MAKØK 3, BLOK 2, 2012/13
WRITTEN EXAM
- SUGGESTED ANSWERS -

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(Note that the answers are "suggested" in the sense that they include more than what can be realistically expected from even the best answer, taking the three-hour constraint into consideration.)

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In a flex-price economy, different steady-state nominal interest rates are always irrelevant for output determination.

A FALSE. As we have seen, it is possible to have cases of the opposite. The example we focused in arises in the Money-in-the-Utility-Function model. If the nominal interest rate is higher, it lower real money demand. If this affect the marginal utility of consumption and/or leisure, it will have real effects in the long run. A specific example is where the marginal utility of consumption increases with real money. Then, a higher nominal interest rate (along with higher inflation) will reduce real money holdings, reduce the marginal utility of consumption, and cause consumers to supply less labor. Output will be lower.

(ii) Consider a sticky-wage model a la Calvo with different labor types. Then, when labor types become closer substitutes, nominal wage inflation is less welfare costly.

A FALSE. When labor types are closer substitutes, a relative wage change caused by wage inflation (together with wage stickiness), will move relative labor demand a lot. Hence there will be *more* labor dispersion, which is *more* welfare costly.

(iii) In a model with forward-looking variables like the basic New Keynesian model, a policy commitment implies a smaller focus on past outcomes compared with discretionary policy.

A FALSE. Under optimization under commitment, the first-order condition includes the lagged output gap (specifically $\pi_t = -(x_t - x_{t-1})$). By including lagged variables, the policymaker can affect expectations about the future today: What you do today affects demand and the output gap today. Next period's variables will be affected, as today is next period's lag. Hence, expectations about the future is affected, which can never be disadvantageous. It is, however, not time consistent as discretionary policy (which therefore do not depend on lagged variables in the basic New-Keynesian model).

QUESTION 2:

Consider the following log-linear model of a closed economy:

$$x_t = \mathrm{E}_t\{x_{t+1}\} - \sigma^{-1}(i_t - \mathrm{E}_t\{\pi_{t+1}\} - r_t^n), \qquad \sigma > 0, \tag{1}$$

$$\pi_t = \beta \mathrm{E}_t\{\pi_{t+1}\} + \kappa x_t + u_t, \qquad 0 < \beta < 1, \quad \kappa > 0, \tag{2}$$

$$i_t = \rho + \phi_\pi \pi_t + \phi_y x_t, \qquad \rho > 0, \quad \phi_\pi > 1, \quad \phi_y > 0, \tag{3}$$

where x_t is the output gap (output's deviation from the flexible-price output), i_t is the nominal interest rate, π_t is goods price inflation and r_t^n is the natural rate of interest given as $r_t^n \equiv \rho + \xi_t$. The term ξ_t, as well as u_t, are mean-zero, i.i.d. exogenous shocks. $\mathrm{E}_t\{.\}$ is the rational-expectations operator conditional upon all information up to and including period t.

(i) Discuss (1) and (2) with focus on the underlying economic mechanisms. What does (3) represent? Explain.

A Here is should be mentioned that (1), the dynamic IS curve, is derived from a log-linearization of consumers' consumption-Euler equations: A higher real interest rate, $i_t -$ $\mathrm{E}_t\left\{\pi_{t+1}\right\}$, make consumers increase future consumption relative to current. The natural rate of interest is the real interest rate under flexible prices, which is consistent with (1) for $x_t = 0$. Equation (2), the New-Keynesian Phillips Curve, is derived from the optimal price-setting decisions of monopolistically competitive firms that operate under price stickiness. Prices are set as a markup over marginal costs, and as the output gap is proportional to marginal costs, it enters (2) positively. The more price rigidity (e.g., the lower a probability of price adjustment under a Calvo price setting scheme), the smaller is κ. Expected future prices are central for price determination, as firms are forward looking, since they acknowledge that the price set today may be effective for some periods. The shock u_t captures inefficient fluctuations in the mark-up; e.g., due to inefficient variations in wages or the mark up itself. Equation (3) is a simple specification for how monetary policy, in terms of nominal interest rate setting, is determined. It is a simple Taylor-type rule where the nominal interest rate in increased (more than one-for-one) when inflation increases, which secures uniqueness in the model. Also, a higher output gap is met by contractionary policy.

(ii) Derive the solutions for x_t and π_t [Hint: Conjecture that the solutions are linear functions of the period's shocks, ε_t and u_t.]

A (Note the typo: ε_t here should of course be ξ_t.) We conjecture

$$\begin{aligned} x_t &= a\xi_t + bu_t, \\ \pi_t &= c\xi_t + du_t, \end{aligned}$$

which due to the nature of the shocks implies that

$$\begin{aligned} \mathrm{E}_t\left\{x_{t+1}\right\} &= 0, \\ \mathrm{E}_t\left\{\pi_{t+1}\right\} &= 0. \end{aligned}$$

Inserting the conjectures and their expected versions, along with the interest rate rule, into (1) and (2), gives

$$\begin{aligned} a\xi_t + bu_t &= -\sigma^{-1}\left(\phi_\pi\left[c\xi_t + du_t\right] + \phi_y\left[a\xi_t + bu_t\right] - \xi_t\right) \\ c\xi_t + du_t &= \kappa\left(a\xi_t + bu_t\right) + u_t. \end{aligned}$$

As this holds for all u_t and ξ_t we obtain

$$\begin{aligned} a &= -\sigma^{-1}\phi_\pi c - \sigma^{-1}\phi_y a + \sigma^{-1}, \\ b &= -\sigma^{-1}\phi_\pi d - \sigma^{-1}\phi_y b, \\ c &= \kappa a, \\ d &= \kappa b + 1. \end{aligned}$$

We then get

$$\begin{aligned} a &= -\sigma^{-1}\phi_\pi \kappa a - \sigma^{-1}\phi_y a + \sigma^{-1}, \\ \left[1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)\right] a &= \sigma^{-1}, \\ a &= \frac{\sigma^{-1}}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)} \end{aligned}$$

and thus

$$c = \frac{\sigma^{-1}\kappa}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)}.$$

We further get

$$\begin{aligned} b &= -\sigma^{-1}\phi_\pi \left(\kappa b + 1\right) - \sigma^{-1}\phi_y b, \\ \left[1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)\right] b &= -\sigma^{-1}\phi_\pi, \\ b &= \frac{-\sigma^{-1}\phi_\pi}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)} \end{aligned}$$

and thus

$$\begin{aligned} d &= \frac{-\sigma^{-1}\phi_\pi \kappa}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)} + 1 \\ &= \frac{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right) - \sigma^{-1}\phi_\pi \kappa}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)} \\ &= \frac{1 + \sigma^{-1}\phi_y}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)} \end{aligned}$$

We can the combine the identified parameters with the conjecture and present the solutions for the output gap and inflation:

$$\begin{aligned} x_t &= \frac{\sigma^{-1}}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)}\xi_t - \frac{\sigma^{-1}\phi_\pi}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)}u_t, \\ \pi_t &= \frac{\sigma^{-1}\kappa}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)}\xi_t + \frac{1 + \sigma^{-1}\phi_y}{1 + \sigma^{-1}\left(\phi_\pi \kappa + \phi_y\right)}u_t, \end{aligned}$$

(iii) What is the role of the parameters ϕ_π and in ϕ_y terms of the output gap and inflation's responses to the shocks? Discuss whether the parameters can be chosen such that the output gap and inflation are stabilized completely?

A First, one observe that if one of the parameters ϕ_π and ϕ_y are approaching infinity, the impact of the shock to the natural rate of interest will be fully neutralized on both the output gap and inflation. This is due to the "divine coincidence" that stabilizing one variable is consistent with stabilizing the other. Here, e.g., responding sufficiently aggressively to the output gap will neutralize the impact of the shock on the output gap, which through the Phillips curve will imply price stability. Secondly, however, one observes that this is not possible in the presence of u_t shocks. Letting ϕ_y approach infinity will stabilize output, but leave inflation unstabilized ($\pi_t = u_t$). Letting ϕ_π approach infinity, on the other hand, will stabilize inflation, but attain this at the cost of unstable output ($x_t = -\kappa^{-1}u_t$). Hence, with this type of shock, a trade off between inflation and output gap stability is present. Hence, parameters cannot be chosen so as to stabilize both. This is only the case if fluctations only arises from shocks to the natural rate of interest.

QUESTION 3:

Consider the model given by (1) and (2) of QUESTION 2, and assume that a welfare-relevant loss function can be written as

$$\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} \left(\pi_t^2 + \alpha_x x_t^2 \right) \right], \qquad \alpha_x > 0. \tag{4}$$

(i) Discuss the economic and model-consistent rationale for such a loss function. Discuss how central aspects as the degree of price rigidity and the elasticity of substitution between monopolistically competitive goods affect the magnitude of α_x.

A This type of loss function can be derived as the second-order Taylor approximation to (the negative of) the representative household's utility function. In the economy, there are welfare losses from firms' monopoly power. Moreover, price rigidities cause losses from aggregate mark-ups being different from the desired markup, and under the Calvo-price structure, staggering cause inefficient dispersion of consumption of various goods. The quadratic terms reflect the costs from fluctuations. Inflation is proportional to the inefficient goods dispersion, and output gap fluctuations are proportional to the fluctuations in the markup gap (that causes inefficient fluctuations in consumption and labor). As more price rigidity makes inflation more costly, it will be synonymous with a lower α_x. For any degree of price rigidity, higher substitutability between goods creates more output dispersion from inflation (goods are closer substitutes so relative price changes move relative demand a lot). This higher inflation cost will also be synonymous with a lower α_x

(ii) Derive the optimal values of x_t and π_t under discretionary policymaking [hence, equation (3) no longer applies]. For this purpose, treat x_t as the policy instrument, and show that the relevant first-order condition for optimal policy together with (2) yield the difference equation

$$\pi_t = \frac{\alpha_x \beta}{\kappa^2 + \alpha_x} \mathrm{E}_t \left\{ \pi_{t+1} \right\} + \frac{\alpha_x}{\kappa^2 + \alpha_x} u_t. \tag{5}$$

A Under discretion, expectations are taken as given, so we have a sequence of one-period minimization problems:

$$\begin{aligned} \min_{x_t} L(\pi_t, x_t) &\equiv \frac{1}{2}\left(\pi_t^2 + \alpha_x x_t^2\right) \\ \text{s.t.} \quad \pi_t &= \kappa x_t + v_t \\ v_t &\equiv \beta \mathrm{E}_t \left\{ \pi_{t+1} \right\} + u_t \end{aligned}$$

The relevant first-order condition is

$$\alpha_x x_t = -\kappa \pi_t.$$

Insert this back into (2) to eliminate x_t:

$$\pi_t = \beta \mathrm{E}_t \left\{ \pi_{t+1} \right\} - \frac{\kappa^2}{\alpha_x} \pi_t + u_t,$$

which is rewritten as

$$\pi_t \left(1 + \kappa^2 / \alpha_x\right) = \beta \mathrm{E}_t \left\{ \pi_{t+1} \right\} + u_t,$$

which can readily be written as (5).

(iii) Show that optimal inflation therefore is uniquely given by

$$\pi_t = \frac{\alpha_x}{\kappa^2 + \alpha_x} u_t,$$

and discuss how and why u_t affects inflation and the output gap. Discuss why shocks to the natural rate of interest do not appear in the solution.

A Since, $\beta / \left(1 + \kappa^2/\alpha_x\right) < 1$ (5) has a unique stationary solution. We find this by conjecturing

$$\pi_t = A u_t$$

which given the assumptions about u_t implies

$$\mathrm{E}_t \left\{\pi_{t+1}\right\} = 0$$

Inserted into difference equation:

$$A u_t = \frac{\alpha_x}{\alpha_x + \kappa^2} u_t,$$

which gives

$$A = \frac{\alpha_x}{\alpha_x + \kappa^2}.$$

So, inflation is

$$\pi_t = \frac{\alpha_x}{\alpha_x + \kappa^2} u_t$$

Output gap follows by using the first-order condition as

$$\begin{aligned} x_t &= -\frac{\kappa}{\alpha_x} \frac{\alpha_x}{\alpha_x + \kappa^2} u_t, \\ &= -\frac{\kappa}{\alpha_x + \kappa^2} u_t. \end{aligned}$$

It is seen that the cost-push shock is "spread" out on both inflation and the output gap. If a positive shock occur, it is optimal to contract policy by letting the output gap become negative, and thereby reduce the inflationary impact of the shock. However, as $\alpha_x > 0$ it is not optimal to fully stabilize inflation. One can indeed see that the output-gap response to is (numerically) decreasing in α_x. The shock to the natural rate of interest does not pose a policy trade off, and its impact on the output gap and inflation can be fully offset by an identical and opposite change in the nominal interest rate.