MakØk3, Fall 2012/2013 (Blok 2)

"Business cycles and monetary stabilization policies"

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Lecture 6, January 7 and 15: **Sticky Wages and Prices** (Galí, Chapter 6)

Introductory remarks

- In the simple New-Keynesian model nominal rigidities takes the form of *goods price rigidities*
  - A crucial break with the classical model of (Galí, 2008, Chapter 2), as monetary policy then plays a role for output determination
- Evidence pointed to the realism of presence of goods price rigidities

- The model's labor market is, however, portrayed as classical in the sense that a flexible nominal wage clears the labor market
  - Unrealistic in itself
  - Precludes, by construction, meaningful talk about unemployment in absence of other labor market frictions
  - The model of Chapter 6, however, does not explicitly consider unemployment, but Galí shows how it can be restated to account for unemployment in his 2011 book

Table 1: Survey evidence on nominal rigidities and other features of wage-setting

	(1)	(2)	(3)	(4)	(5)
	Duration (in months)		Sector Size	Flexibility	Indexation
	Prices	Wages	s_A	τ	γ_w
Austria (AUT)	9.1	12.5	0.35	0.07	0.1
Belgium (BEL)	9.9	12.6	0.45	0.23	0.98
Spain (ESP)	9.7	11.9	0.2	0.12	0.55
France (FRA)	10.1	12	0.46	0.2	0.06
Greece (GRC)	10.2	11.9	0.38	0.34	0.2
Ireland (IRL)	8.5	12.8	0.42	0.15	0.05
Italy (ITA)	9.5	20.3	0.44	0.04	0.02
Netherlands (NLD)	9.1	13.9	0.28	0.11	0
Portugal (PRT)	9.5	12.9	0.19	0.06	0.09
Czech Republic (CZE)	9.7	14.6	0.34	0.12	0.08
Estonia (EST)	10	12.7	0.41	0.21	0.04
Hungary (HUN)	10.7	13.8	0.27	0.03	0.11
Lithuania (LTU)	8.4	11.4	0.47	0.45	0.11
Poland (POL)	9.5	15.4	0.45	0.14	0.07
Slovenia (SVN)	9.6	11.8	0.45	0.28	0.23
Total (ALL)	9.6	14.9	0.37	0.12	0.15

Source: Knell (2010, ECB WP)

- The basic model is therefore extended with nominal wage rigidity
- To understand the implications of nominal wage rigidity, it is introduced in a way akin to goods price rigidities
- This reflects how a macroeconomic research "program" typically develops
  - Start with a simple model
  - If it creates only nonsense, stop the program. If it gives some insights the profession finds valuable in terms of empirical validity and/or policy prescriptions, then continue
  - Extend the simple model with further realistic assumptions (i.e., relax some of the simplifying assumptions)

- Indeed, the introduction of sticky wages came in 2000 (by Erceg, Henderson and Levin), after the basic model was developed in mid-1990s (by King and Wolman, 1996; Yun, 1996; Woodford, 1996; Rotemberg and Woodford, 1997). Made into medium-scale models by, e.g., Smets and Wouters (2003) and Christiano et al. (2005, JPE)
- Ex post, this gradual evolution has pedagogical advantages—one learns better how each extension contributes to the model's properties
  - A medium-scale estimated New-Keynesian model as RAMSES used for monetary policy analysis in Sweden, would be impossible to explain "from scratch"

The basic NK model with sticky wages

- Most assumptions from the basic model of Chapter 3 are retained

New assumptions:

- Regarding the labor market:
  - Each household supplies a distinct type of labor
  - Each type of labor is used by the firms
  - Each household has monopoly power in the determination of their nominal wage (or, are represented by a trade union)—labour suppliers are therefore monopolistic competitors; just as good suppliers (firms)
  - Nominal wage setting is subject to rigidities: Independent of past wage history, a wage setter cannot reset its wage with probability $\theta_w \in [0, 1]$
- Regarding the asset market: Complete markets are assumed (see my expository note on web)
  - Existence of a complete set of state-contingent securities implies *full income insurance*, so marginal utility of income is equalized across households

Optimal behavior by firms

- For each firm $i \in [0,1]$ (producing a unique consumption good), production function is

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{1}$$

where

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} \mathrm{d}j \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}, \qquad \varepsilon_w > 1 \tag{2}$$

with $N_t(i,j)$ being firm i's employment of type-j labor, $j \in [0,1]$.

- In addition to choosing a price (if "allowed" by the Calvo mechanism) and production, a firm must choose the optimal composition of the labor it employs
- Let $W_t(j)$ be the nominal wage of type-j labor
- Optimization is in two stages:
  1) For any choice of $N_t(i)$ choose the $N_t(i,j)$s taking as given the nominal wages
  2) Choose optimal prices as in Chapter 3 (equivalent of choosing $N_t(i)$)

Remark: This mirrors households' consumption choice: First, they choose the composition of consumption goods, then it chooses the consumption aggregate

- The optimal choice of $N_t(i,j)$s follows by cost minimization as

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t}\right)^{-\varepsilon_w} N_t(i), \qquad \text{all } i,j \in [0,1] \tag{3}$$

where

$$W_t \equiv \left[\int_0^1 W_t(i)^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}} \tag{4}$$

- Remark the analogy, or symmetry, with the optimal consumption decisions of households derived in Chapter 3 (Appendix 3.1):

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon_p} C_t \qquad \text{(Eq.1, Chap. 3)}$$

where

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon_p}\, \mathrm{d}i\right]^{\frac{1}{1-\varepsilon_p}}$$

and where ε is replaced by

  * $\varepsilon_p > 1$, the elasticity of substitution between types of goods, distinct from
  * $\varepsilon_w > 1$, the elasticity of substitution between types of labor

- We also get

$$\int_0^1 W_t(j)\, N_t(i,j)\, \mathrm{d}j = W_t N_t(i), \quad \text{analogy to} \quad : \int_0^1 P_t(i)\, C_t(i)\, \mathrm{d}i = P_t C_t$$

- Second step of firm's optimization: Reset price according to

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k \mathrm{E}_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k} \left(Y_{t+k|t} \right) \right) \right\}$$

$$\text{s.t. } Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon_p} C_{t+k} \qquad \text{(Eq. 8, Chap. 3)}$$

where θ has been replaced by

  * θ_p, the probability of firms not being able to reset price, distinct from
  * θ_w, the probability households not being able to reset wage

- In Chapter 3 we saw that log-linearized around a zero-price-inflation steady state this gave the optimal (log) price as

$$p_t^* = \mu^p + (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathrm{E}_t \left\{ mc_{t+k|t} + p_{t+k} \right\}$$

where $\mu^p \equiv \log \mathcal{M}_p = \log \frac{\varepsilon_p}{\varepsilon_p - 1} = -mc$ is log of the desired price markup

- This was shown to give the following inflation-adjustment equation:

$$\pi_t^p = \beta \mathrm{E}_t \left\{ \pi_{t+1}^p \right\} + \lambda_p \widehat{mc}_t, \qquad \lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \qquad \text{(Eq. 16, Chap. 3)}$$

with π_t^p replacing π_t to emphasize that it is price inflation

- To facilitate comparison with upcoming equation for wage dynamics, price dynamics are written in terms of markup fluctuations instead of marginal cost fluctuations:

$$\begin{aligned}
\pi_t^p &= \beta \mathrm{E}_t \left\{\pi_{t+1}^p\right\} + \lambda_p \widehat{mc}_t \\
&= \beta \mathrm{E}_t \left\{\pi_{t+1}^p\right\} + \lambda_p \left(mc_t - mc\right) \\
&= \beta \mathrm{E}_t \left\{\pi_{t+1}^p\right\} + \lambda_p \left(mc_t + \mu^p\right) \\
&\equiv \beta \mathrm{E}_t \left\{\pi_{t+1}^p\right\} - \lambda_p \left(\mu_t^p - \mu^p\right) \\
&= \beta \mathrm{E}_t \left\{\pi_{t+1}^p\right\} - \lambda_p \widehat{\mu}_t^p \qquad (5)
\end{aligned}$$

Optimal behavior by households

- Households, now indexed by $j \in [0,1]$, has utility

$$\mathrm{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t U\left(C_t\left(j\right), N_t\left(j\right)\right) \right\}$$

with the usual consumption index (already used for relative demand schedules in firms' optimization):

$$C_t \equiv \left[\int_0^1 C_t\left(i\right)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \mathrm{d}i \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}}$$

- The new aspect is that each household supplies the unique labor type j, and sets a nominal wage taking into account the firms relative demand for labor types and the wage-Calvo assumption

- A household that resets its wage in period t will choose W_t^* to maximize

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta\theta_w)^k \, U\left(C_{t+k|t}, N_{t+k|t}\right) \right\}$$

subject to

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k}, \qquad N_{t+k} \equiv \int_0^1 N_{t+k}(i)\, di$$

and budget constraint:

$$P_{t+k} C_{t+k|t} + E_{t+k}\left\{ Q_{t+k,t+k+1} D_{t+k+1|t} \right\} \leq D_{t+k|t} + W_t^* N_{t+k|t} - T_{t+k}$$

where $D_{t+k|t}$ is market value of portfolio of state-contingent claims, and $Q_{t+k,t+k+1}$ is the stochastic discount factor (see expository note for details)

- First-order condition for W_t^*:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \,\mathrm{E}_t \left\{ U_c\left(C_{t+k|t}, N_{t+k|t}\right) \frac{\partial C_{t+k|t}}{\partial W_t^*} + U_n\left(C_{t+k|t}, N_{t+k|t}\right) \frac{\partial N_{t+k|t}}{\partial W_t^*} \right\} = 0$$

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \,\mathrm{E}_t \left\{ U_c\left(C_{t+k|t}, N_{t+k|t}\right) \frac{1}{P_{t+k}} \left(N_{t+k|t} + \frac{W_t^* \partial N_{t+k|t}}{\partial W_t^*} \right) + U_n\left(C_{t+k|t}, N_{t+k|t}\right) \frac{\partial N_{t+k|t}}{\partial W_t^*} \right\} = 0$$

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \,\mathrm{E}_t \left\{ N_{t+k|t} \left[U_c\left(C_{t+k|t}, N_{t+k|t}\right) \frac{1}{P_{t+k}} \left(1 + \frac{\partial N_{t+k|t} W_t^*}{\partial W_t^* N_{t+k|t}} \right) + U_n\left(C_{t+k|t}, N_{t+k|t}\right) \frac{\partial N_{t+k|t}}{\partial W_t^*} \frac{1}{N_{t+k|t}} \right] \right\} = 0$$

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \,\mathrm{E}_t \left\{ N_{t+k|t} \left[U_c\left(C_{t+k|t}, N_{t+k|t}\right) \frac{W_t^*}{P_{t+k}} \left(1 + \frac{\partial N_{t+k|t} W_t^*}{\partial W_t^* N_{t+k|t}} \right) + U_n\left(C_{t+k|t}, N_{t+k|t}\right) \frac{\partial N_{t+k|t}}{\partial W_t^*} \frac{W_t^*}{N_{t+k|t}} \right] \right\} = 0$$

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \,\mathrm{E}_t \left\{ N_{t+k|t} \left[U_c\left(C_{t+k|t}, N_{t+k|t}\right) \frac{W_t^*}{P_{t+k}} (1 - \varepsilon_w) - \varepsilon_w U_n\left(C_{t+k|t}, N_{t+k|t}\right) \right] \right\} = 0$$

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \,\mathrm{E}_t \left\{ N_{t+k|t} \left[U_c\left(C_{t+k|t}, N_{t+k|t}\right) \frac{W_t^*}{P_{t+k}} + \mathcal{M}_w U_n\left(C_{t+k|t}, N_{t+k|t}\right) \right] \right\} = 0$$

where $\mathcal{M}_w \equiv \frac{\varepsilon_w}{\varepsilon_w - 1} > 1$ is the desired *wage* markup

- Rewritten as

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \,\mathrm{E}_t \left\{ N_{t+k|t} U_c\left(C_{t+k|t}, N_{t+k|t}\right) \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right\} = 0, \quad MRS_{t+k|t} \equiv -\frac{U_n\left(C_{t+k|t}, N_{t+k|t}\right)}{U_c\left(C_{t+k|t}, N_{t+k|t}\right)}. \tag{8}$$

- In special case of full wage flexibility:

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w MRS_t,$$

i.e., wages are set as a markup over the marginal rate of substitution (introducing an inefficiently high real wage due to the monopoly power of households)

- First-order condition is log-linearized around a zero-wage-inflation steady state, and one gets the (log) optimal wage:

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \, \mathrm{E}_t \left\{ mrs_{t+k|t} + p_{t+k} \right\} \tag{9}$$

where $\mu^w \equiv \log \mathcal{M}_w$.

- Remark the analogy with the previously stated optimal price-setting rule

$$p_t^* = \mu^p + (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k \, \mathrm{E}_t \left\{ mc_{t+k|t} + p_{t+k} \right\}$$

We can therefore derive an analogous wage-inflation schedule

- With the assumed utility function,

$$U(C,N) = \frac{1}{1-\sigma} C^{1-\sigma} - \frac{1}{1+\varphi} N^{1+\varphi},$$

$$MRS = -\frac{U_n}{U_c} = C^{\sigma} N^{\varphi}.$$

- Hence, $mrs_{t+k|t} = \sigma c_{t+k|t} + \varphi n_{t+k|t}$
- Due to the assumption about complete asset markets, $c_{t+k|t} = c_{t+k}$
- Hence $mrs_{t+k|t} = \sigma c_{t+k} + \varphi n_{t+k|t}$, and when the average marginal rate of substitution in the economy is defined as $mrs_{t+k} = \sigma c_{t+k} + \varphi n_{t+k}$ one gets

$$\begin{aligned} mrs_{t+k|t} &= mrs_{t+k} + \varphi \left(n_{t+k|t} - n_{t+k}\right) \\ &= mrs_{t+k} - \varepsilon_w \varphi \left(w_t^* - w_{t+k}\right) \end{aligned}$$

- Note the analogy with the marginal cost expression used for the derivation of price dynamics

$$mc_{t+k|t} = mc_{t+k} - \frac{\alpha \varepsilon_p}{1-\alpha} \left(p_t^* - p_{t+k}\right) \qquad \text{(Eq. 14, Chap. 3)}$$

- The optimal (log) wage then becomes

$$w_t^* = \mu^w + (1-\beta\theta^w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \, \mathrm{E}_t \left\{ mrs_{t+k} - \varepsilon_p \varphi \left(w_t^* - w_{t+k} \right) + p_{t+k} \right\}$$

$$w_t^* \left(1 + (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \, \varepsilon_p \varphi \right) = \mu^w + (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \, \mathrm{E}_t \left\{ mrs_{t+k} + \varepsilon_p \varphi w_{t+k} + p_{t+k} \right\}$$

$$w_t^* \left(1 + \varepsilon_p \varphi \right) = \mu^w + (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \, \mathrm{E}_t \left\{ mrs_{t+k} + \varepsilon_p \varphi w_{t+k} + p_{t+k} \right\}$$

$$w_t^* = \frac{1-\beta\theta_w}{1+\varepsilon_p \varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k \, \mathrm{E}_t \left\{ \mu^w + mrs_{t+k} + \varepsilon_p \varphi w_{t+k} + p_{t+k} \right\}$$

- Let $\mu_t^w \equiv w_t - p_t - mrs_t$ be the economy's average wage markup. Then,

$$\begin{aligned} w_t^* &= \frac{1-\beta\theta_w}{1+\varepsilon_p \varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k \, \mathrm{E}_t \left\{ \mu^w + w_{t+k} - p_{t+k} - \mu_{t+k}^w + \varepsilon_p \varphi w_{t+k} + p_{t+k} \right\} \\ &= \frac{1-\beta\theta_w}{1+\varepsilon_p \varphi} \sum_{k=0}^{\infty} (\beta\theta_w)^k \, \mathrm{E}_t \left\{ \left(1+\varepsilon_p \varphi\right) w_{t+k} - \widehat{\mu}_{t+k}^w \right\} \end{aligned}$$

where $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w$.

- This is the unique stationary solution to the first-order rational expectations difference equation:

$$w_t^* = \beta\theta_w \mathrm{E}_t \left\{ w_{t+1}^* \right\} + (1-\beta\theta_w) \left(w_t - \left(1+\varepsilon_p \varphi\right)^{-1} \widehat{\mu}_t^w \right) \tag{10}$$

- Combined with the log-linear dynamics for aggregate wages from wage index,

$$w_t = \theta_w w_{t-1} + (1 - \theta_w)\, w_t^*, \tag{11}$$

one gets a wage inflation equation

$$\pi_t^w = \beta \mathrm{E}_t \{\pi_{t+1}^w\} - \lambda_w \widehat{\mu}_t^w, \qquad \lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w (1 + \varepsilon_w \varphi)} \tag{12}$$

where $\pi_t^w \equiv w_t - w_{t-1}$ is nominal wage inflation

- Note the analogy with the price inflation curve

- Note that shocks to the economy under wage rigidity cause fluctuations in the wage markup (the wedge between real wages and the marginal rate of substitution), and thus variations in wage inflation [so (12) replaces the condition $w_t - p_t = mrs_t$ in the model with flexible wages]

- Optimal intertemporal allocation of consumption across time is independent of wage setting and is given by

$$c_t = \mathrm{E}_t \{c_{t+1}\} - \sigma^{-1} \left(i_t - \mathrm{E}_t \left\{\pi_{t+1}^p\right\} - \rho\right) \tag{13}$$

Equilibrium

- Again, the equilibrium will be formulated in terms of gaps
- The output gap, $\widetilde{y}_t \equiv y_t - y_t^n$ is again output relative to the natural rate; but now y_t^n is output under flexible prices *and* flexible nominal wages
- The *real wage* gap is defined as

$$\widetilde{\omega}_t = \omega_t - \omega_t^n$$

  with the real wage being

$$\omega_t \equiv w_t - p_t$$

- The natural real wage is found from the definition of marginal costs:

$$\begin{aligned} mc_t &\equiv w_t - p_t - mpn_t \\ &= \omega_t - (y_t - n_t) - \log(1-\alpha) \end{aligned}$$

  which defines ω_t^n as

$$\begin{aligned} \omega_t^n &= \log(1-\alpha) + (y_t^n - n_t^n) - \mu^p \\ &= \log(1-\alpha) + \psi_{ya}^n a_t - \frac{1}{1-\alpha}\left(\psi_{ya}^n a_t - a_t\right) - \mu^p \\ &= \log(1-\alpha) + \psi_{wa}^n a_t - \mu^p, \qquad \psi_{wa}^n \equiv \frac{1-\psi_{ya}^n \alpha}{1-\alpha} \end{aligned}$$

- We now express the price Phillips curve in terms of gaps
- We again use $mc_t \equiv w_t - p_t - mpn_t$ but expressed as average markup ($mc_t = -\mu_t^p$):

$$\mu_t^p = mpn_t - \omega_t$$

$$\widehat{\mu}_t^p = mpn_t - \omega_t - \mu^p$$

$$\widehat{\mu}_t^p = y_t - n_t - \omega_t - \mu^p + \log(1-\alpha)$$

- Since $\omega_t^n = \log(1-\alpha) + (y_t^n - n_t^n) - \mu^p$ we get

$$\begin{aligned}\widehat{\mu}_t^p &= (\widetilde{y}_t - \widetilde{n}_t) - \widetilde{\omega}_t \\ &= -\frac{\alpha}{1-\alpha}\widetilde{y}_t - \widetilde{\omega}_t \end{aligned} \tag{14}$$

- Inserted into price inflation equation (5):

$$\pi_t^p = \beta \mathrm{E}_t \left\{ \pi_{t+1}^p \right\} + \kappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t, \qquad \kappa_p \equiv \frac{\lambda_p \alpha}{1-\alpha} \tag{15}$$

- Similarly, to express the wage Phillips curve in terms of gaps, we use that

$$\mu_t^w = w_t - p_t - mrs_t$$

and thus

$$\begin{aligned} \widehat{\mu}_t^w &= \omega_t - mrs_t - \mu^w \\ &= \widetilde{\omega}_t - (\sigma \widetilde{y}_t + \varphi \widetilde{n}_t) \\ &= \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right) \widetilde{y}_t. \end{aligned} \tag{16}$$

- Inserted into the wage-inflation curve (12):

$$\pi_t^w = \beta \mathrm{E}_t \left\{\pi_{t+1}^w\right\} + \kappa_w \widetilde{y}_t + \lambda_w \widetilde{\omega}_t, \qquad \kappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi}{1-\alpha}\right) \tag{17}$$

- In addition to the two "Phillips curves" we have the usual Euler-equation written in terms of the output gap:

$$\widetilde{y}_t = \mathrm{E}_t \left\{ \widetilde{y}_{t+1} \right\} - \sigma^{-1} \left(i_t - \mathrm{E}_t \left\{ \pi^p_{t+1} \right\} - r^n_t \right) \tag{19}$$

where

$$r^n_t \equiv \rho + \sigma \mathrm{E}_t \left\{ \Delta y^n_{t+1} \right\}$$

is the natural rate of interest

- The model is closed by a specification of monetary policy in the form of a generalized Taylor rule:

$$i_t = \rho + \phi_p \pi^p_t + \phi_w \pi^w_t + \phi_y \widetilde{y}_t + v_t$$

and a definition equation linking real wages and price and nominal wage inflation:

$$\widetilde{\omega}_t = \widetilde{\omega}_{t-1} + \pi^w_t - \pi^p_t - \Delta \omega^n_t$$

- We now have five equations to solve for the five endogenous variables $\widetilde{y}_t$, π^p_t, π^w_t, $\widetilde{\omega}_t$, i_t as functions of the exogenous shocks (a_t and v_t), and given $\widetilde{\omega}_{t-1}$

- To simplify slightly, the nominal interest rate rule is inserted into the Euler-equation, and the system of four equilibrium conditions are written in matrix form as

$$\mathbf{A}_{w,0}\mathbf{x}_t = \mathbf{A}_{w,1}\mathrm{E}_t\left\{\mathbf{x}_{t+1}\right\} + \mathbf{B}_w\mathbf{z}_t$$

$$\mathbf{x}_t \equiv \left[\widetilde{y}_t, \pi_t^p, \pi_t^w, \widetilde{\omega}_{t-1}\right]', \quad \mathbf{z}_t \equiv \left[r_t^n - v_t, \Delta\omega_t^n\right]'$$

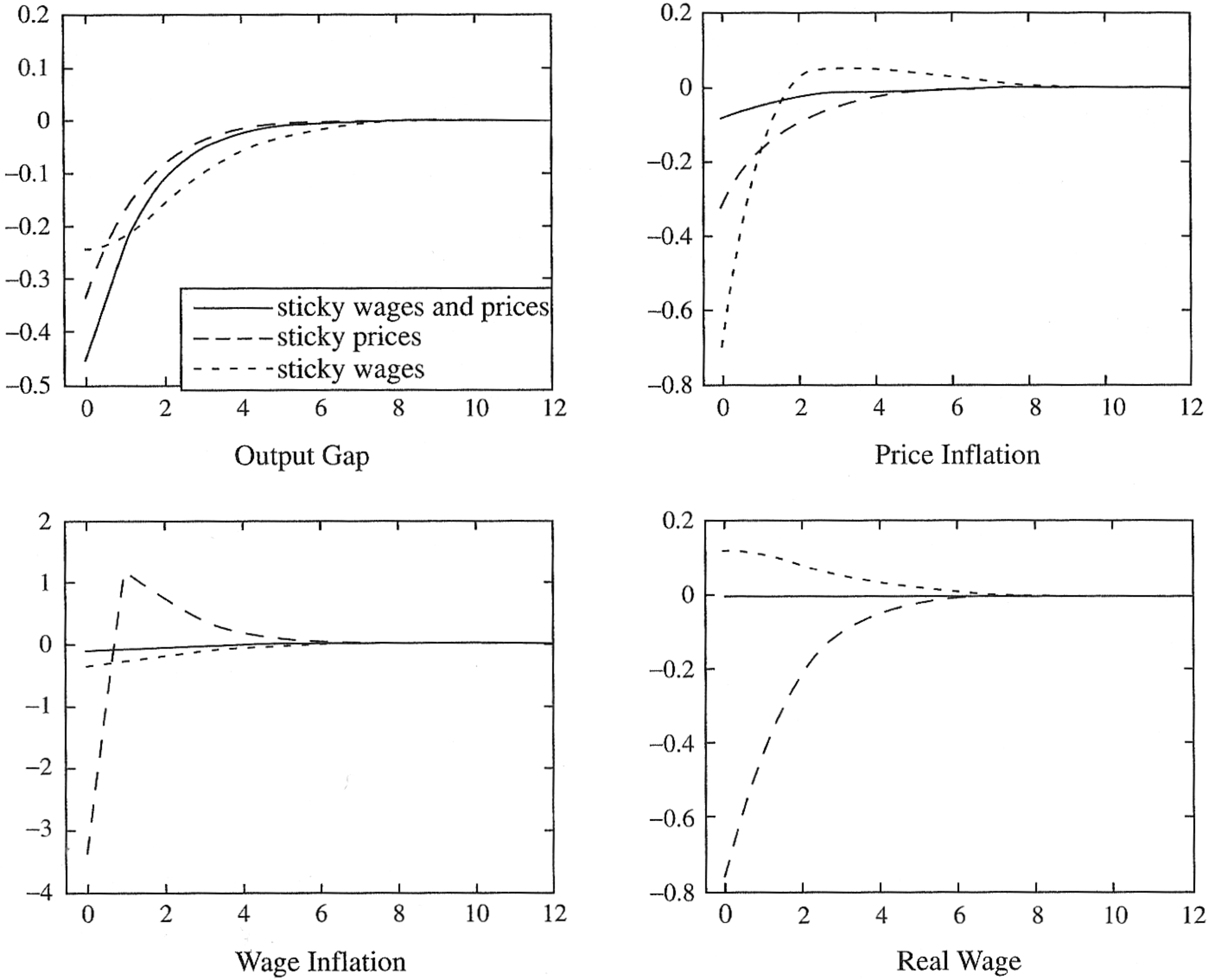
- To examine uniqueness of equilibrium, the relevant matrix is $\mathbf{A}_w \equiv \mathbf{A}_{w,0}^{-1}\mathbf{A}_{w,1}$
  - There are three endogenous variables $\widetilde{y}_t, \pi_t^p, \pi_t^w$ and one predetermined variable $\widetilde{\omega}_{t-1}$.
  - The system of difference equations should thus have three unstable roots and one stable root
  - This corresponds to $\mathbf{A}_w$ having three characteristic roots within the unit circle, and one root outside the unit circle

    (or, had we written the system as $\mathrm{E}_t\left\{\mathbf{x}_{t+1}\right\} = \mathbf{A}_w^{-1}\mathbf{x}_t$...., then the Blanchard and Kahn criterion would state that there should be three roots outside the unit circle; cf. the note on web)

- No analytical solution is available, but numerical analysis indicates that $\phi_p + \phi_w > 1$ is a sufficient condition for equilibrium determinacy—a generalized Taylor principle

- Immediate policy insight: Closing all gaps and having zero inflation rates are not generally feasible:
  - Even if $r_t^n = v_t$, $\widetilde{y}_t = \pi_t^p = \pi_t^w = 0$ will not be possible, as productivity shocks will require changes in real wages
  - So, only if $\Delta\omega_t^n = 0$, is a "gapless" equilibrium possible

Dynamic responses following a contractionary monetary policy shock (with simple Taylor rule based only on price inflation to facilitate comparison with Chapter 3):



Welfare-relevant objective for monetary policy

- Given that all gaps generally cannot be closed, what are the objectives of monetary policy?
- Social planner problem:

$$\max \int_0^1 U\left(C_t\left(j\right), N_t\left(j\right)\right) \mathrm{d}j, \quad \text{all } t$$

s.t. (1), (2) and (6)

- Solution is

$$\begin{aligned} C_t\left(i,j\right) &= C_t, \quad \text{all } i,j \in [0,1] \\ N_t\left(i,j\right) &= N_t\left(j\right) = N_t\left(i\right) = N_t, \quad \text{all } i,j \in [0,1] \\ -\frac{U_{n,t}}{U_{c,t}} &= MPN_t \end{aligned}$$

- Note with monopoly price and wage setting under flexible prices,

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}}\mathcal{M}_w, \qquad P_t = \mathcal{M}_p \frac{\left(1-\tau\right)W_t}{MPN_t}$$

where τ is an employment subsidy

- Let $1-\tau = \left(\mathcal{M}_w\mathcal{M}_p\right)^{-1}$ then $-\frac{U_{n,t}}{U_{c,t}} = MPN_t$ and the flex-price allocation is efficient. This is assumed in the remainder

- By the same method of deriving the approximated welfare under price rigidities only, the welfare loss can be written as a second-order approximation under both price and wage rigidities:

$$\mathbb{W} = -\frac{1}{2}\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\varepsilon_p}{\lambda_p} \left(\pi_t^p\right)^2 + \frac{\varepsilon_w (1 - \alpha)}{\lambda_w} \left(\pi_t^w\right)^2 \right] + \text{t.i.p.} \tag{25}$$

(Measured as a fraction of steady-state consumption.)

- Hence, a trade-off is present between output gap stability, price inflation stability, and wage inflation stability

- The relative weights on each objective have immediate intuition:

  - Price inflation is more costly if the elasticity of substitution between goods are higher (as consumption dispersion will be large)
  - Wage inflation is more costly if the elasticity of substitution between labor types are higher (as labor dispersion will be large)
  - More price (wage) rigidity makes price (wage) inflation more costly as it rises price (wage) dispersion

- Note how the special case of flexible wages ($\lambda_w \to \infty$) makes the wage inflation term vanish (and vice versa for the price inflation term in the special case of flexible prices)

Optimal commitment policy

- Optimal monetary policy is computed under the assumption that commitment is possible (to derive a benchmark against which simple rules are to be compared)
- Following the approach of Chapter 5, one finds the optimal sequences of $\widetilde{y}_t$, π_t^p, π_t^w, $\widetilde{\omega}_t$ to minimize $\mathbb{W}$ subject to the two "Phillips curves" and the dynamic definition of the real wage gap
- From the four first-order conditions and the three equilibrium condition, one solves for the endogenous variables and the three Lagrange multipliers (we need not set up the system here)
- Only in a special case can an analytical solution be attained: When substitution elasticities are proportional and the output gap affects wage and price inflation identically:

$$\varepsilon_p = \varepsilon_w (1 - \alpha), \text{ and } \kappa_p = \kappa_w$$

The optimal policy involves

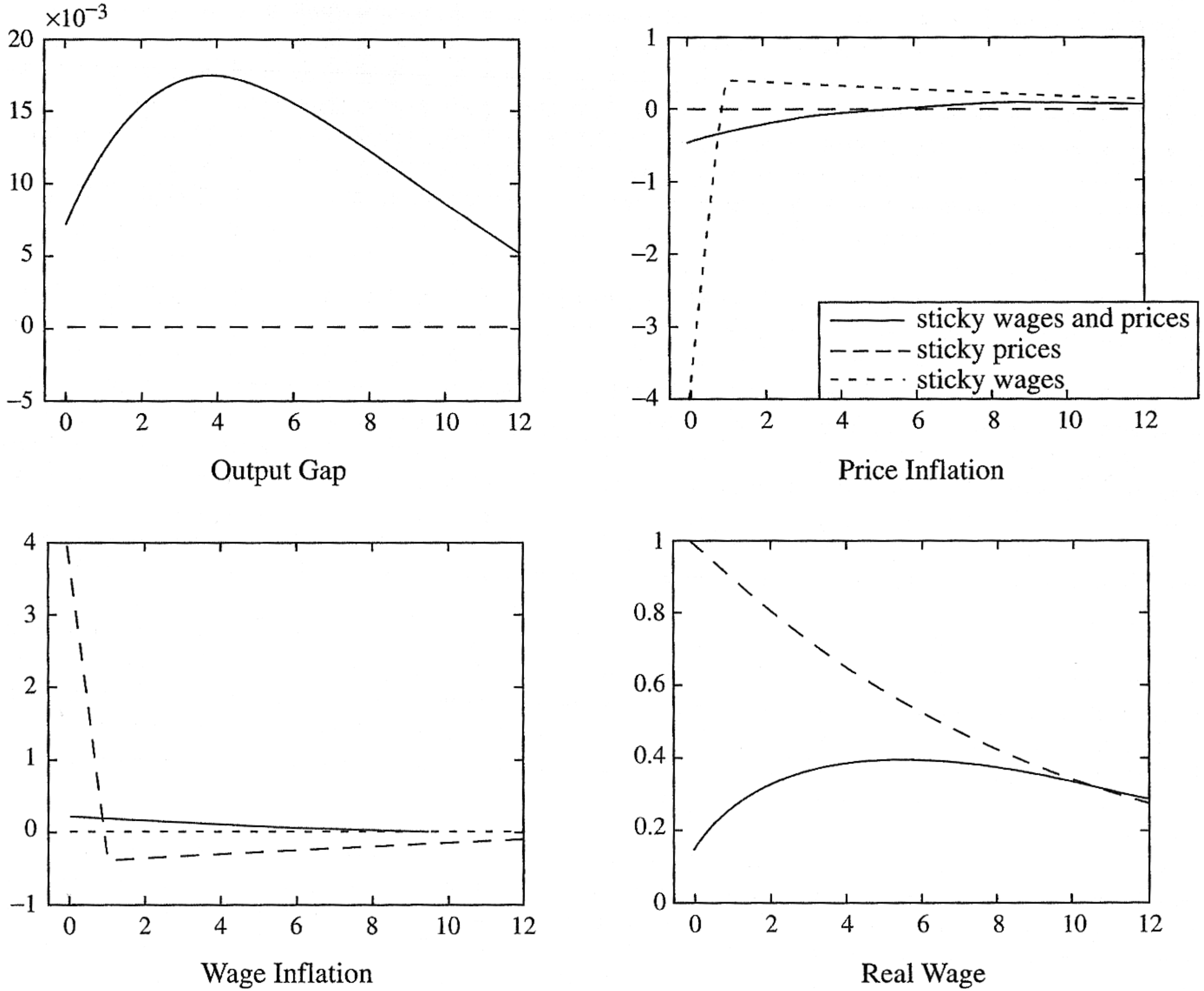
$$\pi_t = 0, \text{ which always implies } \widetilde{y}_t = 0 \text{ irrespective of parameters}$$

where

$$\pi_t \equiv \frac{\lambda_w}{\lambda_p + \lambda_w} \pi_t^p + \frac{\lambda_p}{\lambda_p + \lambda_w} \pi_t^w$$

I.e., a weighted average of inflation rates are fully stabilized; highest weight to price with highest degree of nominal rigidity (lowest λ_z, $z = p, w$)

Dynamic responses following a positive technology shock: Optimal policies (commitment): General case



The performance of simple interest-rate rules

- Following the optimal commitment policies may be complicated
- Simple rules could be a substitute, and their performance are analyzed numerically
  - Three "strict" inflation targeting rules: $\pi_t^p = 0$, or $\pi_t^w = 0$, or $\pi_t = 0$ (which implies $\widetilde{y}_t = 0$)
  - Three Taylor-type rules, called "flexible rules" of the following kind

$$
\begin{aligned}
i_t &= \rho + 1.5\pi_t^p, \\
i_t &= \rho + 1.5\pi_t^w, \\
i_t &= \rho + 1.5\pi_t,
\end{aligned}
$$

- In each case, the model is numerically solved, and the s.d.s of the three welfare-relevant macro-variables are listed along with the welfare losses

Table 6.1 Evaluation of Simple Rules

		Optimal Policy	Strict Rules			Flexible Rules		
			Price	Wage	Composite	Price	Wage	Composite
$\theta_p = \frac{2}{3}$	$\theta_w = \frac{3}{4}$							
	$\sigma(\pi^p)$	0.64	0	0.82	0.66	1.50	1.08	1.12
	$\sigma(\pi^w)$	0.22	0.98	0	0.19	1.05	0.30	0.42
	$\sigma(\widetilde{y})$	0.04	2.38	0.52	0	0.75	1.16	0.01
	$\mathbb{L}$	0.023	0.184	0.034	0.023	0.221	0.081	0.089
$\theta_p = \frac{2}{3}$	$\theta_w = \frac{1}{4}$							
	$\sigma(\pi^p)$	0.29	0	0.82	0.21	1.40	1.45	1.30
	$\sigma(\pi^w)$	1.24	2.91	0	1.63	1.49	0.98	1.25
	$\sigma(\widetilde{y})$	0.19	0.61	0.52	0	0.29	0.68	0.32
	$\mathbb{L}$	0.010	0.038	0.034	0.012	0.097	0.104	0.083
$\theta_p = \frac{1}{3}$	$\theta_w = \frac{3}{4}$							
	$\sigma(\pi^p)$	1.64	0	1.91	1.75	2.58	2.10	2.10
	$\sigma(\pi^w)$	0.11	0.98	0	0.06	1.47	0.07	0.10
	$\sigma(\widetilde{y})$	0.17	2.38	0.27	0	0.87	0.60	0.58
	$\mathbb{L}$	0.016	0.184	0.021	0.017	0.271	0.030	0.031