^2 + \widetilde{y}_t^2\right]$$

  confirms optimality of this outcome

- Reality for monetary policymakers are more complicated: Policymaking involves trade offs
- Introducing trade-offs will emphasize the importance of policy credibility

- Introduce an exogenous inflation variation in the NKPC:

$$\begin{aligned} \pi_t &= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \widetilde{y}_t, \\ &= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa [x_t + (y_t^e - y_t^n)], \qquad & x_t \equiv y_t - y_t^e, \\ &= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa x_t + u_t, \qquad & u_t \equiv \kappa (y_t^e - y_t^n), \end{aligned} \tag{2}$$

  where y_t^e is efficient level of output

- Hence, inefficient fluctuations in the natural rate of output are allowed for (captured by $u_t \neq 0$)

- Associated DIS:

$$x_t = \mathrm{E}_t \{x_{t+1}\} - \sigma^{-1} (i_t - \mathrm{E}_t \{\pi_{t+1}\} - r_t^e), \qquad r_t^e \equiv \rho + \sigma \mathrm{E}_t \{\Delta y_{t+1}^e\} \tag{4}$$

- Assuming that the steady-state output is efficient, the welfare **loss** (i.e., "minus utility") is proportional to

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \alpha_x x_t^2\right],$$

  where $\alpha_x \geq 0$ (In the case of a loss derived from the underlying utility function, $\alpha_x = \kappa/\varepsilon$.)

- Hence, welfare-relevant activity variable is x_t, not $\widetilde{y}_t$

- $y_t^e \neq y_t^n$ can be caused by many types of distortions; e.g., exogenous variations in the desired markup; exogenous changes in wage determination (see Appendix 5.2). Note that $u_t \neq 0$ makes $\pi_t = 0$, $x_t = 0$ *impossible*

Optimal monetary policy under discretion

- The criterion of monetary policy is to minimize

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \alpha_x x_t^2\right],$$

subject to NKPC and DIS and assuming

$$u_t = \rho_u u_{t-1} + \varepsilon_t^u \qquad (3)$$

- Shocks u_t is often called a cost-push shock

- Due to forward-looking nature of model, there is difference between solution under commitment to a policy path, or period-by-period optimization—discretion.
- We examine discretion first:
  - In period t, policy cannot affect expectations about future variables (no persistence in equations)
  - Hence, when optimizing, expected future variables are taken as given

- Solution trick: Treat x_t as the policy instrument, and find i_t compatible with the solution afterwards
- Minimizing

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \alpha_x x_t^2\right]$$

w.r.t. $\{i_t\}_0^\infty$ subject to (2) and (4) is thus equivalent of minimizing

$$\pi_t^2 + \alpha_x x_t^2 + V_t$$

w.r.t. x_t subject to

$$\pi_t = \kappa x_t + v_t$$

taking *as given* V_t and v_t

- The problem thus becomes a sequence of single-period problems. Simple first-order condition:

$$-\alpha_x x_t = \kappa \pi_t, \qquad t = 0, 1, 2, ... \tag{5'}$$

"Lean against the wind" policy: If inflationary pressures arise, contract output ($x_t < 0$) such that the marginal cost (left-hand side) equals the marginal gain (the right hand side)

- The first-order condition can be viewed as a policy rule; a "targeting rule"
(A rule for the nominal interest rate is called an "instrument rule.")

- A tradeoff is present in policy between inflation and output
  - Note that with more nominal rigidity, lower κ, the inflation-output trade-off is "worse": A given reduction in output reduces inflation by less

- Use the first-order condition in the NKPC to eliminate x_t:

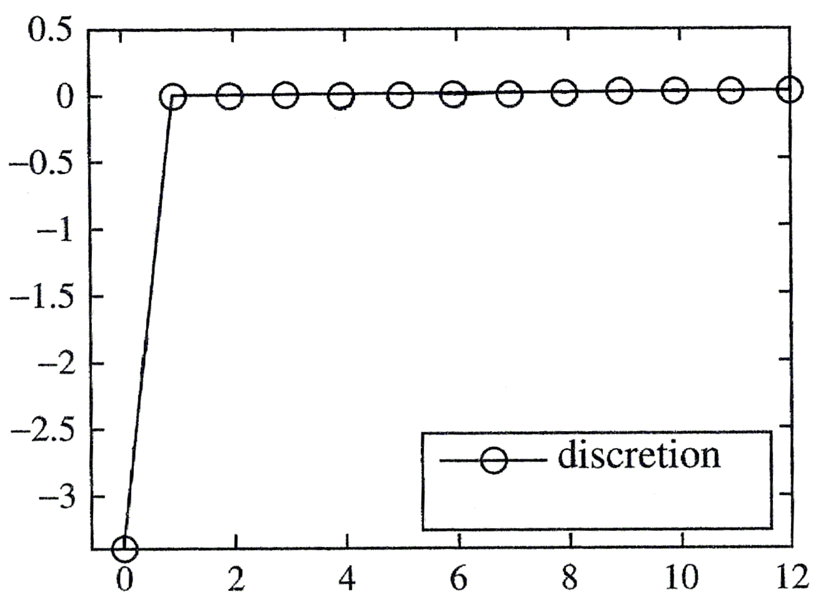
$$\begin{aligned}\pi_t &= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa x_t + u_t \\ \pi_t &= \beta \mathrm{E}_t \{\pi_{t+1}\} - \left(\kappa^2/\alpha_x\right) \pi_t + u_t \\ \pi_t &= \frac{\beta}{1+\kappa^2/\alpha_x} \mathrm{E}_t \{\pi_{t+1}\} + \frac{1}{1+\kappa^2/\alpha_x} u_t\end{aligned}$$

  - First-order expectational difference equation in π_t
  - One unstable root, $\left(1+\kappa^2/\alpha_x\right)/\beta > 1$, secures uniqueness of a stationary solution for inflation
  - Unique x_t follows from first-order condition

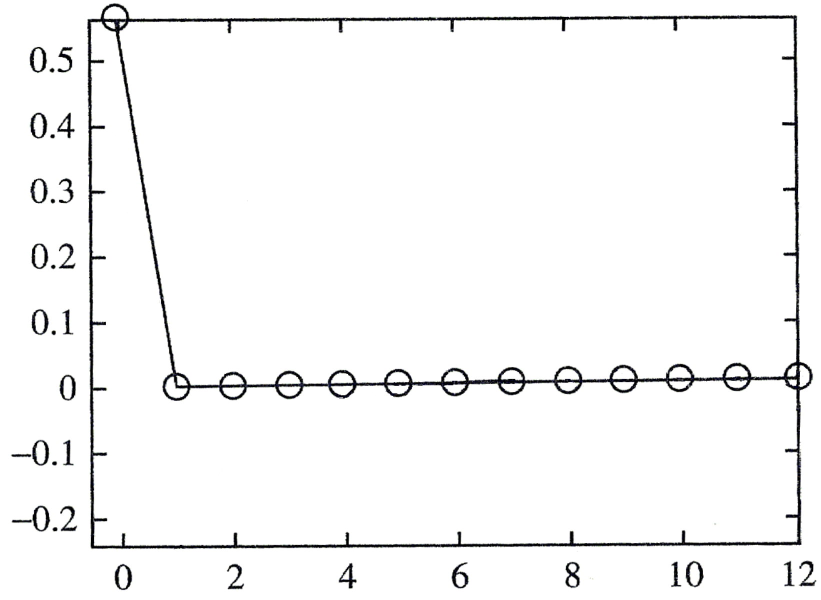
- Solved by the method of undetermined coefficients:

$$\pi_t = \alpha_x \Psi u_t \tag{6}$$

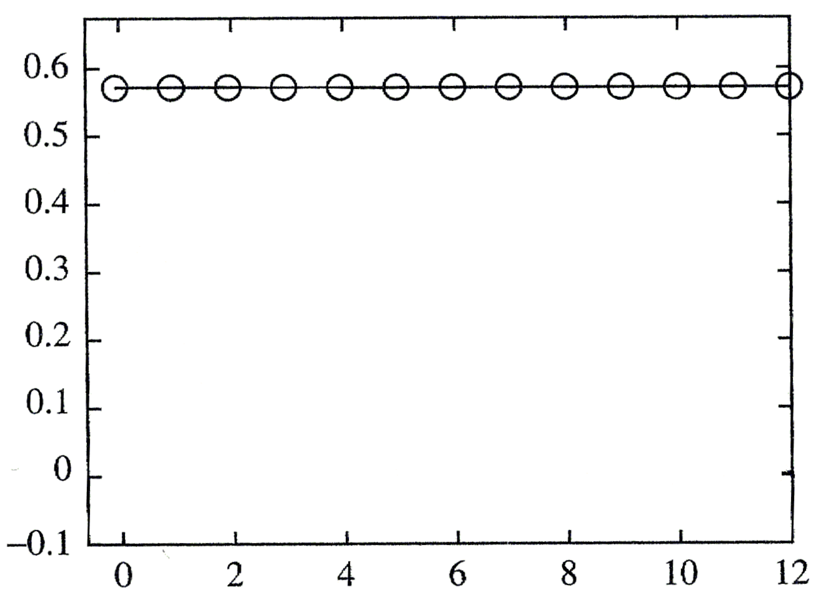
$$x_t = -\kappa \Psi u_t, \qquad \Psi \equiv \frac{1}{\kappa^2 + \alpha_x (1-\beta\rho_u)} \tag{7}$$



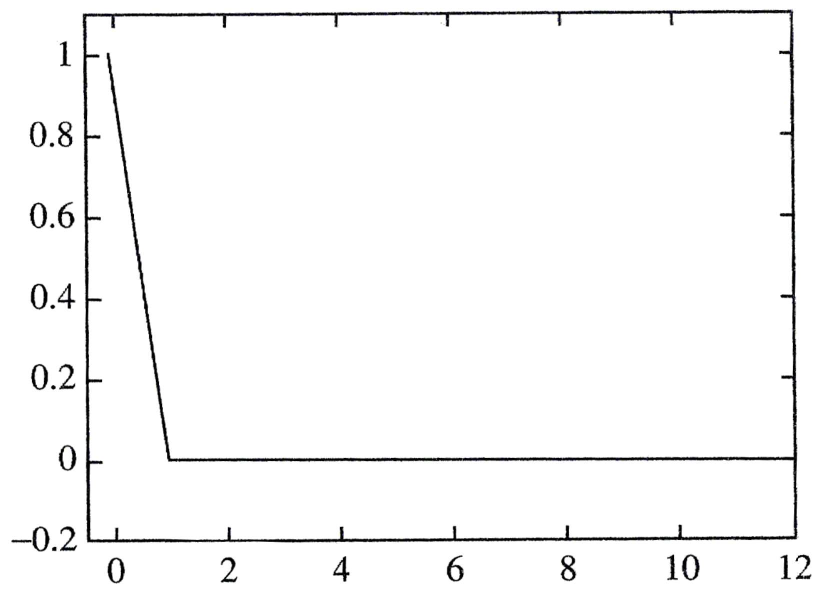
Output Gap



Inflation



Price Level



Cost Push Shock

- Solution for x_t and π_t and the associated solutions for $\mathrm{E}_t\{x_{t+1}\}$ and $\mathrm{E}_t\{\pi_{t+1}\}$ can be used in DIS-curve to find associated solution for the nominal interest rate

  The optimal value of the nominal interest rate can be written in many ways

- If written as a function of exogenous disturbances one gets

$$\begin{aligned} x_t &= \mathrm{E}_t\{x_{t+1}\} - \sigma^{-1}\left(i_t - \mathrm{E}_t\{\pi_{t+1}\} - r_t^e\right) \\ -\kappa\Psi u_t &= -\rho_u\kappa\Psi u_t - \sigma^{-1}\left(i_t - \rho_u\alpha_x\Psi u_t - r_t^e\right) \end{aligned}$$

  and thus

$$i_t = r_t^e + \left(\sigma\kappa\left(1-\rho_u\right) + \rho_u\alpha_x\right)\Psi u_t. \tag{8}$$

  This "instrument rule" leads to indeterminacy (in contrast with the "targeting rule"—the first-order condition)

- If written as function of inflation one gets

$$\begin{aligned} i_t &= r_t^e + \frac{\left(\sigma\kappa\left(1-\rho_u\right) + \rho_u\alpha_x\right)}{\alpha_x}\pi_t \\ &= r_t^e + \phi_\pi\pi_t \qquad \phi_\pi \equiv \frac{\sigma\kappa\left(1-\rho_u\right)}{\alpha_x} + \rho_u \end{aligned}$$

  Uniqueness requires $\phi_\pi > 1$ (here $\sigma\kappa/\alpha_x > 1$)

- One can always design some rule that ensures determinacy and the optimal discretionary outcomes

Optimal monetary policy under commitment

- Problem: find sequences of π_t and x_t that minimize the loss

  Technique: Set up the Lagrangian:

$$\mathcal{L} = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x x_t^2 \right) + \gamma_t \left(\pi_t - \beta \pi_{t+1} - \kappa x_t \right) \right] \right\}$$

  where γ_t are the multipliers on the NKPC

- Optimal paths are solutions to

$$\frac{\partial \mathcal{L}}{\partial x_t} = 0, \qquad t \geq 0, \qquad (*)$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} = 0, \qquad t \geq 0. \qquad (**)$$

$$\mathcal{L} = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x x_t^2 \right) + \gamma_t \left(\pi_t - \beta \pi_{t+1} - \kappa x_t \right) \right] \right\}$$

- Finding (*). We get

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_0} &= 0 \qquad \Leftrightarrow \qquad \alpha_x x_0 - \kappa \gamma_0 = 0, \\
\frac{\partial \mathcal{L}}{\partial x_1} &= 0 \qquad \Leftrightarrow \qquad \beta \alpha_x x_1 - \beta \kappa \gamma_1 = 0, \\
\frac{\partial \mathcal{L}}{\partial x_2} &= 0 \qquad \Leftrightarrow \qquad \beta^2 \alpha_x x_2 - \beta^2 \kappa \gamma_2 = 0, \\
&\vdots
\end{aligned}$$

so,

$$\alpha_x x_t - \kappa \gamma_t = 0, \qquad t \geq 0.$$

$$\mathcal{L} = \mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x x_t^2 \right) + \gamma_t \left(\pi_t - \beta \pi_{t+1} - \kappa x_t \right) \right] \right\}$$

- Finding (**). We get

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial \pi_0} &= 0 \qquad \Leftrightarrow \qquad \pi_0 + \gamma_0 = 0, \\
\frac{\partial \mathcal{L}}{\partial \pi_1} &= 0 \qquad \Leftrightarrow \qquad \beta \pi_1 + \beta \gamma_1 - \beta^0 \beta \gamma_0 = 0, \\
\frac{\partial \mathcal{L}}{\partial \pi_2} &= 0 \qquad \Leftrightarrow \qquad \beta^2 \pi_2 + \beta^2 \gamma_2 - \beta \beta \gamma_1 = 0, \\
\frac{\partial \mathcal{L}}{\partial \pi_3} &= 0 \qquad \Leftrightarrow \qquad \beta^3 \pi_3 + \beta^3 \gamma_3 - \beta^2 \beta \gamma_2 = 0, \\
&\vdots
\end{aligned}$$

implying

$$\begin{aligned}
\pi_0 + \gamma_0 &= 0, \\
\pi_t + \gamma_t - \gamma_{t-1} &= 0, \qquad t \geq 1
\end{aligned}$$

- First-order conditions repeated

$$\alpha_x x_t - \kappa \gamma_t = 0, \qquad t \geq 0.$$

$$\begin{aligned} \pi_0 + \gamma_0 &= 0, \\ \pi_t + \gamma_t - \gamma_{t-1} &= 0, \qquad t \geq 1 \end{aligned}$$

Combined:

$$-\alpha_x x_0 = \kappa \pi_0 \tag{x}$$

$$-\alpha_x (x_t - x_{t-1}) = \kappa \pi_t, \qquad t \geq 1 \tag{xx}$$

- Central implication: Commitment policy involves (for $t \geq 1$)

$$\pi_t = -\frac{\alpha_x}{\kappa}(x_t - x_{t-1})$$

- Hence, inflation and output gap will exhibit *history dependence* or *policy inertia*
- Alternatively, we can write [using (xx) *and* (x)]

$$p_t - p_{-1} = -\frac{\alpha_x}{\kappa} x_t \tag{14'}$$

I.e., commitment policy involves price-level stationarity

- Optimal, as it will affect inflation expectations, and improve the inflation-output gap tradeoff

- Intuition: Consider the case of $\rho_u = 0$

  - If $u_t > 0$, inflation rises, and optimal policy is contractive
  - With policy inertia, next-period policy will also be contractive

    => Next-period inflation is dampened (it becomes negative so the price level falls)

    => Current inflation expectations are dampened

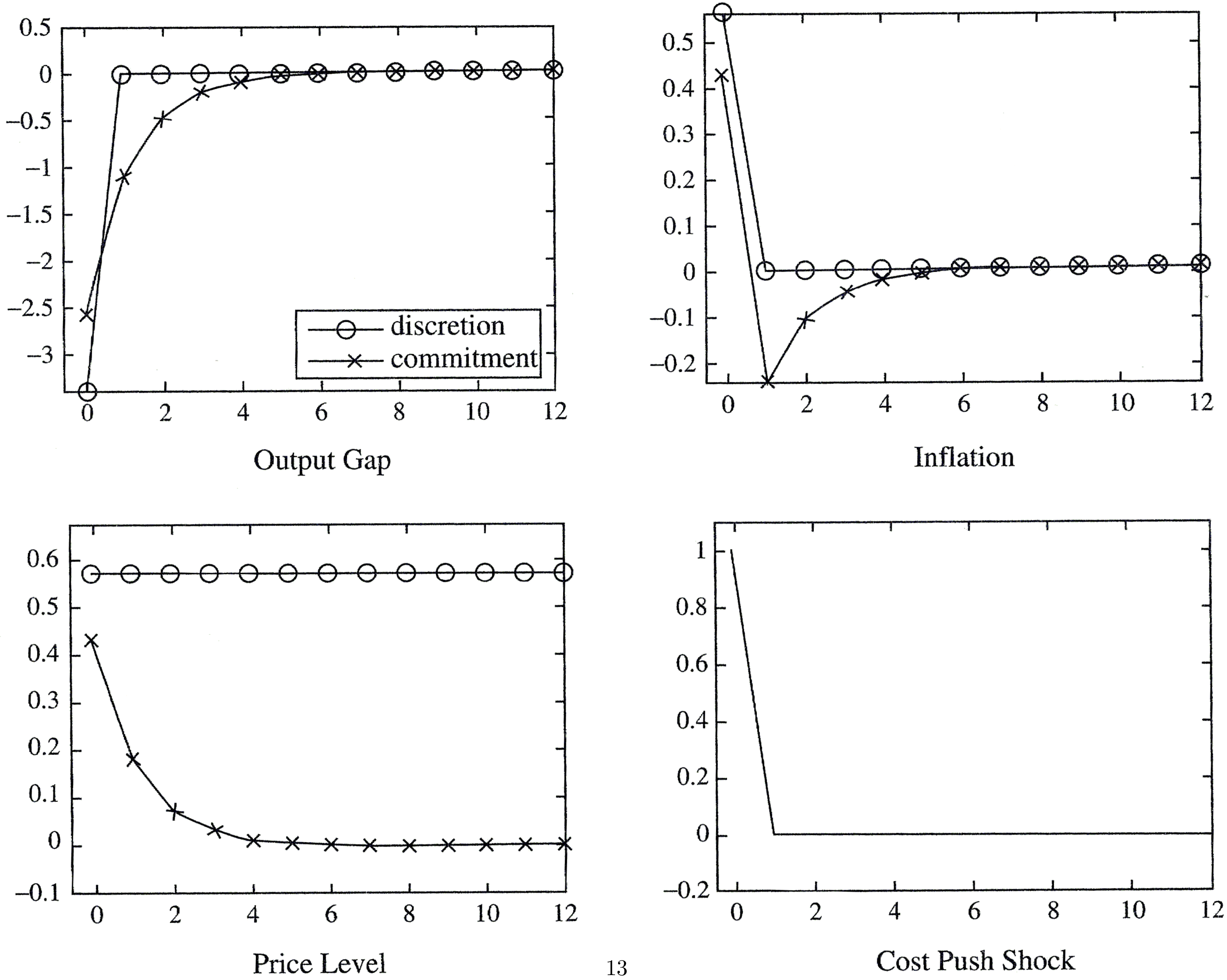
    => Current inflation is reduced

  - Thus a mild, but prolonged contraction provides better inflation stabilization

- Generally, policy inertia improves predictability of future policy, and current variables are easier to affect by smaller current policy adjustments

  Through policy inertia, the central bank is "letting the market do some of the stabilization"

- Or, being able to affect $E_t\{\pi_{t+1}\}$ improves the inflation-output trade off


discretion
commitment
Output Gap
Inflation
Price Level
Cost Push Shock

- Note the credibility problem of the commitment solution. When a cost-push shock has "worn out," it is no longer optimal to contract policy
  - Typical difference between *ex ante* and *ex post* optimality
  - The *time-inconsistency* of optimal policy
  - If the policymaker cannot commit, i.e., affect expectations, it doesn't reap the gains in terms of better stabilization performance
  - Hence, institutional frameworks securing credible commitment policy is desirable

- Difference in first-order conditions for inflation in period 0 and in periods i, $i \geq 1$, show mathematically the time-inconsistency:
  - When period 1 arrives, it is optimal "forget about period 0," set up a new Lagrangian, and aim at:

$$\begin{aligned} \pi_1 + \gamma_1 &= 0, \\ \pi_{1+i} + \gamma_{1+i} - \gamma_i &= 0, \quad i \geq 1. \end{aligned}$$

  Hence, commitment plan at time t is optimal at time t, but **not** at time $t+1$

- Solution for prices and output: Use the NKPC rewritten in terms of price levels:

$$\widehat{p}_t - \widehat{p}_{t-1} = \beta \mathrm{E}_t \left\{ \widehat{p}_{t+1} - \widehat{p}_t \right\} + \kappa x_t + u_t, \qquad \widehat{p}_t \equiv p_t - p_{-1}.$$

- Use the optimality condition:

$$\widehat{p}_t = -\left(\alpha_x / \kappa\right) x_t,$$

to get

$$\widehat{p}_t - \widehat{p}_{t-1} = \beta \mathrm{E}_t \left\{ \widehat{p}_{t+1} - \widehat{p}_t \right\} - \left(\kappa^2 / \alpha_x\right) \widehat{p}_t + u_t,$$

or,

$$\widehat{p}_t = a \widehat{p}_{t-1} + a \beta \mathrm{E}_t \left\{ \widehat{p}_{t+1} \right\} + a u_t, \qquad a \equiv \frac{\alpha_x}{\alpha_x \left(1 + \beta\right) + \kappa^2} < 1.$$

- Solve (second-order) difference equation by method of undetermined coefficients
- Conjecture:

$$\widehat{p}_t = \psi_p \widehat{p}_{t-1} + \psi_u u_t$$

$$\begin{aligned} \mathrm{E}_t \left\{ \widehat{p}_{t+1} \right\} &= \psi_p \widehat{p}_t + \psi_u \rho_u u_t \\ &= \left(\psi_p\right)^2 \widehat{p}_{t-1} + \psi_u \left(\psi_p + \rho_u\right) u_t \end{aligned}$$

- Insert conjecture into difference equation

$$\psi_p \widehat{p}_{t-1} + \psi_u u_t = a\widehat{p}_{t-1} + a\beta \left(\psi_p\right)^2 \widehat{p}_{t-1} + a\beta\psi_u \left(\psi_p + \rho_u\right) u_t + au_t$$

- Coefficients therefore solve

$$\begin{aligned} \psi_p &= a + a\beta \left(\psi_p\right)^2 \\ \psi_u &= a\beta\psi_u \left(\psi_p + \rho_u\right) + a \end{aligned}$$

- First equation identifies ψ_p:

$$\psi_p = \frac{1 - \sqrt{1 - 4a^2\beta}}{2a\beta}, \qquad 0 < \psi_p < 1.$$

(The lower root is the one securing a stationary solution.)

- Second equation identifies ψ_u:

$$\begin{aligned} \psi_u \psi_p &= a\beta\psi_u \left(\left(\psi_p\right)^2 + \rho_u \psi_p\right) + a\psi_p, \\ \psi_u \psi_p &= a\beta\psi_u \left(\psi_p\right)^2 + a\beta\psi_u \rho_u \psi_p + a\psi_p, \\ \psi_u \psi_p &= \psi_u \left(\psi_p - a\right) + a\beta\psi_u \rho_u \psi_p + a\psi_p, \\ 0 &= -\psi_u \left(1 - \beta\rho_u \psi_p\right) + \psi_p, \end{aligned}$$

$$\psi_u = \frac{\psi_p}{1 - \beta\rho_u\psi_p}.$$

- In Galí's notation ($\delta = \psi_p$):

$$\widehat{p}_t = \delta \widehat{p}_{t-1} + \frac{\delta}{1 - \beta \rho_u \delta} u_t \tag{15}$$

- Solution for output follows from this together with (14):

$$\begin{aligned} x_t &= \delta x_{t-1} - \frac{\kappa}{\alpha_x} \frac{\delta}{1 - \beta \rho_u \delta} u_t, \qquad t \geq 1 \\ x_0 &= -\frac{\kappa}{\alpha_x} \frac{\delta}{1 - \beta \rho_u \delta} u_0 \end{aligned} \tag{16}$$

- Emphasizes the history dependence of optimal policy under commitment

- The implied path for the nominal interest rate can again be derived using the DIS curve (and it can be properly designed so as to deliver a unique equilibrium)

The case of “small” steady-state distortions

- Assume that the steady-state is inefficient, i.e., a labor subsidy $\tau = \varepsilon^{-1}$ is not possible
- This has implications for derivation of welfare function and the nature of optimal policy under discretion and commitment

- From last lecture:

$$\frac{U_t - U}{U_c C} \simeq \widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2 + \frac{U_n N}{U_c C\,(1-\alpha)}\left(\widehat{y}_t + \frac{1}{2}\frac{\varepsilon}{\Theta}\text{var}_i\left\{p_t\,(i)\right\} + \frac{1+\varphi}{2\,(1-\alpha)}\left(\widehat{y}_t - a_t\right)^2\right) + \text{t.i.p.}$$

- In an efficient steady state:

$$-\frac{U_n}{U_c} = MPN = (1-\alpha)\,\frac{Y}{N} = (1-\alpha)\,\frac{C}{N} \qquad \Longrightarrow \qquad \frac{U_n N}{U_c C\,(1-\alpha)} = -1$$

making the linear $\widehat{y}_t$ terms *vanish*

- In an *inefficient* steady state where $1-\Phi \;\equiv \mathcal{M}^{-1} < 1$:

$$-\frac{U_n}{U_c} = MPN\,(1-\Phi) = (1-\alpha)\,\frac{C}{N}\,(1-\Phi) \qquad \Longrightarrow \qquad \frac{U_n N}{U_c C\,(1-\alpha)} = -\,(1-\Phi)$$

making the linear $\widehat{y}_t$ terms *stay*

- We get

$$\frac{U_t - U}{U_c C} \simeq \widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2 - (1-\Phi)\left(\widehat{y}_t + \frac{1}{2}\frac{\varepsilon}{\Theta}\text{var}_i\{p_t(i)\} + \frac{1+\varphi}{2(1-\alpha)}(\widehat{y}_t - a_t)^2\right) + \text{t.i.p.}$$
$$= \Phi\widehat{y}_t + \frac{1-\sigma}{2}\widehat{y}_t^2 - \frac{1}{2}\left(\frac{\varepsilon}{\Theta}\text{var}_i\{p_t(i)\} + \frac{1+\varphi}{1-\alpha}(\widehat{y}_t - a_t)^2\right) + \text{t.i.p.}$$

as Φ is "small," which means that it is of first order (hence, Φ multiplied by squared terms vanish)

- Using the *Lemma 2* from Appendix in Chapter 4, one gets; cf. Appendix 5.1:

$$\mathbb{W} = \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\Phi\widehat{x}_t - \frac{\varepsilon}{2\lambda}\pi_t^2 - \frac{1}{2}\left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right)\widehat{x}_t^2\right],$$

where

$$\widehat{x}_t \equiv x_t - x,$$

with $x \equiv y^n - y^e < 0$ measuring the inefficiency of steady-state output.

- In absence of inefficiencies, $\Phi = 0$ and $\widehat{x}_t = x_t$ and we have the analysis from before
- With inefficiencies, raising $\widehat{x}_t$ above zero has welfare benefits (obviously)

Optimal policy

- The loss function under (small) steady-state distortions become

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x \widehat{x}_t^2 \right) - \Lambda \widehat{x}_t \right].$$

- The NKPC has the same qualitative representation as before:

$$\begin{aligned}
\pi_t &= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \widetilde{y}_t, \\
&= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \left[x_t + (y_t^e - y_t^n) \right], \\
&= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \left[x_t - x + (y_t^e - y_t^n) + x \right], \\
&= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \left[\widehat{x}_t + (y_t^e - y_t^n) + y^n - y^e \right], \\
&= \beta \mathrm{E}_t \{\pi_{t+1}\} + \kappa \widehat{x}_t + u_t, \qquad u_t \equiv \kappa \left[(y_t^e - y^e) - (y_t^n - y^n) \right]. \qquad (18)
\end{aligned}$$

- Optimal policy in terms of shock responses will be qualitatively the same as under an efficient steady state
- Average policies will, however, be vastly different
  - Under discretion the incentive to increase $\widehat{x}_t$ is feeding into inflation expectations and inflation resulting in a permanent, inefficient, *inflation bias* (like in Barro and Gordon models of the 1980s)
  - Under commitment, the incentive will only be temporarily exploited and inflation will gradually converge to zero