Inflation Targets and Contracts with Uncertain Central Banker Preferences: Computations for Additional Types of Preference Uncertainty

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Abstract

Some additional computations examine to which extent the quantitative and qualitative nature of the results in “Inflation Targets and Contracts with Uncertain Central Banker Preferences” (*Journal of Money Credit, and Banking* 30 (August, 1998)) is robust to the introduction of uncertainty about other parameters in the central banker's loss function.

1. Basics

The social loss function, shared by the government, is:

$$L^G = \frac{1}{2}\mathrm{E}\left[\lambda\left(y-\overline{y}\right)^2 + \left(\pi-\overline{\pi}\right)^2\right],$$

while output obeys

$$y = \pi - \mathrm{E}\left[\pi\right] - \epsilon. \tag{1}$$

The central banker has the following preferences:

$$L^G = \frac{1}{2}\left[\left(\lambda-\alpha\right)\left(y-y^t\right)^2 + \left(1+\alpha\right)\left(\pi-\pi^t\right)^2\right],$$

where y^t and π^t are (to the government and the public) unknown output and inflation targets, respectively (α and the rest of the notation is as described in the paper). We specify that:

$$\begin{aligned} y^t &= \overline{y} + \nu, \\ \pi^t &= \overline{\pi} + \omega, \end{aligned} \tag{2}$$

where ν and ω are mean-zero preference shocks with variances σ_ν^2 and σ_ω^2, respectively. We assume that α, ϵ, ν and ω are all independent of each other.

2. The optimal linear contract

Under the linear contract, the central banker minimises $L^{CB} + f\left(\pi-\overline{\pi}\right)$ s.t. (1), taking as given inflation expectations. The following first-order condition emerges:

$$\left(\lambda-\alpha\right)\left(\pi - \mathrm{E}\left[\pi\right] - \epsilon - y^t\right) + \left(1+\alpha\right)\left(\pi-\pi^t\right) + f = 0,$$

which yields the optimal inflation response:

$$\begin{aligned} \pi &= \frac{\lambda-\alpha}{1+\lambda}\left(\mathrm{E}\left[\pi\right] + \epsilon + y^t\right) + \frac{1+\alpha}{1+\lambda}\pi^t - \frac{1}{1+\lambda}f \\ &= \pi^t + \frac{\lambda-\alpha}{1+\lambda}\left(y^t - \left(\pi^t - \mathrm{E}\left[\pi\right] - \epsilon\right)\right) - \frac{1}{1+\lambda}f. \end{aligned}$$

Taking expectations on both sides we get:

$$\begin{aligned}
\mathrm{E}[\pi] &= \frac{\lambda}{1+\lambda}\left(\mathrm{E}[\pi]+\mathrm{E}[\epsilon]+\mathrm{E}\left[y^t\right]\right)-\frac{1}{1+\lambda}\left(\mathrm{E}[\alpha]\,\mathrm{E}[\pi]+\mathrm{E}[\alpha\epsilon]+\mathrm{E}\left[\alpha y^t\right]\right)\\
&\quad+\frac{1}{1+\lambda}\mathrm{E}\left[\pi^t\right]+\frac{1}{1+\lambda}\mathrm{E}\left[\alpha\pi^t\right]-\frac{1}{1+\lambda}f\\
&= \frac{\lambda}{1+\lambda}\left(\mathrm{E}[\pi]+\overline{y}\right)+\frac{1}{1+\lambda}\overline{\pi}-\frac{1}{1+\lambda}f.
\end{aligned}$$

Hence,

$$\mathrm{E}[\pi]=\lambda\overline{y}+\overline{\pi}-f,$$

which is the same as equation (6) in the paper. Inserting these expectations into the central banker's reaction function, we get actual inflation for a given contract as:

$$\begin{aligned}
\pi &= \frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+\overline{\pi}-f+\epsilon+y^t\right)\\
&\quad+\frac{1+\alpha}{1+\lambda}\pi^t-\frac{1}{1+\lambda}f\\
&= \frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+\overline{\pi}+\epsilon+y^t\right)+\frac{1+\alpha}{1+\lambda}\pi^t-\frac{1}{1+\lambda}\left(1+\lambda-\alpha\right)f. \qquad (3)
\end{aligned}$$

Correspondingly, actual output will be given by:

$$\begin{aligned}
y &= \frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+\overline{\pi}+\epsilon+y^t\right)+\frac{1+\alpha}{1+\lambda}\pi^t-\frac{1}{1+\lambda}\left(1+\lambda-\alpha\right)f-\left(\lambda\overline{y}+\overline{\pi}-f\right)-\epsilon\\
&= -\frac{1+\alpha}{1+\lambda}\left(\lambda\overline{y}+\overline{\pi}-\pi^t+\epsilon\right)+\frac{\alpha}{1+\lambda}f+\frac{\lambda-\alpha}{1+\lambda}y^t. \qquad (4)
\end{aligned}$$

The government's choice of the optimal contract follows from solving:

$$\begin{aligned}
&\min_f \frac{1}{2}\mathrm{E}\left[\lambda\left(y-\overline{y}\right)^2+\left(\pi-\overline{\pi}\right)^2\right],\\
\text{s.t.}\quad \pi &= \frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+\overline{\pi}+\epsilon+y^t\right)+\frac{1+\alpha}{1+\lambda}\pi^t-\frac{1}{1+\lambda}\left(1+\lambda-\alpha\right)f,\\
y &= -\frac{1+\alpha}{1+\lambda}\left(\lambda\overline{y}+\overline{\pi}-\pi^t+\epsilon\right)+\frac{\alpha}{1+\lambda}f+\frac{\lambda-\alpha}{1+\lambda}y^t.
\end{aligned}$$

The relevant first-order condition is

$$\mathrm{E}\left[\lambda\frac{\alpha}{1+\lambda}\left(y-\overline{y}\right)-\frac{1}{1+\lambda}\left(1+\lambda-\alpha\right)\left(\pi-\overline{\pi}\right)\right]=0$$

$$\Longrightarrow \mathrm{E}\left[\lambda\alpha\left(y-\overline{y}\right)-\left(1+\lambda-\alpha\right)\pi\right]=-\left(1+\lambda\right)\overline{\pi},$$

which, by use of the expressions for y and π, becomes

$$\begin{aligned}
&\mathrm{E}\left[-\lambda\alpha\frac{1+\alpha}{1+\lambda}\left(\lambda\overline{y}+\overline{\pi}-\pi^t+\epsilon\right)+\lambda\alpha\frac{\alpha}{1+\lambda}f+\lambda\alpha\frac{\lambda-\alpha}{1+\lambda}y^t-\lambda\alpha\overline{y}\right]\\
&-\mathrm{E}\left[\left(1+\lambda-\alpha\right)\frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+\overline{\pi}+\epsilon+y^t\right)+\left(1+\lambda-\alpha\right)\frac{1+\alpha}{1+\lambda}\pi^t-\frac{1}{1+\lambda}\left(1+\lambda-\alpha\right)^2f\right]\\
=\;&-\left(1+\lambda\right)\overline{\pi},
\end{aligned}$$

or,

$$\begin{aligned}
&\mathrm{E}\left[-\lambda\alpha\left(1+\alpha\right)\left(\lambda\overline{y}+\overline{\pi}-\pi^t+\epsilon\right)+\lambda\alpha^2f+\lambda\alpha\left(\lambda-\alpha\right)y^t-\lambda\alpha\left(1+\lambda\right)\overline{y}\right]\\
&-\mathrm{E}\left[\left(1+\lambda-\alpha\right)\left(\lambda-\alpha\right)\left(\lambda\overline{y}+\overline{\pi}+\epsilon+y^t\right)+\left(1+\lambda-\alpha\right)\left(1+\alpha\right)\pi^t-\left(1+\lambda-\alpha\right)^2f\right]\\
=\;&-\left(1+\lambda\right)^2\overline{\pi},
\end{aligned}$$

or,

$$\begin{aligned} & \mathrm{E}\left[-\left(\lambda\overline{y}+\overline{\pi}+\epsilon\right)\left(\lambda\alpha\left(1+\alpha\right)+\left(1+\lambda-\alpha\right)\left(\lambda-\alpha\right)\right)-\lambda\alpha\left(1+\lambda\right)\overline{y}\right] \\ & +\mathrm{E}\left[y^{t}\left(\lambda\alpha\left(\lambda-\alpha\right)-\left(1+\lambda-\alpha\right)\left(\lambda-\alpha\right)\right)+\pi^{t}\left(\lambda\alpha\left(1+\alpha\right)-\left(1+\lambda-\alpha\right)\left(1+\alpha\right)\right)\right] \\ & +\mathrm{E}\left[\left(\lambda\alpha^{2}+\left(1+\lambda-\alpha\right)^{2}\right)f\right] \\ = & -\left(1+\lambda\right)^{2}\overline{\pi}, \end{aligned}$$

or,

$$\begin{aligned} & \mathrm{E}\left[-\left(\lambda\overline{y}+\overline{\pi}+\epsilon\right)\left(1+\lambda\right)\left(\alpha^{2}+\lambda-\alpha\right)-\lambda\alpha\left(1+\lambda\right)\overline{y}\right] \\ & +\mathrm{E}\left[y^{t}\left(\lambda-\alpha\right)\left(1+\lambda\right)\left(\alpha-1\right)+\pi^{t}\left(1+\alpha\right)\left(1+\lambda\right)\left(\alpha-1\right)+\left(\lambda\alpha^{2}+\left(1+\lambda-\alpha\right)^{2}\right)f\right] \\ = & -\left(1+\lambda\right)^{2}\overline{\pi}, \end{aligned}$$

or (where we have already imposed that $\mathrm{E}[\lambda\alpha\left(1+\lambda\right)\overline{y}]=0$),

$$\begin{aligned} & \mathrm{E}\left[-\left(\lambda\overline{y}+\overline{\pi}+\epsilon\right)\left(\alpha^{2}+\lambda-\alpha\right)+y^{t}\left(\lambda-\alpha\right)\left(\alpha-1\right)+\pi^{t}\left(1+\alpha\right)\left(\alpha-1\right)+\left(\alpha^{2}+1+\lambda-2\alpha\right)f\right] \\ = & -\left(1+\lambda\right)\overline{\pi}. \end{aligned}$$

Solving out the terms yield:

$$\begin{aligned} & \mathrm{E}\left[-\lambda\overline{y}\left(\alpha^{2}+\lambda-\alpha\right)-\overline{\pi}\left(\alpha^{2}+\lambda-\alpha\right)-\epsilon\left(\alpha^{2}+\lambda-\alpha\right)\right] \\ & +\mathrm{E}\left[y^{t}\left(\lambda\alpha-\alpha^{2}-\lambda+\alpha\right)+\pi^{t}\left(\alpha^{2}-1\right)\right]+\mathrm{E}\left[\left(\alpha^{2}+1+\lambda-2\alpha\right)f\right] \\ = & -\left(1+\lambda\right)\overline{\pi}. \end{aligned}$$

Taking expectations yields:

$$\begin{aligned} & -\lambda\overline{y}\sigma_{\alpha}^{2}-\lambda^{2}\overline{y}-\overline{\pi}\sigma_{\alpha}^{2}-\lambda\overline{\pi}-\overline{y}\sigma_{\alpha}^{2}-\lambda\overline{y}+\overline{\pi}\sigma_{\alpha}^{2}-\overline{\pi}+\left(1+\lambda+\sigma_{\alpha}^{2}\right)f \\ = & -\left(1+\lambda\right)\overline{\pi}, \end{aligned}$$

Solving for f then yields:

$$f^{*}=\frac{\left(1+\lambda\right)\left(\lambda+\sigma_{\alpha}^{2}\right)\overline{y}}{1+\lambda+\sigma_{\alpha}^{2}}, \tag{5}$$

which is the solution we also find in the paper (conform equation (13) in the paper).

The solutions for inflation and output under the optimal linear contract follow upon substitution of the right hand side of (5) for f into (3) and (4), respectively. This yields:

$$\pi=\overline{\pi}-\left(\frac{\alpha+\sigma_{\alpha}^{2}}{1+\lambda+\sigma_{\alpha}^{2}}\right)\overline{y}+\left(\frac{\lambda-\alpha}{1+\lambda}\right)\left(\epsilon+\upsilon\right)+\left(\frac{1+\alpha}{1+\lambda}\right)\omega, \tag{6}$$

$$y=-\left(\frac{\alpha}{1+\lambda+\sigma_{\alpha}^{2}}\right)\overline{y}-\left(\frac{1+\alpha}{1+\lambda}\right)\epsilon+\left(\frac{\lambda-\alpha}{1+\lambda}\right)\upsilon+\left(\frac{1+\alpha}{1+\lambda}\right)\omega. \tag{7}$$

Note that, as before, under the optimal contract $\mathrm{E}[\pi]<\overline{\pi}$. Because the contract affects the variances of output and inflation, the choice of optimal contract will take both these variances, as well as the average inflation performance, into consideration. This Brainard (1967) related result thus holds irrespective of what type of uncertainty we impose in the central banker's preferences.

The equilibrium welfare loss follows upon substitution of (6) and (7) into

$$L^{G}=\frac{1}{2}\mathrm{E}\left[\lambda\left(y-\overline{y}\right)^{2}+\left(\pi-\overline{\pi}\right)^{2}\right].$$

If we work this out, we obtain:

$$L^{G}=\frac{\left(1+\lambda\right)\left(\lambda+\sigma_{\alpha}^{2}\right)}{2\left(1+\lambda+\sigma_{\alpha}^{2}\right)}\overline{y}^{2}+\frac{\lambda+\sigma_{\alpha}^{2}}{2\left(1+\lambda\right)}\sigma_{\epsilon}^{2}+\frac{\lambda^{2}+\sigma_{\alpha}^{2}}{2\left(1+\lambda\right)}\sigma_{\upsilon}^{2}+\frac{1+\sigma_{\alpha}^{2}}{2\left(1+\lambda\right)}\sigma_{\omega}^{2}. \tag{8}$$

3. The optimal inflation target

Under the Svensson (1997) type of inflation target regime, the central banker minimises the following loss function:

$$\frac{1}{2}\left[(\lambda-\alpha)\left(y-y^t\right)^2+(1+\alpha)\left(\pi-\pi^b\right)^2\right],$$

s.t. (1). The following first-order condition emerges:

$$(\lambda-\alpha)\left(\pi-\mathrm{E}\left[\pi\right]-\epsilon-y^t\right)+(1+\alpha)\left(\pi-\pi^b\right)=0,$$

leading to the following reaction function:

$$\begin{aligned}\pi &= \frac{\lambda-\alpha}{1+\lambda}\left(\mathrm{E}\left[\pi\right]+\epsilon+y^t\right)+\frac{1+\alpha}{1+\lambda}\pi^b \\ &= \pi^b+\frac{\lambda-\alpha}{1+\lambda}\left(y^t-\left(\pi^b-\mathrm{E}\left[\pi\right]-\epsilon\right)\right).\end{aligned}$$

Taking expectations on both sides we get

$$\mathrm{E}\left[\pi\right]=\frac{1}{1+\lambda}\pi^b+\frac{\lambda}{1+\lambda}\left(\mathrm{E}\left[\pi\right]+\overline{y}\right),$$

from which expected inflation emerges as

$$\mathrm{E}\left[\pi\right]=\pi^b+\lambda\overline{y}.$$

This is the same as equation (17) in the paper.

Inserting these expectations into the central banker's reaction function yields for actual inflation:

$$\begin{aligned}\pi &= \frac{\lambda-\alpha}{1+\lambda}\left(\pi^b+\lambda\overline{y}+\epsilon+y^t\right)+\frac{1+\alpha}{1+\lambda}\pi^b \\ &= \pi^b+\frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+y^t+\epsilon\right).\end{aligned} \tag{9}$$

Actual output then follows as

$$\begin{aligned}y &= \pi^b+\frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+y^t+\epsilon\right)-\left(\pi^b+\lambda\overline{y}\right)-\epsilon \\ &= \frac{\lambda-\alpha}{1+\lambda}y^t-\frac{1+\alpha}{1+\lambda}\left(\lambda\overline{y}+\epsilon\right)\end{aligned} \tag{10}$$

To find the optimal inflation target the government then solves

$$\min_{\pi^b}\frac{1}{2}\mathrm{E}\left[\lambda\left(y-\overline{y}\right)^2+\left(\pi-\overline{\pi}\right)^2\right],$$

$$\begin{aligned}\text{s.t.}\quad \pi &= \pi^b+\frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+y^t+\epsilon\right), \\ y &= \frac{\lambda-\alpha}{1+\lambda}y^t-\frac{1+\alpha}{1+\lambda}\left(\lambda\overline{y}+\epsilon\right).\end{aligned}$$

The relevant first-order condition is

$$\mathrm{E}\left[\pi^b+\frac{\lambda-\alpha}{1+\lambda}\left(\lambda\overline{y}+y^t+\epsilon\right)-\overline{\pi}\right]=0,$$

or,

$$\pi^b+\frac{\lambda}{1+\lambda}\left(\lambda\overline{y}+\overline{y}\right)-\overline{\pi}=0,$$

which yields the following optimal choice for the inflation target:

$$\pi^{b*}=\overline{\pi}-\lambda\overline{y}, \tag{11}$$

which is the same as equation (20) in the paper. Substitute (11) into (9). The equilibrium outcomes for inflation and output can be written as, respectively:

$$\pi = \bar{\pi} - \alpha \overline{y} + \left(\frac{\lambda - \alpha}{1+\lambda}\right)(\epsilon + \upsilon),$$

$$y = -\alpha \overline{y} - \left(\frac{1+\alpha}{1+\lambda}\right)\epsilon + \left(\frac{\lambda - \alpha}{1+\lambda}\right)\upsilon. \tag{12}$$

Substituting (12) into the government's loss function, society's welfare loss is given by:

$$L^G = \frac{1}{2}\left[\lambda + \sigma_\alpha^2 (1+\lambda)\right]\overline{y}^2 + \frac{\lambda + \sigma_\alpha^2}{2(1+\lambda)}\sigma_\epsilon^2 + \frac{\lambda^2 + \sigma_\alpha^2}{2(1+\lambda)}\sigma_\upsilon^2. \tag{13}$$

Hence, a linear contract is strictly preferred to an inflation target if and only if:

$$\frac{(1+\lambda)\left(\lambda + \sigma_\alpha^2\right)}{\left(1+\lambda+\sigma_\alpha^2\right)}\overline{y}^2 + \frac{1+\sigma_\alpha^2}{(1+\lambda)}\sigma_\omega^2 < \left[\lambda + \sigma_\alpha^2 (1+\lambda)\right]\overline{y}^2. \tag{14}$$

Because the first term on the left hand side is smaller than the term on the right hand side, if $\overline{y}^2/\sigma_\omega^2$ is relatively large, a linear contract is preferred to an inflation target. The opposite is true if $\overline{y}^2/\sigma_\omega^2$ is relatively small, that is, if the uncertainty about the central banker's preferred inflation rate is relatively large (in contrast to a linear inflation contract, inflation targeting then removes this relative important source of uncertainty).

4. The optimal combination of a target and a linear inflation contract

The central banker now minimises the loss function:

$$\frac{1}{2}\left[(\lambda - \alpha)\left(y - y^t\right)^2 + (1+\alpha)\left(\pi - \pi^b\right)^2\right] + f\left(\pi - \pi^b\right).$$

subject to (1), taking as given inflation expectations. The first-order condition is now given by:

$$(\lambda - \alpha)\left(\pi - \mathrm{E}[\pi] - \epsilon - y^t\right) + (1+\alpha)\left(\pi - \pi^b\right) + f = 0.$$

Hence, the central banker's reaction function is:

$$\begin{aligned}\pi &= \frac{\lambda - \alpha}{1+\lambda}\left(\mathrm{E}[\pi] + \epsilon + y^t\right) + \frac{1+\alpha}{1+\lambda}\pi^b - \frac{1}{1+\lambda}f \\ &= \pi^b + \frac{\lambda - \alpha}{1+\lambda}\left(y^t - \left(\pi^b - \mathrm{E}[\pi] - \epsilon\right)\right) - \frac{1}{1+\lambda}f.\end{aligned}$$

Taking expectations on both sides yields

$$\mathrm{E}[\pi] = \frac{\lambda}{1+\lambda}\left(\mathrm{E}[\pi] + \overline{y}\right) + \frac{1}{1+\lambda}\pi^b - \frac{1}{1+\lambda}f,$$

Hence,

$$\mathrm{E}[\pi] = \pi^b + \lambda\overline{y} - f.$$

Substitute this back into the central banker's reaction function to give:

$$\begin{aligned}\pi &= \frac{\lambda - \alpha}{1+\lambda}\left((1+\lambda)\overline{y} + \pi^b - f + \epsilon + \nu\right) + \frac{1+\alpha}{1+\lambda}\pi^b - \frac{1}{1+\lambda}f \\ &= \mathrm{E}[\pi] - \alpha\left(\overline{y} - \frac{1}{1+\lambda}f\right) + \left(\frac{\lambda - \alpha}{1+\lambda}\right)(\epsilon + \nu),\end{aligned}$$

where we have made use of (2). Combining this and the expression for $\mathrm{E}[\pi]$ with (1) yields:

$$y = -\alpha\left(\overline{y} - \frac{1}{1+\lambda}f\right) + \left(\frac{\lambda - \alpha}{1+\lambda}\right)\nu - \left(\frac{1+\alpha}{1+\lambda}\right)\epsilon.$$

The optimal arrangement eliminates the inflation bias and minimises the variances of output and inflation. Therefore, it is given by:

$$f^{**} = (1+\lambda)\overline{y}, \qquad \pi^{b**} = \overline{\pi} + \overline{y},$$

which is the same as the optimal arrangement in Section 6 of the paper (conform equation (25) in the paper).

5. The Optimal Combination Regime with Quadratic Punishments

The central banker now minimises

$$\begin{aligned} & \frac{1}{2}\left[(\lambda-\alpha)\left(y-y^t\right)^2+(1+\alpha)\left(\pi-\pi^b\right)^2\right]+f\left(\pi-\pi^b\right)+\frac{1}{2}q\left(\pi-\pi^b\right)^2 \\ = & \frac{1}{2}\left[(\lambda-\alpha)\left(y-y^t\right)^2+(1+\alpha+q)\left(\pi-\pi^b\right)^2\right]+f\left(\pi-\pi^b\right), \end{aligned}$$

subject to (1), taking inflation expectations as given. The first-order condition is:

$$(\lambda-\alpha)\left(\pi-\mathrm{E}\left[\pi\right]-\epsilon-y^t\right)+(1+\alpha+q)\left(\pi-\pi^b\right)+f=0.$$

Hence, the central banker's reaction function is:

$$\begin{aligned} \pi &= \frac{\lambda-\alpha}{1+\lambda+q}\left(\mathrm{E}\left[\pi\right]+\epsilon+y^t\right)+\frac{1+\alpha+q}{1+\lambda+q}\pi^b-\frac{1}{1+\lambda+q}f \\ &= \pi^b+\frac{\lambda-\alpha}{1+\lambda+q}\left(y^t-\left(\pi^b-\mathrm{E}\left[\pi\right]-\epsilon\right)\right)-\frac{1}{1+\lambda+q}f. \end{aligned}$$

Taking expectations on both sides of the equation and solving yields:

$$\mathrm{E}\left[\pi\right]=\pi^b+\frac{\lambda}{1+q}\overline{y}-\frac{1}{1+q}f.$$

Substitute this into the expression for inflation and work out:

$$\begin{aligned} \pi &= \frac{\lambda-\alpha}{1+\lambda+q}\left(\frac{1+\lambda+q}{1+q}\overline{y}+\pi^b-\frac{1}{1+q}f+\epsilon+\nu\right)+\frac{1+\alpha+q}{1+\lambda+q}\pi^b-\frac{1}{1+\lambda+q}f \\ &= \mathrm{E}\left[\pi\right]-\frac{\alpha}{1+q}\left(\overline{y}-\frac{1}{1+\lambda+q}f\right)+\left(\frac{\lambda-\alpha}{1+\lambda+q}\right)(\epsilon+\nu). \end{aligned}$$

Using the expression for $\mathrm{E}[\pi]$, the expression for π and (1), we find for output:

$$y=-\frac{\alpha}{1+q}\left(\overline{y}-\frac{1}{1+\lambda+q}f\right)+\left(\frac{\lambda-\alpha}{1+\lambda+q}\right)\nu-\left(\frac{1+\alpha+q}{1+\lambda+q}\right)\epsilon.$$

To solve for the optimal delegation arrangement, we choose f and π^b optimally as functions of q. This yields:

$$f^{***}=(1+\lambda+q)\,\overline{y}, \qquad \pi^{b***}=\overline{\pi}+\overline{y},$$

which are the same expressions as in the paper (as a function of q), conform equation (29) in the paper. Using these expressions in the expressions for inflation and output, the government's loss is given by:

$$\frac{1}{2}\lambda\overline{y}^2+\mathrm{E}\left\{\lambda\left[\left(\frac{\lambda-\alpha}{1+\lambda+q}\right)\nu-\left(\frac{1+\alpha+q}{1+\lambda+q}\right)\epsilon\right]^2+\frac{1}{2}\left[\left(\frac{\lambda-\alpha}{1+\lambda+q}\right)(\epsilon+\nu)\right]^2\right\}.$$

Hence, the optimal value for q minimises:

$$\frac{1}{2}\lambda\overline{y}^2+\frac{\lambda\left[\lambda+(1+q)^2\right]+(1+\lambda)\,\sigma_\alpha^2}{2\,(1+\lambda+q)^2}\sigma_\epsilon^2+\frac{(1+\lambda)\left(\lambda^2+\sigma_\alpha^2\right)}{2\,(1+\lambda+q)^2}\sigma_\nu^2. \qquad (15)$$

Because the third term is monotonically decreasing in q, it is immediately clear that the optimal value for q, which we denote by $\tilde{q}$, is larger than $q^{***}\equiv(1+\lambda)\,\sigma_\alpha^2/\lambda^2$, the optimal value in the paper, conform equation (30) in the paper. Indeed, straightforward minimisation of this expression with respect to q yields:

$$\tilde{q}=\frac{(1+\lambda)\,\sigma_\alpha^2}{\lambda^2}+\frac{(1+\lambda)\left(\lambda^2+\sigma_\alpha^2\right)}{\lambda\sigma_\epsilon^2}\sigma_\nu^2>\frac{(1+\lambda)\,\sigma_\alpha^2}{\lambda^2}.$$

Clearly, equilibrium welfare losses are higher than in the absence of uncertainty about the preferred output level by the central bank: for $q=\tilde{q}$ the second term in (15) is higher than for $q=q^{***}$. Moreover, for $q=\tilde{q}$, the third term in (15) is positive, while this term disappears if $\sigma_\nu^2=0$. Hence, delegation is relatively less attractive than if the central banker's preferred output level is certain.