State Manipulation and Asymptotic Inefficiency in a Dynamic Model of Monetary Policy

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Technical Appendices (not intended for publication)

F. Derivation of (9)

First, we need to solve (8). It will be convenient to restate the problem, and for that purpose, note that

$$\frac{1}{2}(1-\delta)\left[u_t^2+\lambda\varphi_t^2\right]+\delta L_T(u_t)+\delta^{T+1}L^r(\pi;\rho^T u_t)$$

can be rewritten [using (6) and (4)] as

$$\frac{1}{2}(1-\delta)\left(1+\frac{\delta\gamma_T}{1-\delta}+\frac{\left(\delta\rho^2\right)^{T+1}+\delta^{T+1}\rho^{2T}\lambda\pi}{1-\delta\rho^2}\right)u_t^2+\frac{1}{2}(1-\delta)\lambda\varphi_t^2,$$

which by the definitions of y_T and $\hat{y}_T$ becomes:

$$\frac{1}{2}(1-\delta)\left[\hat{y}_T u_t^2+\lambda\varphi_t^2\right].$$

Hence, problem (8) can be restated as

$$L^d(\pi;u_{t-1})=\min_{\varphi_t}\left\{\frac{1}{2}(1-\delta)\left[\hat{y}_T u_t^2+\lambda\varphi_t^2\right]\text{ s.t. } u_t=\rho u_{t-1}+\beta(\pi u_{t-1}-\varphi_t)\right\}. \qquad \text{(F.1)}$$

From the relevant first-order condition, $-(1-\delta)\beta\hat{y}_T\left(\rho u_{t-1}+\beta(\pi u_{t-1}-\varphi_t)\right)+(1-\delta)\lambda\varphi_t=0$, we recover φ_t as

$$\varphi_t=\frac{\beta\left(\rho+\beta\pi\right)\hat{y}_T}{\lambda+\beta^2\hat{y}_T}u_{t-1}, \qquad \text{(F.2)}$$

and the associated unemployment rate as

$$u_t = \frac{\lambda(\rho + \beta\pi)}{\lambda + \beta^2 \hat{y}_T} u_{t-1}. \tag{F.3}$$

Inserting (F.2) and (F.3) back into (F.1), then gives

$$\begin{aligned} L^d(\pi; u_{t-1}) &= \frac{1}{2}(1-\delta)\left[\hat{y}_T\left(\frac{\lambda(\rho+\beta\pi)}{\lambda+\beta^2\hat{y}_T}u_{t-1}\right)^2 + \lambda\left(\frac{\beta(\rho+\beta\pi)\hat{y}_T}{\lambda+\beta^2\hat{y}_T}u_{t-1}\right)^2\right], \\ &= \frac{1}{2}(1-\delta)\frac{\lambda\hat{y}_T(\rho+\beta\pi)^2}{\lambda+\beta^2\hat{y}_T}u_{t-1}^2, \\ &= \frac{1}{2}(1-\delta)\frac{\lambda\beta^2\hat{y}_T}{\lambda+\beta^2\hat{y}_T}\left(\frac{\rho}{\beta}+\pi\right)^2 u_{t-1}^2, \end{aligned}$$

which is equation (9).

G. Proof that (D.3) applies and that $\kappa_{0,T}$ is bounded in δ for any given T

First, we find the MPE of a T-period game, when (13) applies. Equilibrium behavior of the MA is described by a reaction function $\pi_t(\pi_t^e)$ defined as

$$\pi_t(\pi_t^e, u_{t-1}) = \underset{\pi_t}{\operatorname{argmin}}\left\{\frac{1}{2}(1-\delta)\left[u_t^2 + \lambda\pi_t^2\right] + \delta\widetilde{L}_{T-1}(u_t) \text{ s.t. } (13),\ \pi_t^e \text{ given}\right\}, \quad T > 1.$$

The relevant first-order condition is, using the definition of $\widetilde{L}_T$, (D.2),

$$(1-\delta)\left[-\beta u_t + \lambda\pi_t\right] - \delta\beta\left(\gamma_{1,T-1} + \gamma_{2,T-1}u_t\right) = 0.$$

Applying $\pi_t^e = \pi_t$, we then find

$$(1-\delta)\left[-\beta\left((1-\rho)u_n + \rho u_{t-1}\right) + \lambda\pi_t\right] - \delta\beta\left(\gamma_{1,T-1} + \gamma_{2,T-1}\left((1-\rho)u_n + \rho u_{t-1}\right)\right) = 0,$$

from which we obtain the MPE inflation rate conditional on $\gamma_{1,T-1}$ and $\gamma_{2,T-1}$:

$$\pi_t = \frac{\delta\beta\gamma_{1,T-1}}{(1-\delta)\lambda} + \frac{\beta}{\lambda}x_{T-1}\left((1-\rho)u_n + \rho u_{t-1}\right), \quad T > 1,$$

where $x_{T-1} \equiv 1 + \delta\gamma_{2,T-1}/(1-\delta)$.

We then find the relevant Riccati equations to solve for the unknown parameters $\gamma_{0,T}$,

$\gamma_{1,T}$ and $\gamma_{2,T}$ by noting that along the equilibrium path,

$$\begin{aligned}
\widetilde{L}_T(u_{t-1}) &= \frac{1}{2}(1-\delta)\left[u_t^2 + \lambda \pi_t^2\right] + \delta \widetilde{L}_{T-1}(u_t), \quad T > 1, \\
u_t &= (1-\rho)u_n + \rho u_{t-1}, \\
\pi_t &= \frac{\delta \beta \gamma_{1,T-1}}{(1-\delta)\lambda} + \frac{\beta}{\lambda} x_{T-1}\left((1-\rho)u_n + \rho u_{t-1}\right).
\end{aligned}$$

This means that

$$\begin{aligned}
& \gamma_{0,T} + \gamma_{1,T} u_{t-1} + \gamma_{2,T} \frac{u_{t-1}^2}{2} \qquad \text{(G.1)} \\
= & \frac{1}{2}(1-\delta)\left[\left((1-\rho)u_n + \rho u_{t-1}\right)^2 + \lambda \left(\frac{\delta\beta\gamma_{1,T-1}}{(1-\delta)\lambda} + \frac{\beta}{\lambda} x_{T-1}\left((1-\rho)u_n + \rho u_{t-1}\right)\right)^2\right] \\
& + \delta \left[\gamma_{0,T-1} + \gamma_{1,T-1}\left((1-\rho)u_n + \rho u_{t-1}\right) + \gamma_{2,T-1}\frac{\left((1-\rho)u_n + \rho u_{t-1}\right)^2}{2}\right], \quad T > 1,
\end{aligned}$$

Equating coefficients to u_{t-1}^2 on both sides of (G.1) gives the first Riccati equation:

$$\frac{\gamma_{2,T}}{2} = \frac{1}{2}(1-\delta)\left[\rho^2 + \lambda\left(\frac{\beta\rho}{\lambda}x_{T-1}\right)^2\right] + \frac{\delta\rho^2\gamma_{2,T-1}}{2}, \quad T > 1,$$

which after some manipulation can be written as a difference equation in x_T:

$$x_T = 1 + \frac{\beta^2}{\lambda}\delta\rho^2 x_{T-1}^2 + \delta\rho^2 x_{T-1}, \quad T > 1. \qquad \text{(G.2)}$$

Note that this is precisely the same as that for y_T, cf. equation (A.2).

Equating coefficients to u_{t-1} in (G.1) gives

$$\begin{aligned}
\gamma_{1,T} &= (1-\delta)\left[\rho(1-\rho)u_n + \frac{\beta^2\rho x_{T-1}}{\lambda}\left(x_{T-1}(1-\rho)u_n + \frac{\delta\gamma_{1,T-1}}{(1-\delta)}\right)\right] \\
& \quad + \delta\rho\left[\gamma_{1,T-1} + \gamma_{2,T-1}(1-\rho)u_n\right],
\end{aligned}$$

or,

$$\begin{aligned}
\gamma_{1,T} &= (1-\delta)\rho(1-\rho)u_n + (1-\delta)\frac{\beta^2\rho x_{T-1}^2}{\lambda}(1-\rho)u_n + \frac{\beta^2\rho x_{T-1}}{\lambda}\delta\gamma_{1,T-1} \\
& \quad + \delta\rho\gamma_{1,T-1} + \delta\rho\gamma_{2,T-1}(1-\rho)u_n, \\
&= (1-\delta)\rho(1-\rho)u_n + (1-\delta)\frac{\beta^2\rho x_{T-1}^2}{\lambda}(1-\rho)u_n + \frac{\beta^2\rho x_{T-1}}{\lambda}\delta\gamma_{1,T-1} \\
& \quad + \delta\rho\gamma_{1,T-1} + (1-\delta)\rho\frac{\delta\gamma_{2,T-1}}{1-\delta}(1-\rho)u_n,
\end{aligned}$$

$$
\begin{aligned}
&= (1-\delta)\,\rho(1-\rho)u_n + (1-\delta)\,\frac{\beta^2 \rho x_{T-1}^2}{\lambda}\,(1-\rho)\,u_n + \frac{\beta^2 \rho x_{T-1}}{\lambda}\delta\gamma_{1,T-1} \\
&\quad +\delta\rho\gamma_{1,T-1} + (1-\delta)\,\rho\,(x_{T-1}-1)\,(1-\rho)u_n, \\
&= (1-\delta)\,\frac{\beta^2 \rho x_{T-1}^2}{\lambda}\,(1-\rho)\,u_n + \frac{\beta^2 \rho x_{T-1}}{\lambda}\delta\gamma_{1,T-1} + +\delta\rho\gamma_{1,T-1} \\
&\quad + (1-\delta)\,\rho x_{T-1}(1-\rho)u_n, \\
&= (1-\delta)\,\rho\,(1-\rho)\left[\frac{\beta^2 \rho x_{T-1}^2}{\lambda} + x_{T-1}\right] u_n + \frac{\beta^2 \rho x_{T-1}}{\lambda}\delta\gamma_{1,T-1} + +\delta\rho\gamma_{1,T-1},
\end{aligned}
$$

leading to a difference equation in $\gamma_{1,T}$:

$$
\gamma_{1,T} = (1-\delta)\,\rho\,(1-\rho)\left[\frac{\beta^2 \rho x_{T-1}^2}{\lambda} + x_{T-1}\right] u_n + \left[\frac{\beta^2 x_{T-1}}{\lambda} + 1\right]\delta\rho\gamma_{1,T-1}, \quad T > 1. \qquad \text{(G.3)}
$$

Equating the constants on both sides of (G.1), yields

$$
\begin{aligned}
\gamma_{0,T} &= \frac{1}{2}\,(1-\delta)\left[(1-\rho)^2 u_n^2 + \lambda\left(\left[\frac{\delta\beta\gamma_{1,T-1}}{(1-\delta)\,\lambda}\right]^2 + \frac{\beta^2}{\lambda^2}x_{T-1}^2\,(1-\rho)^2 u_n^2\right.\right. \\
&\quad \left.\left. +2\frac{\delta\beta^2\gamma_{1,T-1}}{(1-\delta)\,\lambda^2}x_{T-1}(1-\rho)u_n\right)\right] + \delta\gamma_{0,T-1} + \delta\gamma_{1,T-1}\,(1-\rho)\,u_n \\
&\quad +\frac{1}{2}\delta\gamma_{2,T-1}\,(1-\rho)^2 u_n^2,
\end{aligned}
$$

or,

$$
\begin{aligned}
\gamma_{0,T} &= \frac{1}{2}\,(1-\delta)\,(1-\rho)^2 u_n^2 + \frac{\delta^2\beta^2\gamma_{1,T-1}^2}{2\,(1-\delta)\,\lambda} + \frac{1}{2}\,(1-\delta)\,\frac{\beta^2}{\lambda}x_{T-1}^2\,(1-\rho)^2 u_n^2 \\
&\quad +\frac{\delta\beta^2\gamma_{1,T-1}}{\lambda}x_{T-1}(1-\rho)u_n + \delta\gamma_{0,T-1} + \delta\gamma_{1,T-1}\,(1-\rho)\,u_n \\
&\quad +\frac{1}{2}\delta\gamma_{2,T-1}\,(1-\rho)^2 u_n^2, \\
&= \frac{1}{2}\,(1-\delta)\,(1-\rho)^2 u_n^2 + \frac{\delta^2\beta^2\gamma_{1,T-1}^2}{2\,(1-\delta)\,\lambda} + \frac{1}{2}\,(1-\delta)\,\frac{\beta^2}{\lambda}x_{T-1}^2\,(1-\rho)^2 u_n^2 \\
&\quad +\frac{\delta\beta^2\gamma_{1,T-1}}{\lambda}x_{T-1}(1-\rho)u_n + \delta\gamma_{0,T-1} + \delta\gamma_{1,T-1}\,(1-\rho)\,u_n \\
&\quad +\frac{1}{2}\,(1-\delta)\left(\frac{\delta\gamma_{2,T-1}}{1-\delta}\right)(1-\rho)^2 u_n^2, \\
&= \frac{1}{2}\,(1-\delta)\,(1-\rho)^2 u_n^2 + \frac{\delta^2\beta^2\gamma_{1,T-1}^2}{2\,(1-\delta)\,\lambda} + \frac{1}{2}\,(1-\delta)\,\frac{\beta^2}{\lambda}x_{T-1}^2\,(1-\rho)^2 u_n^2 \\
&\quad +\frac{\delta\beta^2\gamma_{1,T-1}}{\lambda}x_{T-1}(1-\rho)u_n + \delta\gamma_{0,T-1} + \delta\gamma_{1,T-1}\,(1-\rho)\,u_n \\
&\quad +\frac{1}{2}\,(1-\delta)\,(x_{T-1}-1)\,(1-\rho)^2 u_n^2,
\end{aligned}
$$

leading to a difference equation in $\gamma_{0,T}$:

$$\gamma_{0,T} = \frac{\delta^2\beta^2\gamma_{1,T-1}^2}{2(1-\delta)\lambda} + \frac{1}{2}(1-\delta)\frac{\beta^2}{\lambda}x_{T-1}^2(1-\rho)^2 u_n^2 + \frac{\delta\beta^2\gamma_{1,T-1}}{\lambda}x_{T-1}(1-\rho)u_n$$
$$+\delta\gamma_{0,T-1} + \delta\gamma_{1,T-1}(1-\rho)u_n + \frac{1}{2}(1-\delta)x_{T-1}(1-\rho)^2 u_n^2, \quad T>1. \quad \text{(G.4)}$$

In order to characterize the solution to the system of Riccati equations, (G.2), (G.3) and (G.4), we need first to find the relevant boundary conditions of the difference equations, i.e., $\gamma_{2,1}$, $\gamma_{1,1}$, and $\gamma_{0,1}$, respectively. These are found by considering the MPE of a 1-period game, where the loss of the future is given by $\delta\widetilde{L}^r(0,u_t)$, i.e., is characterized by a reversion to the zero inflation rule. In such a game the MA minimizes $\frac{1}{2}(1-\delta)[u_t^2+\lambda\pi_t^2]+\delta\widetilde{L}^r(0,u_t)$ s.t. (13), π_t^e given. By use of (D.1), the relevant first-order condition reads: $-\beta(1-\delta)u_t+(1-\delta)\lambda\pi_t-\delta\beta(c_1+c_2u_t)=0$. Applying that $\pi_t^e=\pi_t$ in equilibrium, this condition yields the MPE inflation rate

$$\pi_t|_{T=1} = \frac{\delta\beta}{(1-\delta)\lambda}c_1 + \frac{\beta}{\lambda}\left(1+\frac{\delta c_2}{1-\delta}\right)[(1-\rho)u_n+\rho u_{t-1}]. \quad \text{(G.5)}$$

Combining (G.5) with the fact that $u_t=(1-\rho)u_n+\rho u_{t-1}$ in equilibrium, we recover the loss of the 1-period MPE as

$$\begin{aligned}&\widetilde{L}_1(u_{t-1})\\ =\ &\frac{1}{2}(1-\delta)\Big[((1-\rho)u_n+\rho u_{t-1})^2\\ &+\lambda\left(\frac{\delta\beta}{(1-\delta)\lambda}c_1+\frac{\beta}{\lambda}\left(1+\frac{\delta c_2}{1-\delta}\right)[(1-\rho)u_n+\rho u_{t-1}]\right)^2\Big],\end{aligned}$$

from which it follows that

$$\gamma_{2,1} = (1-\delta)\rho^2\left[1+\frac{\beta^2}{\lambda}\left(1+\frac{\delta c_2}{1-\delta}\right)^2\right], \quad \text{(G.6)}$$

$$\gamma_{1,1} = (1-\delta)\rho\left[(1-\rho)u_n+\frac{\beta^2}{\lambda}\left(1+\frac{\delta c_2}{1-\delta}\right)^2(1-\rho)u_n + \frac{\delta\beta^2 c_1}{(1-\delta)\lambda}\left(1+\frac{\delta c_2}{1-\delta}\right)\right], \quad \text{(G.7)}$$

$$\gamma_{0,1} = \frac{1}{2}(1-\delta)\left[(1-\rho)^2u_n^2+\frac{\delta^2\beta^2}{(1-\delta)^2\lambda}c_1^2+\frac{\beta^2}{\lambda}\left(1+\frac{\delta c_2}{1-\delta}\right)^2(1-\rho)^2u_n^2 + 2\frac{\delta\beta^2c_1}{(1-\delta)\lambda}\left(1+\frac{\delta c_2}{1-\delta}\right)(1-\rho)u_n\right], \quad \text{(G.8)}$$

are the relevant boundary conditions on, respectively, (G.2), (G.3), and (G.4). By use of the definitions of c_1 and c_2 these expressions are further reduced to:

$$\gamma_{2,1} = (1-\delta)\rho^2\left[1+\frac{\beta^2}{\lambda(1-\delta\rho^2)^2}\right], \tag{G.9}$$

$$\gamma_{1,1} = \rho(1-\rho)\left[1+\frac{\beta^2}{\lambda(1-\delta\rho^2)^2(1-\delta\rho)}\right](1-\delta)u_n, \tag{G.10}$$

$$\gamma_{0,1} = \frac{1}{2}(1-\rho)^2\left[1+\frac{\beta^2}{\lambda(1-\delta\rho^2)^2(1-\delta\rho)^2}\right](1-\delta)u_n^2. \tag{G.11}$$

where (G.9) can be re-written in terms of x_1:

$$x_1 = 1+\delta\rho^2\left[1+\frac{\beta^2}{\lambda(1-\delta\rho^2)^2}\right]. \tag{G.12}$$

Note that this is precisely the same as y_1 for $\pi = 0$, cf. (A.4).

Having derived these boundary conditions, we can now characterize the solution to the system of Riccati equations. First note the recursive structure of (G.2), (G.3), and (G.4). Equation (G.2) determines alone $\gamma_{2,T}$ [together with boundary condition (G.12)]. Then, given this solution, equation (G.3) alone determines $\gamma_{1,T}$ [together with boundary condition (G.10)]. Then, given these solutions, equation (G.4) finally determines $\gamma_{0,T}$ [together with boundary condition (G.11)]. Since the determination of $\gamma_{2,T}$ follows from the determination of x_T, and since the determination of the latter exactly mimics that of y_T we can immediately apply the arguments of Appendix A to conclude that x_T is positive and bounded for all T.

Next we consider the properties of the solution to $\gamma_{1,T}$. For convenience, we rewrite the first-order difference equation (G.3) as

$$\gamma_{1,T} = \Omega_{T-1}(1-\delta)u_n + \Psi_{T-1}\gamma_{1T-1}, \quad T>1, \tag{G.13}$$

with

$$\Omega_{T-1} \equiv \rho(1-\rho)\left[\frac{\beta^2\rho x_{T-1}^2}{\lambda}+x_{T-1}\right], \quad \Psi_{T-1} \equiv \left[\frac{\beta^2 x_{T-1}}{\lambda}+1\right]\delta\rho,$$

both being bounded in δ for any T because x_{T-1} is. Equation (G.13) has the solution:

$$\gamma_{1,T} = \sum_{i=1}^{T-1}\left(\left[\prod_{j=1}^{i-1}\Psi_{T-j}\right]\Omega_{T-i}\right)(1-\delta)u_n + \prod_{j=1}^{T-1}\Psi_{T-j}\gamma_{1,1},$$

which by use of (G.10) becomes

$$\gamma_{1,T} = \left\{ \sum_{i=1}^{T-1} \left(\left[\prod_{j=1}^{i-1} \Psi_{T-j} \right] \Omega_{T-i} \right) + \prod_{j=1}^{T-1} \Psi_{T-j} \rho (1-\rho) \left[1 + \frac{\beta^2}{\lambda (1-\delta\rho^2)^2 (1-\delta\rho)} \right] \right\} (1-\delta) u_n.$$

In more condensed form, the solution can therefore be written as

$$\gamma_{1,T} = \kappa_{1,T} (1-\delta) u_n, \tag{G.14}$$

where

$$\kappa_{1,T} \equiv \sum_{i=1}^{T-1} \left(\left[\prod_{j=1}^{i-1} \Psi_{T-j} \right] \Omega_{T-i} \right) + \prod_{j=1}^{T-1} \Psi_{T-j} \rho (1-\rho) \left[1 + \frac{\beta^2}{\lambda (1-\delta\rho^2)^2 (1-\delta\rho)} \right]$$

is a bounded function of δ for any T since the Ω_js and Ψ_js are.

Now we can examine the properties of the solution to $\gamma_{0,T}$. We first, for convenience, rewrite the first-order difference equation (G.4), using (G.14), as

$$\begin{aligned} \gamma_{0,T} \;=\; & \left[\frac{\delta^2 \beta^2 \kappa_{1,T-1}^2}{2\lambda} + \frac{1}{2} \frac{\beta^2}{\lambda} x_{T-1}^2 (1-\rho)^2 + \frac{\delta \beta^2 \kappa_{1,T-1}}{\lambda} x_{T-1} (1-\rho) \right] (1-\delta) u_n^2 \\ & + \delta \gamma_{0,T-1} + \delta \kappa_{1,T-1} (1-\delta)(1-\rho) - \rho u_n^2 + \frac{1}{2} (1-\delta) x_{T-1} (1-\rho)^2 u_n^2, \quad T > 1. \end{aligned}$$

This can be reduced in the following way:

$$\gamma_{0,T} = \Phi_{T-1} (1-\delta) u_n^2 + \delta \gamma_{0,T-1} \tag{G.15}$$

where

$$\begin{aligned} \Phi_{T-1} \;\equiv\; & \frac{\delta^2 \beta^2 \kappa_{1,T-1}^2}{2\lambda} + \frac{1}{2} \frac{\beta^2}{\lambda} x_{T-1}^2 (1-\rho)^2 + \frac{\delta \beta^2 \kappa_{1,T-1}}{\lambda} x_{T-1} (1-\rho) \\ & + \delta \kappa_{1,T-1} (1-\rho) + \frac{1}{2} x_{T-1} (1-\rho)^2 \end{aligned}$$

is bounded in δ and T because both x_{T-1} and $\kappa_{1,T-1}$ are. Equation (G.15) has the solution:

$$\gamma_{0,T} = \sum_{i=1}^{T-1} \delta^{i-1} \Phi_{T-i} (1-\delta) u_n^2 + \delta^{T-1} \gamma_{0,1},$$

which by use of (G.11) becomes

$$\gamma_{0,T} = \left\{ \sum_{i=1}^{T-1} \delta^{i-1} \Phi_{T-i} + \delta^{T-1} \frac{1}{2} (1-\rho)^2 \left[1 + \frac{\beta^2}{\lambda (1-\delta\rho^2)^2 (1-\delta\rho)^2} \right] \right\} (1-\delta) u_n^2,$$

In more condensed form, the solution can be written as

$$\gamma_{0,T} = \kappa_{0,T} (1-\delta) u_n^2, \tag{G.16}$$

where

$$\kappa_{0,T} \equiv \sum_{i=1}^{T-1} \delta^{i-1} \Phi_{T-i} + \delta^{T-1} \frac{1}{2} (1-\rho)^2 \left[1 + \frac{\beta^2}{\lambda (1-\delta\rho^2)^2 (1-\delta\rho)^2} \right]$$

is a bounded function of δ for any given T since the Φ_js are. This proves the form of the solution of $\gamma_{0,T}$ as stated by (D.3) as well as the claim made in the proof of Proposition 3 that $\kappa_{0,T}$ is bounded in δ.

H. Proof that (E.4) applies and that μ_T is bounded in δ for any given T

For the purpose of proving the claim, we need to determine the parameter θ_T of the function $\bar{L}_T(u_{t-1})$ as given by (E.3). Now this function must satisfy

$$\bar{L}_T(u_{t-1}) = \mathrm{E}_{t-1} \min_{\pi_t} \left\{ \frac{1}{2} (1-\delta) \left[u_t^2 + \lambda \pi_t^2 \right] + \delta \bar{L}_{T-1} (u_{t-1}) \quad \text{s.t. (14), } \pi_t^e \text{ given} \right\}, \quad T > 1.$$

From the relevant first-order condition (using the conjectured form of $\bar{L}_T$), $-(1-\delta)\beta u_t + (1-\delta)\lambda\pi_t - \delta\beta\gamma_{T-1} u_t = 0$, we recover, by applying the expectations are formed rationally:

$$\pi_t^e = \frac{\beta}{\lambda} y_{T-1} \rho u_{t-1}.$$

The actual MPE inflation rate, conditional on γ_{T-1}, is the subsequently found as

$$\pi_t = \frac{\beta}{\lambda} y_{T-1} \rho u_{t-1} + \frac{\beta y_{T-1}}{\lambda + \beta^2 y_{T-1}} \varepsilon_t, \tag{H.1}$$

so the MPE unemployment rate becomes

$$u_t = \rho u_{t-1} + \frac{\lambda}{\lambda + \beta^2 y_{T-1}} \varepsilon_t. \tag{H.2}$$

We then solve for the unknown parameters of the function $\bar{L}_T$. In equilibrium, the following recursion holds:

$$L_T(u_{t-1}) = \mathrm{E}_{t-1} \left\{ \frac{1}{2} (1-\delta) \left[u_t^2 + \lambda \pi_t^2 \right] + \delta \bar{L}_{T-1} (u_{t-1}) \right\}, \quad T > 1,$$

which by use of (E.3), (H.1) and (H.2) is equivalent of

$$\begin{aligned}
\frac{\gamma_T}{2}u_{t-1}^2+\frac{\theta_T}{2}\sigma^2 &= \mathrm{E}_{t-1}\left\{\frac{1}{2}(1-\delta)\left[\left(\rho u_{t-1}+\frac{\lambda}{\lambda+\beta^2 y_{T-1}}\varepsilon_t\right)^2\right.\right. \\
&\quad \left.+\lambda\left(\frac{\beta}{\lambda}y_{T-1}\rho u_{t-1}+\frac{\beta y_{T-1}}{\lambda+\beta^2 y_{T-1}}\varepsilon_t\right)^2\right] \\
&\quad \left.+\frac{\delta\gamma_{T-1}}{2}\left(\rho u_{t-1}+\frac{\lambda}{\lambda+\beta^2 y_{T-1}}\varepsilon_t\right)^2+\frac{\delta\theta_{T-1}}{2}\sigma^2\right\} \\
&= \frac{1}{2}(1-\delta)\left[1+\frac{\beta^2}{\lambda}y_{T-1}^2\right]\rho^2 u_{t-1}^2+\frac{1}{2}(1-\delta)\frac{\lambda\left(\lambda+\beta^2 y_{T-1}^2\right)}{\left(\lambda+\beta^2 y_{T-1}\right)^2}\sigma^2 \\
&\quad +\frac{\delta\gamma_{T-1}}{2}\rho^2 u_{t-1}^2+\frac{\delta\gamma_{T-1}\lambda^2}{2\left(\lambda+\beta^2 y_{T-1}\right)^2}\sigma^2+\frac{\delta\theta_{T-1}}{2}\sigma^2.
\end{aligned}$$

Equating coefficients on both sides to u_{t-1}^2 leads to (A.2), so the determination of y_T and, thus, γ_T, is unaltered by the introduction of supply shocks. Equating coefficients to σ^2 gives a difference equation in θ_T:

$$\theta_T=(1-\delta)\frac{\lambda\left(\lambda+\beta^2 y_{T-1}^2\right)}{\left(\lambda+\beta^2 y_{T-1}\right)^2}+\frac{\delta\gamma_{T-1}\lambda^2}{\left(\lambda+\beta^2 y_{T-1}\right)^2}+\delta\theta_{T-1},\quad T>1,$$

or,

$$\theta_T=(1-\delta)\frac{\lambda y_{T-1}}{\lambda+\beta^2 y_{T-1}}+\delta\theta_{T-1},\quad T>1. \tag{H.3}$$

Equation (H.3) has the solution

$$\theta_T=\sum_{i=1}^{T-1}\delta^{i-1}\left[\frac{\lambda y_{T-i}}{\lambda+\beta^2 y_{T-i}}\right](1-\delta)+\delta^{T-1}\theta_1. \tag{H.4}$$

To complete the solution, however, we need to find the relevant boundary condition on (H.3), i.e., θ_1. We must therefore consider the MPE of a 1-period game, where the loss of the future is given by $\delta\bar{L}^r(s^r;u_t)$. In this game, the MA minimizes $\frac{1}{2}(1-\delta)\left[u_t^2+\lambda\pi_t^2\right]+\delta\bar{L}^r(s^r;u_t)$ s.t. (H.2), π_t^e given. By use of (E.1), the relevant first-order condition reads: $-(1-\delta)\beta u_t+(1-\delta)\lambda\pi_t-\delta(1-\delta)\beta\frac{\rho^2}{1-\delta\rho^2}u_t=0$. Applying that expectations are rational, we therefore have $-(1-\delta)\beta\rho u_{t-1}++(1-\delta)\lambda\pi_t^e-\delta(1-\delta)\beta\frac{\rho^2}{1-\delta\rho^2}\rho u_{t-1}=0$, or,

$$\pi_t^e|_{T=1}=\frac{\beta}{\lambda(1-\delta\rho^2)}\rho u_{t-1}.$$

Applying this, we find that actual MPE inflation is

$$\pi_t|_{T=1} = \frac{\beta}{\lambda\left(1-\delta\rho^2\right)}\rho u_{t-1} + \frac{\beta}{\beta^2+\lambda\left(1-\delta\rho^2\right)}\varepsilon_t. \tag{H.5}$$

In consequence, MPE unemployment is

$$u_t|_{T=1} = \rho u_{t-1} + \frac{\lambda\left(1-\delta\rho^2\right)}{\beta^2+\lambda\left(1-\delta\rho^2\right)}\varepsilon_t. \tag{H.6}$$

The expected loss of the MPE in the 1-period game is therefore, using (H.5) and (H.6),

$$\begin{aligned}\bar{L}_1 \quad = \quad & \mathrm{E}_{t-1}\left\{\frac{1}{2}\left(1-\delta\right)\left(\rho u_{t-1} + \frac{\lambda\left(1-\delta\rho^2\right)}{\beta^2+\lambda\left(1-\delta\rho^2\right)}\varepsilon_t\right)^2\right.\\ & \left. + \lambda\left(\frac{\beta}{\lambda\left(1-\delta\rho^2\right)}\rho u_{t-1} + \frac{\beta}{\beta^2+\lambda\left(1-\delta\rho^2\right)}\varepsilon_t\right)^2\right\},\end{aligned}$$

from which we immediately recover, by the conjecture (E.3),

$$\theta_1 = \frac{1}{2}\left(1-\delta\right)\frac{\lambda\left(\beta^2+\lambda\left(1-\delta\rho^2\right)^2\right)}{\left(\beta^2+\lambda\left(1-\delta\rho^2\right)\right)^2} \tag{H.7}$$

Inserting this back into (H.4), we can express θ_T as

$$\theta_T = \mu_T\left(1-\delta\right),$$

where

$$\mu_T \equiv \sum_{i=1}^{T-1}\delta^{i-1}\left[\frac{\lambda y_{T-i}}{\lambda+\beta^2 y_{T-i}}\right] + \delta^{T-1}\frac{1}{2}\frac{\lambda\left(\beta^2+\lambda\left(1-\delta\rho^2\right)^2\right)}{\left(\beta^2+\lambda\left(1-\delta\rho^2\right)\right)^2},$$

is a bounded function of δ for any given T as y_T is. This proves the form of the solution of θ_T as stated by (E.4) as well as the claim made in the proof of Proposition 4 that μ_T is bounded in δ.