Monetary Policy Frameworks and Real Equilibrium Determinacy*

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Abstract

In a simple "prototype" model of monetary policymaking, I examine the issue of real equilibrium determinacy under targeting and instrument rules. The former framework involves minimization of a loss function (under discretion or commitment), whereas the latter involves commitment to an interest rate rule. While instrument rules only lead to determinacy under certain conditions, the targeting rules under consideration always secure determinacy. Within an extended model, I argue that econometric estimations of nominal interest rate response functions may tell little about the economy's stability properties. Instead, they could reveal whether targeting-rule based policy is performed under discretion or commitment.

Keywords: Monetary policy; targeting rules, interest rate rules, real equilibrium determinacy; rules vs. discretion.

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1. Introduction

Since Sargent and Wallace (1975) showed that an interest rate peg rendered the price-level indeterminate in a rational expectations IS-LM-AS model, there has been a lot of research in the issue of designing monetary policy in order to secure determinate rational expectations equilibria. This has recently been witnessed in the new line of research in models of the type dubbed "The New Neo-Classical Synthesis" (Goodfriend and King, 1997).[1] These models feature micro-founded, forward-looking private-sector behavior and nominal rigidities, and the latter feature often turns the issue into one of real indeterminacy. I.e., existence of an infinity of well-behaved rational expectations equilibria for, say, inflation and output.

To ensure real determinacy (and thus exclude the potential for inefficient, self-fulfilling fluctuations — sunspot equilibria), some restrictions are typically required on the nominal interest rate. In this strand of literature, it usually involves having the nominal interest rate satisfy what Woodford (1999c) has labelled the "Taylor principle." This principle states that a central bank following a policy rule in conformity with the celebrated Taylor (1993) rule, should respond sufficiently aggressive towards inflation and/or output. This will imply that any sunspot-driven increase in inflation will be met by a contractive policy, which depresses output and, through the inflation adjustment mechanism, also inflation. The self-fulfilling nature of the inflation increase is then avoided, and determinacy is secured.[2]

A current disagreement in the literature is how to appropriately model monetary policymaking (both from a prescriptive and descriptive point of view). One strand of the literature considers the performance of instrument rules for the nominal interest rate, e.g., of the Taylor-type, and assumes that the central bank commits to following the instrument rule (this is the case for most of the contributions in Taylor, 1999). Another strand views the monetary policymaking process as one where the central bank is optimizing, typically by minimizing some well-defined loss function (e.g., defined over inflation and output). This is a situation that Svensson (1999, 2001b) characterizes as a central bank adhering to a targeting rule (see also Rogoff, 1985, and Walsh, 1998, Chapter 8). Many pros and cons of either model framework have been put forth (see Svensson, 2001b, for a discussion), and it is the purpose of this paper to contribute to this debate by comparing instrument and targeting rules in terms of their stability properties; i.e., in terms of their ability to ensure real determinacy.

[1] See, e.g., several contributions in the volume edited by Taylor (1999).

[2] In a Taylor-type rule where only inflation, or inflation expectations, is the argument, the sufficient restriction is usually that the coefficient on inflation exceeds one; this amount to an "active" Taylor rule in the language of Leeper (1991).

In a simple "prototype" model of monetary policymaking within the "Synthesis" paradigm, I therefore examine the issue of real equilibrium determinacy under either monetary policy framework. It is found — as is well known — that instrument rules only lead to determinacy under certain conditions, but it turns out that the targeting rule under consideration always secure determinacy (this holds irrespective of whether the central bank is optimizing under discretion or commitment). The general intuition behind this better performance of targeting rules is that they circumvent a problem that instrument rules suffer from. By being mechanical behavioral rules, they are subject to what one could label a "reverse Lucas critique," in the sense that any change in private sector behavior (say, a sunspot-driven increase in inflation) is not triggering a change in policy behavior. This seems, from a methodologically point of view, somewhat odd, as great care is usually put into securing that private sector behavior is micro-founded and thus not subject to the Lucas (1976) critique. The targeting rule framework, on the other hand, treats the central bank as an optimizing entity just as the rest of the economy's agents, and this implies that changes in private sector behavior will lead to changes in policymaking. In the model I examine, it is precisely this immunity to a "reverse Lucas critique," that drives the determinacy results under targeting rules. If, e.g., a sunspot-driven change in inflation was to occur, an optimizing central bank aiming at inflation stability, would react to this change in order to minimize its loss function. As a consequence, such non-fundamental equilibria are ruled out.[3]

As a by-product of the above analysis, it is found that equilibrium relationships between the nominal interest rate and macroeconomic variables may tell little about the economy's stability properties. This is further demonstrated within an extended model from which "data" are extracted from stochastic simulations of determinate equilibria. Estimations of conventional Taylor-type rules then turn out to sometimes suggest equilibrium indeterminacy by revealing interest rate functions, which are not in conformity with the Taylor principle.[4] This is particularly prevalent when monetary policy is conducted under commitment. While econometric estimations of nominal interest rate response functions thus may be uninformative regarding the determinacy issue, they may provide information

[3]My results are thus supportive of the following statement by Svensson (2001b): "Macroeconomics long ago stopped modelling private economic agents as following mechanical rules for consumption, saving, production and investment decisions; instead they are now normally modeled as optimizing agents (...) It is long overdue to acknowledge that modern central banks are, as least when it comes to the inflation targeters, optimizing to at least the same extent as private economic agents" (p. 51).

[4]Recently, Clarida *et al.* (2000) have argued, based on estimated Taylor rules, that lack of conformity with the Taylor principle in the pre-Volcker/Greenspan period, suggests that the US economy was subject to expectations-driven fluctuations (Orphanides, 2001, and Perez, 2001, question the identified lack of Taylor principle using real-time data).

on whether optimization-based policy is performed under discretion or commitment.

It should be emphasized that the results presented here are cast within a particular class of economic models (which, however, it is fair to say have had significant impact in recent literature). Other models of monetary policymaking, e.g., do not necessarily endorse active Taylor instrument rules as ones securing determinacy. E.g., models with money in the production function, models with loans constraints, limited participation models, or models of the fiscal theory of the price level determination. On examples of determinacy issues with Taylor-type instrument rules in such models, see, e.g., Benhabib *et al.* (2001), Carlstrom and Fuerst (1999), Christiano and Gust (1999) and Woodford (1996), respectively. An interesting avenue for future research would be to examine the properties of targeting rules within these models.

The remainder of the paper is structured as follows. Subsection 2.1 presents the simple model, and Subsection 2.2 defines the monetary policy frameworks under consideration. Subsections 2.3 and 2.4, respectively, considers determinacy under discretionary and commitment targeting rules. Section 3 exemplifies, in an extended model, how estimations of interest rate response functions may give misleading answers regarding an economy's stability properties. Section 4 summarizes and the Appendix contains various proofs.

2. Real equilibrium determinacy and policy rules in a simple model

2.1. The model

This section presents a stripped-down version of a model belonging to the "The New Neo-Classical Synthesis" class. The present version corresponds closely to that used by, e.g., Clarida *et al.* (1999) and Woodford (1999b). Time is discrete, and aggregate demand in the closed economy in periods $t = 1, 2, ..., \infty$ is represented by an intertemporal "IS curve:"

$$x_t = \mathrm{E}_t x_{t+1} - \sigma \left(i_t - \mathrm{E}_t \pi_{t+1} \right) + \mu_t, \quad \sigma > 0, \tag{1}$$

where x_t is the output gap measured as the log deviation of output, y_t, from the natural rate (flex-price level), y_t^n. The (short) nominal interest rate, i_t, is taken to be the monetary policy instrument. The inflation rate is π_t (the log difference of prices between $t-1$ and t). E_t is the expectations operator conditional upon all information up to, and including, period t. This expression approximates the Euler equation characterizing optimal aggregate consumption choices. Parameter σ can thus be interpreted as the rate of intertemporal substitution (times the steady-state ratio of real interest rate sensitive demand to total demand). The variable μ_t comprises interest-insensitive spending and expected (log) changes in the natural rate of output. Aggregate supply is modelled by an expectations-augmented "Phillips curve:"

$$\pi_t = \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t + \varepsilon_t, \quad \kappa > 0, \quad 0 < \beta < 1, \tag{2}$$

The structure of (2) resembles what Roberts (1995) has labelled the "New Keynesian Phillips Curve" and it can be derived from a variety of supply-side models.[5] Inflation is increasing with the output gap as prices are set as a mark-up over real marginal costs, which are increasing with the output gap. Higher expected future inflation raises current inflation, as price setters under this sticky-price formulation cannot fully adjust to current shocks; hence, to protect the discounted stream of real profits, expected future prices become important — to an extent determined by the discount factor β. The term ε_t is a shock, often labelled a "cost-push" shock, cf. Clarida *et al.* (1999), and it comprises any variation in π_t unexplained by the output gap and expected future inflation. It is assumed to follow an AR(1) process, i.e., $\varepsilon_t = \rho \varepsilon_{t-1} + \xi_t$, where $0 \leq \rho < 1$ and ξ_t is a white-noise innovation.

This model is one where an interest rate peg would lead to an indeterminate equilibrium (this will be formally seen below). To understand this, consider a sunspot-driven increase in inflation expectations. As this does not affect the nominal interest rate, the real interest rate falls. This stimulates demand and the output gap. Through the interaction of the IS- and Phillips-curves, this implies an increase in current inflation that is larger than the increase in expected inflation. The increase in expected inflation therefore initiates an increase in output and inflation, which is followed by the variables' gradual return to steady state. As the increase in inflation expectations is of arbitrary size, one cannot pin down a unique non-explosive rational expectations equilibrium. The economy is consequently vulnerable to expectations-driven fluctuations, i.e., sunspot fluctuations.

The description of the model ends with a characterization of the preferences of society. These are captured by the loss function

$$L = \mathrm{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\lambda x_t^2 + \pi_t^2 \right], \quad \lambda > 0, \tag{3}$$

reflecting the conventional idea that variations in inflation as well as the output gap are disliked.[6]

[5] For example, it approximates the aggregate pricing equation emerging from monopolistically competitive firms' optimal behavior in Calvo's (1983) model of staggered price determination (see, e.g., Rotemberg and Woodford, 1998, for a derivation).

[6] With the model (1)-(2), L may represent a second-order approximation to the (negative of the) utility

2.2. Monetary policy frameworks

In defining the monetary policy frameworks to be considered, I follow the terminology of Svensson (1999, 2001b). First, a *targeting rule* is a framework there the central bank chooses the path of interest rates $\{i_t\}_{t=1}^{\infty}$ so as to minimize a loss function $\sum_{t=1}^{\infty} L_t(\omega_{1t}, \omega_{2t}, ..., \omega_{nt})$ subject to (1) and (2), where $\omega_{1t}, \omega_{2t}, ..., \omega_{nt}$ are goal variables. More specifically, this rule is what Svensson labels a *general* targeting rule; i.e., a full commitment to a loss function. This corresponds to the establishment of a monetary policy regime with clearly defined constitutional mandates about which goal variables should be stabilized, at which values they are stabilized around, and how much relative weight each goal variable should receive in policy conduct. Svensson also defines a *specific* targeting rule as an explicit description of conditions for the relationships between (target) variables, which policymaking should aim to satisfy. Typically, these conditions are first-order conditions associated with the minimization of an appropriate loss function (see also Svensson and Woodford, 1999, for more discussion of taxonomy). For my purpose, it does not matter whether a general or specific targeting rule is examined, when it is the relevant first-order condition(s), which must be satisfied under either definition. This is because it is such conditions, which are crucial for the results on real determinacy; see below.

Secondly, an *instrument rule* is a framework where the central bank's path of interest rates is given by a commitment to $\{r_t(\psi_{1t}, \psi_{2t}, ..., \psi_{mt})\}_{t=1}^{\infty}$, where r_t is some function, and $\psi_{1t}, \psi_{2t}, ..., \psi_{mt}$ are variables to which the interest rate responds in period t; these may represent any lags or expected leads of variables. Associated with these distinct frameworks, I define rational expectations equilibria for the current example by:

Definition 1 (Targeting rule equilibrium). *A targeting rule equilibrium (TRE) is a rational expectations equilibrium satisfying*

a) The necessary and sufficient conditions determining the optimal choice of $\{i_t\}_{t=1}^{\infty}$, *where optimization may be conducted either under commitment or discretion*

b) Equations (1) and (2)

Definition 2 (Instrument rule equilibrium). *An instrument rule equilibrium (IRE) is a rational expectations equilibrium satisfying equations (1) and (2), with* $r_t(\psi_{1t}, \psi_{2t}, ..., \psi_{mt})$ *substituted for* i_t, $t = 1, 2, ..., \infty$.

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of a representative agent in the economy. See Woodford (1999c), where it is subsumed that some fiscal (tax) policy keeps the natural rate at the efficient level (by counteracting the distortion caused by monopolistic competition). With this assumption, the appropriate target value for the output gap is zero.

2.3. Real determinacy with a targeting rule under discretionary policy

A natural targeting rule framework within this model, is the case where the loss function to be minimized is simply L. This is often referred to as "flexible" inflation targeting; cf. Svensson (1999). Focusing here on the case of discretionary policymaking, I consider the case where the central bank solves

$$\min_{\{i_t\}_{t=1}^{\infty}} \mathrm{E}_1 \left[\sum_{t=1}^{\infty} \beta^{t-1} \left(\lambda x_t^2 + \pi_t^2 \right) \right], \quad \text{s.t.} \quad (1) \text{ and } (2), \tag{4}$$

and where minimization is performed on a period-by-period basis. The resulting equilibrium outcomes for inflation and the output gap are summarized by the following proposition:

Proposition 1. *The targeting rule described by the problem (4) under discretion, results in a* unique, *non-explosive TRE, characterized by*

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda (1 - \beta \rho)} \varepsilon_t, \qquad x_t = -\frac{\kappa}{\kappa^2 + \lambda (1 - \beta \rho)} \varepsilon_t. \tag{5}$$

Proof. *Since the only predetermined variables in period t are μ_t and ε_t, it follows that a time-consistent equilibrium must satisfy the recursion $\ell_t(\mu_t, \varepsilon_t) = \min_{i_t} \{\lambda x_t^2 + \pi_t^2 + \beta E_t \left[\ell_t \left(\mu_{t+1}, \varepsilon_{t+1} \right) \right] \}$, where ℓ_t is the associated "value" function, and where the minimization is performed subject to (1) and (2). In a time-consistent equilibrium, π_t and x_t will be therefore be (potentially time-varying and non-unique) functions of μ_t and ε_t. Deciding on i_t, $E_t \pi_{t+1}$ and $E_t x_{t+1}$ can therefore be taken as parametrically given. Substituting (1) and (2) into the per-period loss function, one recovers the necessary (and sufficient) first-order condition for optimal policy:*

$$\lambda x_t + \kappa \pi_t = 0. \tag{6}$$

Use (6) in (2) to obtain the following expectational difference equation for inflation: $\pi_t = \beta \left(1 + \kappa^2/\lambda\right)^{-1} E_t \pi_{t+1} + \left(1 + \kappa^2/\lambda\right)^{-1} \varepsilon_t$. Since π_t is free and the eigenvalue, $\left(1 + \kappa^2/\lambda\right)/\beta$, is strictly greater than one, it follows by use of Blanchard and Kahn (1980) that π_t, given by (5) is the unique, *non-explosive rational expectations solution to this difference equation. By (6), the unique solution for x_t follows.* ■

Proposition 1 demonstrates that under a targeting rule, the economy exhibits real equilibrium determinacy. I.e., the rational expectations solutions for output gap and inflation are unique functions of the fundamentals of the economy, here just represented by the predetermined variable ε_t. The first-order condition for optimal policy under the targeting rule gives a mathematical clue as to why this is so.

Consider the situation where a sunspot driven increase in inflation and the output gap was occurring. Obviously, a central bank operating with the aim of price and output gap stability — as implied by the envisaged targeting rule — would immediately raise the interest rate in order to bring inflation and output towards their targets. In consequence, the imagined increase in inflation and the output gap cannot be a rational expectations equilibrium. The first-order condition (6) simply precludes such equilibria. The condition shows that the central bank sets the interest rate so as to equate the marginal loss in terms of output gap and inflation to zero. Note that this implies that in equilibrium any co-movement between the output gap and inflation is *negative*. Consider then a candidate equilibrium, where the relationship between inflation and the output gap is given by $x_t = (\omega/\kappa)\,\pi_t$, where ω is some parameter. Inserting this relationship into (2) yields the difference equation $\pi_t = \beta(1-\omega)^{-1}\mathrm{E}_t\pi_{t+1} + (1-\omega)^{-1}\varepsilon_t$. This to leads to infinitely many solutions for π_t only if $\beta(1-\omega)^{-1}$ is numerically greater than one. A necessary condition for this is $\omega > 0$. I.e., in a sunspot equilibrium in the model, the co-movement between inflation and the output gap is *positive*. This, however, contradicts the first-order condition guiding optimal monetary policy under the targeting rule; hence, equilibrium determinacy prevails.[7]

In terms of the policy instrument, the explanation for the determinacy of the TRE is somewhat more involved. Note, however, first that the equilibrium value of the nominal interest rate can be expressed as a function of the predetermined variables according to the following lemma:

Lemma 1. *The targeting rule described by the problem (4) under discretion, results in an* equilibrium path *for the nominal interest rate, which is a* unique *function of the predetermined variables:*

$$i_t = \frac{\kappa(1-\rho)+\sigma\lambda\rho}{\sigma\left[\kappa^2+\lambda(1-\beta\rho)\right]}\varepsilon_t + \frac{1}{\sigma}\mu_t. \tag{7}$$

[7] One may claim that it is unrealistic that the central bank can assure that (6) holds in any period, since it does not control x_t or π_t, but rather i_t. First note that (6) is in principle not different from any other first-order condition arising from optimal public policy, e.g., optimal tax policy in a simple public finance problem equating the marginal loss of taxes in terms of of private consumption with the gains of public consumption (this condition will not involve the tax rate). Hence, (6) appears not less realistic than such conditions. Secondly, if one acknowledges imperfect control explicitly (by, say, introducing a stochastic control error in policy), the condition (6) would hold in expected value. It is easy to show, analogously to the proof of Proposition 1, that the expected values of inflation and the output gap would be determinate. Actual values would thus differ from these to an extent determined by the *exogenous* imperfect control.

Proof. *Lead π_t and x_t as given by (5) and take period t expectations to get*

$$E_t \pi_{t+1} = \frac{\lambda \rho}{\kappa^2 + \lambda (1 - \beta \rho)} \varepsilon_t, \qquad E_t x_{t+1} = -\frac{\kappa \rho}{\kappa^2 + \lambda (1 - \beta \rho)} \varepsilon_t, \tag{8}$$

which together with x_t inserted into (1) yields (7). ■

The expression for i_t as given by (7) reflects that under discretion, it is feasible (and obviously, optimal) to offset any impact of the μ_t-shock upon the economy. This shock does not present a trade-off in policy, and this explains why the equilibrium values of the output gap and inflation are independent of this shock. An ε_t-shock, however, induces such a trade-off, implying an interest rate response that "spreads out" the impact upon the output gap and inflation (to an extent determined, among other parameters, by the preference for output stability relative to inflation stability, λ).

Now, to facilitate an understanding about why indeterminacy is ruled out under a targeting rule, it is instructive to consider the case where the monetary framework is one of an instrument rule, and the instrument is set exactly according to (7). The case is described in the following lemma:

Lemma 2. *An instrument rule characterized by (7), results in infinitely many non-explosive IRE.*

Proof. *See Appendix A.*

Lemma 2 is a standard example of real equilibrium indeterminacy in models of this type, when the interest rate rule is "passive." By not instructing the central bank to respond to, say, inflation appropriately, self-fulfilling equilibria can occur. The intuition for this phenomenon is well known, and corresponds to the one provided in Subsection 2.1 for the case of a pure interest rate peg. This reflects that interest rate responses towards exogenous variables (here, the shocks) have no consequences for determinacy analysis (cf. Svensson and Woodford, 1999).

The question is then why such equilibrium indeterminacy does not prevail under the targeting rule framework where the interest rate follows precisely the same path? To answer the question informally, the reason is that the passivity imbedded in the instrument rule characterized by (7), which leads to indeterminacy in an IRE (with a non-optimizing central bank), is *absent* under a targeting rule where the central bank acts *actively* to achieve its goals by inducing (6) and thus inducing determinacy. Clearly, a central bank with mandates for inflation and output stability would change its behavior from (7), if suddenly, e.g., inflation and output gap expectations rose for no fundamental reason. Otherwise, one would have a case of a "reverse Lucas critique," where changes in private sector behavior is associated with unchanged policymaker behavior. Nevertheless, the described activism under a targeting rule may very well lead to an interest rate path, which *in equilibrium* looks passive [as (7) surely does].

This last observation suggests that examining an equilibrium relationship between the nominal interest rate and macroeconomic variables may not yield the appropriate answer to if and why determinacy prevails under a targeting rule. Indeed, before presenting the more formal argument for determinacy in terms of the nominal interest rate setting, it is instructive first to present an argument, which at first glance *may* seem to offer an answer for determinacy, but after closer scrutiny does not.[8] For this purpose, the following lemma on the equilibrium relationship(s) between the nominal interest rate and the model's endogenous variables is useful:

Lemma 3. *The targeting rule described by the problem (4) under discretion, results in infinitely many* equilibrium relations *between the nominal interest rate and the predetermined and endogenous variables:*

$$i_t = \sum_{i=0}^{\infty} b_i E_t \pi_{t+i} - \sum_{i=0}^{\infty} a_i E_t x_{t+i} + c\varepsilon_t + \frac{1}{\sigma}\mu_t, \tag{9}$$

where the only restriction on the coefficients $\{a_i\}_{i=0}^{\infty}$, $\{b_i\}_{i=0}^{\infty}$ *and* c *is*

$$\sigma\lambda \sum_{i=0}^{\infty} b_i \rho^i + \sigma\kappa \sum_{i=0}^{\infty} a_i \rho^i + c\sigma \left[\kappa^2 + \lambda\left(1-\beta\rho\right)\right] = \kappa\left(1-\rho\right) + \sigma\lambda\rho. \tag{10}$$

Proof. *In order for the nominal interest rate to be consistent with the TRE values of current and expected future variables, it follows by (9) and Lemma 1 that the following relation must be satisfied:*

$$\sum_{i=0}^{\infty} b_i E_t \pi_{t+i} - \sum_{i=0}^{\infty} a_i E_t x_{t+i} + c\varepsilon_t = \frac{\kappa\left(1-\rho\right) + \sigma\lambda\rho}{\sigma\left[\kappa^2 + \lambda\left(1-\beta\rho\right)\right]}\varepsilon_t.$$

One can then substitute in the TRE values of the contemporaneous and expected future variables [i.e., use (5)], in order to rewrite this relation as

$$\sum_{i=0}^{\infty} b_i \rho^i \frac{\lambda}{\kappa^2 + \lambda\left(1-\beta\rho\right)}\varepsilon_t + \sum_{i=0}^{\infty} a_i \rho^i \frac{\kappa}{\kappa^2 + \lambda\left(1-\beta\rho\right)}\varepsilon_t + c\varepsilon_t = \frac{\kappa\left(1-\rho\right) + \sigma\lambda\rho}{\sigma\left[\kappa^2 + \lambda\left(1-\beta\rho\right)\right]}\varepsilon_t.$$

[8] While firstly presenting an inadequate answer may rightfully appear odd, it will serve to highlight the issue of (potential) inferences about the determinacy issue from estimated interest rate relations investigated further in Section 3.

If this is to hold for every realization of ε_t, it follows that the coefficients $\{a_i\}_{i=0}^{\infty}$, $\{b_i\}_{i=0}^{\infty}$ and c must satisfy (10). ■

The lemma shows that one can represent the unique TRE as *equilibrium relations* between the nominal interest rate, the predetermined variables and any current or expected leads of the endogenous variables. However, as there is only one restriction on this representation, (10), one can only express the nominal interest rate as a unique function of *one* endogenous variable (note that all representations involve the same unique response to the μ_t-shock).

One of these representations is the one presented by Clarida *et al.* (1999). It expresses the nominal interest rate as a function of the one-period ahead inflation expectations, and thus depicts a forward-looking Taylor rule where there is no response towards the output gap. In the terminology of Lemma 3, this corresponds to a case where $a_i = 0$, all i, $b_0 = 0$, $b_i = 0$, all $i > 1$, $c = 0$, and where b_1 is determined by (10). This results in the following expression for the nominal interest rate:

$$i_t = \left[1 + \frac{\kappa(1-\rho)}{\sigma\lambda\rho}\right] \mathrm{E}_t\pi_{t+1} + \frac{1}{\sigma}\mu_t. \tag{11}$$

Note that the coefficient on expected inflation is strictly greater than one, implying that any increase in expected inflation is associated with an increase in the real interest rate. The nominal interest rate is thus in conformity with the "Taylor principle" (or, alternatively, it is an "active" Taylor rule). This implies that the explanation for indeterminacy of an IRE under the interest rate rule (7) does not apply. Immediately, one could therefore think that since the nominal interest rate can be expressed as (11), uniqueness of the TRE has been explained. Indeed, as shown in Appendix B, $b_1 > 1$ is a necessary condition for an IRE with an interest rate rule of the form $i_t = b_1 \mathrm{E}_t\pi_{t+1} + (1/\sigma)\mu_t$ to be determinate. However, the appendix also shows that if $b_1 > 1 + 2(1+\beta)/(\sigma\kappa)$, then the IRE is indeterminate. This is a reflection of the well-known phenomenon that an interest rate rule can be "too active." In that case, the economy is vulnerable to expectation-driven fluctuations. E.g., an arbitrary increase in $\mathrm{E}_t\pi_{t+1}$ can be self-fulfilling, as it implies a huge fall in demand and output and, hence, current inflation, because of the strong increase in the real interest rate. The economy will then "zig-zag" back to steady state (e.g., $\mathrm{E}_t\pi_{t+2}$ would be below steady state, which is consistent with $\mathrm{E}_t\pi_{t+1}$ being above, as the real interest rate in $t+1$ would fall strongly thereby pushing up output and inflation above steady state in $t+1$).[9]

[9] In accordance with the discussion following Proposition 1, the co-movement of inflation and the output gap will be positive on this path towards steady state.

In terms of (11), it means that whenever $\kappa(1-\rho)/(\lambda\rho) > 1 + 2(1+\beta)/\kappa$ (which cannot be ruled out) an IRE would be indeterminate. Hence, the fact that the value of the nominal interest rate can be expressed as (11) is not the explanation for the uniqueness of the TRE (as uniqueness did not require any restrictions on the parameters of the model). To further see that looking at a particular equilibrium representation may give misleading information about the economy's stability properties, consider the case where i_t is formulated as a function of *current* inflation only [i.e., all parameters $\{a_i\}_{i=0}^{\infty}$, $\{b_i\}_{i=0}^{\infty}$ and c in (10) except b_0 are zero]. Then, by Lemma 3, it follows that $i_t = [\rho + \kappa(1-\rho)/(\sigma\lambda)]\,\pi_t + (1/\sigma)\,\mu_t$. As evident, the coefficient on π_t can be greater or smaller than one, which implies that an IRE associated with this interest rule would be indeterminate for some parameter constellations (e.g., for those implying b_0 being sufficiently lower than one). Finally, note that one might as well express i_t as a function of the one-period ahead expectation of the output gap. It immediately follows by (6) and (11) that the interest rate expression in this format would become $i_t = -[\lambda/\kappa + (1-\rho)/(\sigma\rho)]\,\mathrm{E}_t x_{t+1} + (1/\sigma)\,\mu_t$. Appendix C shows that an IRE under this interest rule is always indeterminate.

The above demonstrates that an examination of the equilibrium relationship(s) between the nominal interest rate and macroeconomic variables could not provide information about the economy's stability properties; as such empirical investigations of interest rate functions may reveal little about stability properties. Yet, Proposition 1 has shown that the rational expectations equilibrium under the examined targeting rule is always determinate. The explanation, in terms of the policy instrument i_t, is that underlying the first-order condition (6) is a *reaction function* which expresses then nominal interest rate as a unique function of the shocks and *any* private sector expectations.

Lemma 4. *Let π_{t+1}^e and x_{t+1}^e, respectively, denote the private sector's subjective expectations about the period $t+1$ inflation rate and output gap (formed in period t). The targeting rule described by the problem (4) under discretion, results in a* unique *expression for the nominal interest rate, which holds for* any *values of π_{t+1}^e and x_{t+1}^e:*

$$i_t = \frac{\sigma(\lambda+\kappa^2)+\kappa\beta}{\sigma(\lambda+\kappa^2)}\pi_{t+1}^e + \frac{1}{\sigma}x_{t+1}^e + \frac{\kappa}{\sigma(\lambda+\kappa^2)}\varepsilon_t + \frac{1}{\sigma}\mu_t. \qquad (12)$$

Proof. *Substitute x_t and π_t as given by (1) and (2), respectively (with π_{t+1}^e replacing $\mathrm{E}_t\pi_{t+1}$ and x_{t+1}^e replacing $\mathrm{E}_t x_{t+1}$), into the first-order condition (6). As (1) and (2) hold for any values of π_{t+1}^e and x_{t+1}^e, it follows that the interest rate resulting from the substitution will be uniquely given by (12) for any values of π_{t+1}^e and x_{t+1}^e, when (6) applies, i.e., when* *the targeting rule is described by the problem (4) under discretion.* ■

This lemma shows why there is determinacy under the considered targeting rule with reference to the policy instrument i_t. It follows from (12) that optimal behavior of the central bank implies responses towards expectations which render any other expectations than those depending on fundamentals only [i.e., (8)] for non-rational or inconsistent with a non-explosive rational expectations equilibrium.

Consider, first the case where output gap expectations, e.g., exceed those given by fundamentals only. I.e., the case where $x^e_{t+1} > \mathrm{E}_t x_{t+1}$, with $\mathrm{E}_t x_{t+1}$ given by (8). By (12), any such divergence between x^e_{t+1} and $\mathrm{E}_t x_{t+1}$ results in an increase of the interest rate by $1/\sigma$, which neutralizes the impact on x_t entirely. The value of x_t will thus remain at its value dependent on fundamentals only, i.e., (5).[10] Moving one period forward, it follows that the central bank again through (12) secures x_{t+1} to be consistent with its value dependent on fundamentals only. Hence, any divergence between x^e_{t+1} and $\mathrm{E}_t x_{t+1}$ given by (8) cannot be a rational expectations equilibrium.

Consider then the case where inflation expectations, e.g., exceed those given by fundamentals only. I.e., the case where $\pi^e_{t+1} > \mathrm{E}_t \pi_{t+1}$ with $\mathrm{E}_t \pi_{t+1}$ given by (8). By (12), any such divergence between π^e_{t+1} and $\mathrm{E}_t \pi_{t+1}$ results in an increase of the interest rate by $[\sigma(\lambda + \kappa^2) + \kappa\beta] / [\sigma(\lambda + \kappa^2)]$. As this increase is greater than one, the real interest rate increases; in particular, by $\kappa\beta / [\sigma(\lambda + \kappa^2)]$.[11] This lowers the current output gap by $\kappa\beta / (\lambda + \kappa^2)$; cf. (1). The net effect on current inflation is therefore, cf. (2), $\beta - \kappa^2\beta / (\lambda + \kappa^2) = \beta\lambda / (\lambda + \kappa^2) < 1$. Hence, the effect is an increase in current inflation above its fundamental value, which is *smaller* than the increase in inflation expectations. Proceeding forwards in time, it follows that if the increase in inflation expectations, above those given by fundamentals only, is to be a rational expectations equilibrium, it will be one where inflation in increasing through time (and the output gap is decreasing through time). However, such explosive rational expectations equilibria are ruled out in this model. Hence, the targeting rule secures equilibrium determinacy by the implicit reaction of the interest rate to deviations from non-fundamental based expectations.

In equilibrium, the nominal interest rate will evolve according to (7) which in an IRE yields indeterminacy, but the associated TRE will be determinate due to the "threat" to fight non-fundamental changes in expectations, which is imbedded in the decision making

[10] Such a change in expectations thus works analogously to a μ-shock, i.e., it presents no trade-off for the central bank.

[11] Note that the coefficient on π^e_{t+1} in (12) thus conforms with the Taylor principle, but also that it is smaller than the upper limit identified on b_1 in the determinacy analysis of the forward-looking Taylor rule $i_t = b_1 \mathrm{E}_t \pi_{t+1} + (1/\sigma)\mu_t$ above.

of an optimizing central bank.[12]

The expression for the nominal interest rate (12), is mathematically equivalent (up to unimportant constants) to the instrument rule recently proposed by Evans and Honkapohja (2001). They consider the stability properties of adaptive learning rules within a non-rational expectations version of the model used here. They do not make the distinction between targeting and instrument rules, and focus essentially on the latter (they, however, do not use this terminology), and show that (7) leads to indeterminacy (and instability under adaptive learning). They argue that this results from all agents having rational expectations (p. 11), and that departure from rational expectations and implementation of (12) as an interest rate rule leads to determinacy (and stability under adaptive learning).

The difference between their paper and mine arises in terms of interpretation. I argue that optimization by a central bank (i.e., adherence to a targeting rule) *implies* determinacy, because the associated first-order condition *implies* a reaction function like (12) for any expectations. In the associated unique rational expectations equilibrium, however, one can express the nominal interest rate as (7) (or in infinitely many other ways, cf. Lemma 3). In contrast, when analyzing the stability properties of (7) in a rational expectations context, Evans and Honkapohja do not explicitly use the first-order condition (6) [although they use it, together with the assumption of a unique rational expectations equilibrium, to derive (7)]. Indeed, of the infinitely many rational expectations equilibria they then show exist when the central bank adopts (7), just one satisfies the first-order condition; namely the equilibrium depending on fundamentals only. Hence, I emphasize that it is the first-order condition, reflecting the central bank's reactions to non-fundamentals-based-equilibrium expectations, that secures a unique rational expectations equilibrium.

In a similar vein, Svensson and Woodford (1999) advocate that in order to ensure a unique rational expectations equilibrium (in a model related to that of Subsection 2.1, but with transmission lags), one must specify an instrument rule that responds to out-of-equilibrium behavior. They also consider targeting rules, but perform determinacy analyses within an IRE (in my terminology). They suggest a type of instrument rule, which "punishes" deviations from the specific targeting rule (i.e., the optimality condition).

[12] An analogy with the well-known Barro and Gordon (1983) set-up may be enlightening. In this type of model, the central bank chooses π_t to minimize (3), subject to a Lucas-style supply equation, $x_t = \pi_t - \pi_t^e - x^*$, $x^* > 0$, taking as given subjective inflation expectations, π_t^e. *In* the rational expectations equilibrium, i.e., where $\pi_t^e = \mathrm{E}_{t-1}\pi_t$, one has that $\pi_t = \lambda x^*$. It would from this expression, of course, be erroneous to conclude that monetary policy is passive towards a (non-rational) change in inflation expectations, as this would ignore the reaction function, $\pi_t = [\lambda/(1+\lambda)](\pi_t^e + x^*)$, which is crucial in the derivation of the rational expectations equilibrium.

Within the model considered here, such an instrument rule would be the equivalent of

$$i_t = w\left(\pi_t + \frac{\lambda}{\kappa}x_t\right) + \frac{\kappa(1-\rho)+\sigma\lambda\rho}{\sigma\left[\kappa^2+\lambda(1-\beta\rho)\right]}\varepsilon_t + \frac{1}{\sigma}\mu_t, \tag{13}$$

where w is an arbitrary constant [this is consistent with Lemma 3, when one constrains c to equal $[\kappa(1-\rho)+\sigma\lambda\rho]\ /\ \{\sigma[\kappa^2+\lambda(1-\beta\rho)]\}$ and uses (10) to identify the linear equilibrium relation between π_t and x_t]. In equilibrium, $\pi_t + (\lambda/\kappa)x_t = 0$ by (6), and the interest rate becomes identical to (7). By suitable choice of w (typically $w > 1$), determinacy of the associated IRE — implementing the discretionary equilibrium — is assured. I believe that implementation of (13) for determinacy is unnecessary, as the optimization imbedded in the targeting rule approach delivers determinacy through its implicit commitment to fight non-fundamentals-based-equilibrium expectations as argued above [since (13) directly incorporates the first-order condition, Svensson and Woodford note that their analogous specification is "somewhat in the spirit of a targeting rule" (p. 43)].[13]

2.4. Real determinacy with a targeting rule under commitment policy

This sub-section considers the case where the central bank under the targeting rule has the ability to commit to a policy plan for the future. I.e., the assumption of discretionary, period-by-period optimization is now abandoned in favor of an assumption that the bank acts in accordance with commitment under a "timeless perspective" (Woodford, 1999b).[14] It turns out that the issue of determinacy is not qualitatively affected by this assumption, but that the resulting equilibrium relationships between the nominal interest rate and endogenous variables even more strongly demonstrate that inference about equilibrium determinacy from these can be misleading.

First, the following proposition characterizes the equilibrium outcomes under the targeting rule (4), and where policy is conducted according to commitment under the timeless perspective:

[13]Note that, as argued by Svensson (2001b), an instrument rule like (13) is more complicated than the associated specific targeting rule, $\pi_t + (\lambda/\kappa)x_t = 0$, as the former involves ε_t (which could correspond to what Svensson interprets as "judgement in monetary policy"), μ_t as well as more parameters. It could therefore be more difficult to verify by the private sector whether the central bank has lived up to its intentions under (13) than under the targeting rule.

[14]This concept elegantly circumvent the arbitrariness of the initial date in the optimization problem, which would otherwise imply that the bank in the next period would have the incentive to "re-commit" (as this date is then an arbitrary initial date). The timeless perspective involves commitment to a policy pattern that the central bank "would have wished to commit itself to at a date far in the past" (Woodford, 1999b, p. 293), implying that the initial date has no meaning. See McCallum and Nelson (2000) for further discussion.

Proposition 2. *The targeting rule described by the problem (4) under commitment, results in a* unique, *non-explosive TRE, characterized by*

$$\begin{aligned} x_t &= \chi x_{t-1} - \varphi\varepsilon_t, \qquad \pi_t = \frac{\lambda}{\kappa}(1-\chi)x_{t-1} + \frac{\lambda}{\kappa}\varphi\varepsilon_t, \\ 0 &< \chi \equiv \frac{1+\beta^{-1}(1+\kappa^2/\lambda) - \sqrt{\left(1+\beta^{-1}(1+\kappa^2/\lambda)\right)^2 - 4\beta^{-1}}}{2} < 1, \\ \varphi &\equiv \frac{\kappa\chi}{\lambda(1-\chi\beta\rho)} > 0. \end{aligned} \tag{14}$$

Proof. *In solving for the commitment policy, set up the Lagrangian (cf. Currie and Levine, 1993; Woodford, 1999a):*

$$\mathcal{L} = E_1\left[\sum_{t=1}^{\infty}\beta^{t-1}\left(\lambda x_t^2 + \pi_t^2\right) + 2\vartheta_t\left(\pi_t - \beta E_t\pi_{t+1} - \kappa x_t - \varepsilon_t\right)\right],$$

where ϑ_t is (half the) multiplier on (2). [There is no need to include the constraint (1), as the multiplier on this will be zero.] From the first-order conditions $\lambda x_t - \vartheta_t\kappa = 0$ and $\pi_t + \vartheta_t - \vartheta_{t-1} = 0$ one obtains the optimality condition

$$\pi_t + \frac{\lambda}{\kappa}(x_t - x_{t-1}) = 0. \tag{15}$$

[Under a "non-timeless perspective" this condition would not hold in period 1, *where instead (6) would apply. This has no implications for determinacy, as under (6) equilibrium is also unique if future values are; cf. Proposition 1.] Use (15) together with (2) to obtain a second-order difference equation in x_t, $E_t x_{t+1} = \left(1+\beta^{-1}(1+\kappa^2/\lambda)\right)x_t - \beta^{-1}x_{t-1} + \left(\beta^{-1}\kappa/\lambda\right)\varepsilon_t$, which can be written in matrix form as*

$$\begin{bmatrix} \varepsilon_{t+1} \\ x_t \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 \\ 0 & 0 & 1 \\ \beta^{-1}\kappa/\lambda & -\beta^{-1} & \left(1+\beta^{-1}(1+\kappa^2/\lambda)\right) \end{bmatrix}\begin{bmatrix} \varepsilon_t \\ x_{t-1} \\ x_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ 0 \\ 0 \end{bmatrix}.$$

The real and positive roots of the system are ρ and

$$\frac{1+\beta^{-1}(1+\kappa^2/\lambda) \pm \sqrt{\left(1+\beta^{-1}(1+\kappa^2/\lambda)\right)^2 - 4\beta^{-1}}}{2}.$$

This implies that two roots are smaller that 1 *and one is above. As the system involves two predetermined variables (ε_t and x_{t-1}) and one jump variable (x_t), this implies by use of Blanchard and Kahn (1980) that the system identifies a unique, non-explosive rational* *expectations equilibrium solution for x_t. The solution is easily recovered by the method of undetermined coefficients. Combining this solution with (15), the unique solution for π_t follows.* ■

This proposition mirrors Proposition 1 for the case of discretionary policymaking by showing that a targeting rule with commitment policy also leads to a determinate rational expectations equilibrium, where inflation and the output gap are unique functions of fundamentals only (here, x_{t-1} and ε_t).

Note the difference of the solution with the one under discretion. In contrast with (5), the solution under commitment, (14), differs because the lagged value of the output gap appears. This reflects the optimality of "history dependent" policy, or "inertial policy," as stressed by Woodford (1999a,b). In this model, such behavior induces a more favorable inflation-output gap trade-off in the following sense. Consider the case where the economy is hit by a temporary (positive) cost-push shock. By letting the associated contractive policy persist into the future, the central bank lowers inflation expectations, which dampens current inflation and thus helps to stabilize the cost-push shock. Hence, a given reduction in inflation can be attained at a lower cost in terms of the current output gap; if the reduction in the output gap persists. This is indeed reflected by the solution (14), where a temporary $\varepsilon_t > 0$ lowers x_t, but also all x_{t+i} and π_{t+i}, $i > 0$ (to a decaying extent). Albeit optimal, the solution is time-inconsistent, as the central bank in period $t+1$ (absent the ability of sticking to its commitment), has the incentive to stop contracting the economy from then on.

As under discretion, one can characterize the path of the nominal interest rate as function of the predetermined variables:

Lemma 5. *The targeting rule described by the problem (4) under commitment, results in an* equilibrium path *for the nominal interest rate, which is a* unique *function of the predetermined variables x_{t-1}, ε_t and μ_t:*

$$i_t = -\frac{\chi}{\sigma}(1-\chi)\left(1-\frac{\lambda\sigma}{\kappa}\right)x_{t-1} + \frac{\varphi}{\sigma}(1-\chi-\rho)\left(1-\frac{\lambda\sigma}{\kappa}\right)\varepsilon_t + \frac{1}{\sigma}\mu_t. \tag{16}$$

Proof. *Lead π_t and x_t as given by (14), apply the value of x_t, and take period t expectations to get*

$$\begin{aligned} E_t\pi_{t+1} &= \frac{\lambda}{\kappa}(1-\chi)\chi x_{t-1} - \frac{\lambda}{\kappa}\varphi(1-\chi-\rho)\varepsilon_t, \\ E_t x_{t+1} &= \chi^2 x_{t-1} - \varphi(\chi+\rho)\varepsilon_t \end{aligned}$$

which together with x_t inserted into (1) yields (16). ■

One sees again that the μ_t-shock is completely neutralized, thus explaining its absence in the solutions for inflation and the output gap. As the ε_t-shock entails an inflation-output gap trade-off, it is not fully neutralized, and (16) compared with (7), shows how the nominal interest rate under commitment depends upon history in order to induce the optimal inertia in policymaking.

Paralleling Lemma 2, which describes the case of discretion, consider what happens if the monetary framework is one of an instrument rule, and i_t is set according to (16):

Lemma 6. *An instrument rule characterized by (16), may result in infinitely many non-explosive IRE.*
Proof. *See Appendix D.* ■

This lemma reflects once more that instrument rules that are "passive" may induce real equilibrium indeterminacy of IRE.[15] In order to address why there, in contrast, always is determinacy under a targeting rule (implying the same instrument path), it is again instructive first to look at the relationship between the nominal interest rate and the endogenous variables of the model. One will then — more clearer than under discretion — see that such relationships, which may appear in data, say little, if any, about the stability properties of the economy. The following lemma applies:

Lemma 7. *The targeting rule described by the problem (4) under commitment, results in infinitely many* equilibrium relations *between the nominal interest rate and the predetermined and endogenous variables:*

$$i_t = \sum_{i=0}^{\infty} \overline{b}_i E_t \pi_{t+i} - \sum_{i=0}^{\infty} \overline{a}_i E_t x_{t+i} + \overline{c}\varepsilon_t - d x_{t-1} + \frac{1}{\sigma}\mu_t, \tag{17}$$

where the only restrictions on the coefficients $\{\overline{a}_i\}_{i=0}^{\infty}$, $\left\{\overline{b}_i\right\}_{i=0}^{\infty}$, $\overline{c}$ and d are

$$\sum_{i=0}^{\infty} \overline{b}_i \frac{\lambda}{\kappa} (1-\chi)\chi^i - \sum_{i=0}^{\infty} \overline{a}_i \chi^{i+1} - d = -\frac{\chi}{\sigma}(1-\chi)\left(1 - \frac{\lambda\sigma}{\kappa}\right), \tag{18}$$

$$-\sum_{i=0}^{\infty} \overline{b}_i \frac{\lambda}{\kappa}\varphi \left[(1-\chi)\sum_{j=0}^{i-1}\chi^{i-j-1}\rho^j - \rho^i\right] + \varphi \sum_{i=0}^{\infty} \overline{a}_i \sum_{j=0}^{i-1} \chi^{i-j}\rho^j + \overline{c} = \frac{\varphi}{\sigma}(1-\chi-\rho)\left(1 - \frac{\lambda\sigma}{\kappa}\right). \tag{19}$$

[15] Although, as explained in the proof, determinacy may be possible for parameter configurations securing that the coefficient on the lagged output gap is not negative. If the coefficient is sufficiently positive, determinacy prevails as the interest rate increase in response to a higher lagged output gap, invokes a sufficiently contractionary policy stance such that self-fulfilling inflationary booms are ruled out.

Proof. *In order for the nominal interest rate to be consistent with the TRE values of current and expected future variables, it follows by (17) and Lemma 5 that the following relation must be satisfied:*

$$\begin{aligned}
&\sum_{i=0}^{\infty}\overline{b}_i E_t\pi_{t+i} - \sum_{i=0}^{\infty}\overline{a}_i E_t x_{t+i} + \overline{c}\varepsilon_t - dx_{t-1} \\
= \; & -\frac{\chi}{\sigma}(1-\chi)\left(1-\frac{\lambda\sigma}{\kappa}\right)x_{t-1} + \frac{\varphi}{\sigma}(1-\chi-\rho)\left(1-\frac{\lambda\sigma}{\kappa}\right)\varepsilon_t.
\end{aligned} \tag{20}$$

One can then successively forward the TRE expressions for inflation and the output gap, (14), in order to find

$$\begin{aligned}
E_t\pi_{t+i} &= \frac{\lambda}{\kappa}(1-\chi)\chi^i x_{t-1} - \frac{\lambda}{\kappa}\varphi\left((1-\chi)\sum_{j=0}^{i-1}\chi^{i-j-1}\rho^j - \rho^i\right)\varepsilon_t, \\
E_t x_{t+i} &= \chi^{i+1}x_{t-1} - \varphi\sum_{j=0}^{i-1}\chi^{i-j}\rho^j\varepsilon_t,
\end{aligned}$$

which inserted into (20) yields

$$\begin{aligned}
&\sum_{i=0}^{\infty}\overline{b}_i\left[\frac{\lambda}{\kappa}(1-\chi)\chi^i x_{t-1} - \frac{\lambda}{\kappa}\varphi\left((1-\chi)\sum_{j=0}^{i-1}\chi^{i-j-1}\rho^j - \rho^i\right)\varepsilon_t\right] \\
&-\sum_{i=0}^{\infty}\overline{a}_i\left[\chi^{i+1}x_{t-1} - \varphi\sum_{j=0}^{i-1}\chi^{i-j}\rho^j\varepsilon_t\right] + \overline{c}\varepsilon_t - dx_{t-1} \\
= \; & -\frac{\chi}{\sigma}(1-\chi)\left(1-\frac{\lambda\sigma}{\kappa}\right)x_{t-1} + \frac{\varphi}{\sigma}(1-\chi-\rho)\left(1-\frac{\lambda\sigma}{\kappa}\right)\varepsilon_t.
\end{aligned}$$

If this is to hold for every value of x_{t-1} and ε_t, it follows that the coefficients $\{\overline{a}_i\}_{i=0}^{\infty}$, $\left\{\overline{b}_i\right\}_{i=0}^{\infty}$, $\overline{c}$ and d must satisfy (18) and (19). ■

This lemma shows that, just as under discretion, the unique TRE nominal interest rate can be represented as infinitely many equilibrium relationships between the predetermined variables and current or expected leads of the endogenous variable. In contrast with discretion, the difference is that two restrictions on the relationships must be satisfied since there are two predetermined variables (apart from μ_t, which does not play a role; cf. above). In consequence, one can express the nominal interest rate as a unique function of *two* endogenous variables.

As the purpose of this exercise is to show that equilibrium relationships tell little about determinacy, consider the representation which most closely corresponds to (11); i.e., the often presented equilibrium representation under discretion. This would now be a case of $i_t = \bar{b}_1 \mathrm{E}_t \pi_{t+1} + \bar{a}_0 x_t + (1/\sigma)\, \mu_t$, and where $\bar{b}_1$ and $\bar{a}_0$ are identified by (18) and (19). In other words, a forward-looking Taylor rule where the output gap now matters. The following expression emerges by applying Lemma 7:

$$i_t = \left(1 - \frac{\kappa}{\lambda\sigma}\right) \mathrm{E}_t \pi_{t+1} + \frac{1}{\sigma}\mu_t. \tag{21}$$

Hence, $\bar{a}_0 = 0$, $\bar{b}_1 < 1$ and one cannot rule out that $\bar{b}_1 < 0$. That is, in equilibrium — in a determinate equilibrium — there may be a *negative* relationship between the nominal interest rate and inflation expectations, and it is *always* the case that the Taylor principle fails as also Clarida *et al.* (1999) emphasize [it thus follows trivially by Appendix B that an IRE under (21) is always indeterminate]. A reason is that under a history-dependent policy, a contraction today (say, towards a positive and temporary ε_t) is expected to be followed by a contractive stance in the future. This reduces inflation expectations. If equilibrium policy indeed is to be contractive in period t, the real interest rate must increase, so the nominal interest rate must either increase (in which case the correlation between i_t and $\mathrm{E}_t\pi_{t+1}$ is negative), or decrease less than inflation expectations do (in which case the correlation between i_t and $\mathrm{E}_t\pi_{t+1}$ is positive but less than one-for-one).[16]

Therefore, an equilibrium representation like (11) which *could* seem to explain determinacy under the discretionary targeting rule (as the Taylor principle was satisfied), clearly offers no clue about determinacy under a targeting rule with commitment policy.[17] Moreover, this example emphasizes further that observed relationships between the nominal interest rate and macroeconomic variables offer little insights about determinacy.

Like with the case of discretion, the intuition for determinacy with a targeting rule under commitment, is found by expressing the implied interest rate reaction function towards any expectations. The analogue of Lemma 4 is under the case of commitment given by

Lemma 8. *The targeting rule described by the problem (4) under commitment, results*

[16] Remark the reverse analogy when a deflationary shock occurs. One would then have that a commitment policy could be consistent with an increase in the nominal interest rate, as the future expected continued expansion leads to an even higher increase in the inflation expectations thus reducing the real interest rate. This way one can get an economy out of a deflationary situation while not violating the zero bound on nominal interest rates, as is indeed contained in Svensson's (2001a) proposal to save (open) economies from a liquidity trap.

[17] I have recently received Nakornthab (2000), which shows that the Taylor principle also fails in this type of model under price-level targeting and nominal income growth targeting; regimes considered by, respectively, Vestin (2000) and Jensen (2001). The analogy arises as these regimes offer ways of approaching the commitment solution even under discretionary policymaking. Nakornthab (2000) argues, with reference to this particular equilibrium relationship, that those regimes are vulnerable to indeterminacy. This is confirmed by Appendix B, *if* the equilibrium relationship is interpreted as an instrument rule.

in a unique *expression for the nominal interest rate, which holds for* any *values of the expected future inflation and output gap:*

$$i_t = \frac{\sigma(\lambda+\kappa^2)+\kappa\beta}{\sigma(\lambda+\kappa^2)}\pi^e_{t+1} + \frac{1}{\sigma}x^e_{t+1} + \frac{\kappa}{\sigma(\lambda+\kappa^2)}\varepsilon_t + \frac{1}{\sigma}\mu_t - \frac{1}{\sigma}\frac{\lambda}{\lambda+\kappa^2}x_{t-1}. \tag{22}$$

Proof. *Substitute (1) and (2) (with π^e_{t+1} replacing $\mathrm{E}_t\pi_{t+1}$ and x^e_{t+1} replacing $\mathrm{E}_t x_{t+1}$) into the optimality condition (15). As (1) and (2) hold for any π^e_{t+1} and x^e_{t+1}, it follows that the nominal interest rate resulting from the substitution will be uniquely given by (22) for any values of π^e_{t+1} and x^e_{t+1} when (15) applies; i.e., when the targeting rule is described by (4) under commitment.* ■

As evident this presents the nominal interest rate as an implied reaction function similar to the one under discretionary policymaking (12), with the only difference being the appearance of the lagged output gap. In terms of explaining the determinacy result, the arguments following Lemma 4 thus apply under commitment as well.

3. Estimated interest rate response functions under targeting rules

Clearly, the model used in the previous section is too simple to portray a realistic scenario for monetary policymaking. In this section, the model of Section 2.1 is therefore amended in a number of directions so as to further stress how estimated interest rate functions may not convey information about the economy's stability properties. First, it is assumed that the output gap is predetermined one period ahead, and inflation is predetermined two periods ahead. In consequence, monetary policy affects the output gap with a one-period lag, and inflation with a two-period lag. This seems to be broadly consistent with empirical findings, which show that the effects of monetary policy are first present in output (after some time) and then later in the inflation rate; see Walsh (1998, Chapter 1) and the references provided there. Secondly, I introduce endogenous persistence in both the output gap and inflation equations. Again, this is mainly empirically motivated; it is easy to reject empirically that output and inflation does not depend on their past values. See, e.g., Fuhrer (2000) or Rudebusch and Svensson (1999) on demand persistence in US data, and, e.g., Fuhrer and Moore (1995), Roberts (1997, 1998) or Galí and Gertler (1999) on inflation persistence in US data.

With these alterations, the equations for the output gap and inflation becomes, respectively,

$$x_{t+1} = \theta x_t + (1-\theta)\mathrm{E}_t x_{t+2} - \sigma(\mathrm{E}_t i_{t+1} - \mathrm{E}_t\pi_{t+2}) + g'_{t+1}, \quad 0 \leq \theta < 1, \tag{23}$$

$$\pi_{t+2} = \phi\pi_{t+1} + (1-\phi)\beta \mathrm{E}_t\pi_{t+3} + \kappa \mathrm{E}_t x_{t+2} + \varepsilon_{t+2}, \quad 0 \leq \phi < 1, \tag{24}$$

with

$$g'_{t+1} \equiv \theta y_t^n + g_{t+1} - y_{t+1}^n + (1-\theta)\,\mathrm{E}_t y_{t+2}^n,$$

and where $g_{t+1} = \gamma_g g_t + \xi_{t+1}^g$, $0 \leq \gamma_g < 1$, is a demand shock, and $y_{t+1}^n = \gamma_y y_t^n + \xi_{t+1}^y$, $0 \leq \gamma_y < 1$, is the stochastic (log of) natural rate (thus capturing technology shocks). The innovations ξ_{t+1}^g and ξ_{t+1}^y (and ξ_{t+1}) are i.i.d. and white noise. Note that the parameters θ and ϕ, respectively, quantifies the degree of endogenous persistence in demand and inflation. Furthermore, remark that the described transmission lags of monetary policy is achieved as demand in period $t+1$ is decided in period t (through the period t expectation of the real interest rate in period $t+1$), and prices in period $t+2$ are set in period t (through the period t expectation of the inflation rate in period $t+3$ and the expectation of the period $t+2$ output gap). An interest rate decision in period t thus cannot affect demand in period t, but only demand period $t+1$ (given, of course, that the decision has implications for the period $t+1$ interest rate) and thus inflation in period $t+2$.

In this more elaborate version, the model cannot be solved analytically. Instead, I follow standard practice and express the model in state-space form and adopt conventional numerical solution algorithms under discretionary and commitment policies, i.e., targeting rules. See, e.g., Backus and Driffill (1986), Currie and Levine (1993) and Svensson (1994, Appendix) on these methods. These algorithms generally perform very well, but for the current model, convergence (under discretion) requires that the control variable enters the loss function. Such convergence problems are also reported by Svensson (2000), who therefore adds a small loss of interest changes to the loss function. The loss function is therefore amended to

$$\widetilde{L} = \mathrm{E}_0 \sum_{t=1}^{\infty} \beta^{t-1} \left[\lambda x_t^2 + \pi_t^2 + \upsilon i_t^2 + \varsigma\,(i_t - i_{t-1})^2\right], \quad \upsilon \geq 0, \quad \varsigma \geq 0, \tag{25}$$

which allows for losses of either nominal interest rate variability *per se* or losses from interest rate changes. In state-space form, the model is then written as

$$\begin{bmatrix} \mathbf{X}_{t+1} \\ \mathrm{E}_t \boldsymbol{\chi}_{t+1} \end{bmatrix} = \mathbf{A} \begin{bmatrix} \mathbf{X}_t \\ \boldsymbol{\chi}_t \end{bmatrix} + \mathbf{B}^2 \mathrm{E}_t i_{t+2} + \mathbf{B}^1 \mathrm{E}_t i_{t+1} + \mathbf{B} i_t + \begin{bmatrix} \boldsymbol{\xi}_{t+1} \\ \mathbf{0}_{3\times 1} \end{bmatrix}, \tag{26}$$

where $\mathbf{A}$ is a 10×10 matrix, $\mathbf{B}^2$, $\mathbf{B}^1$, $\mathbf{B}$ are 10×1 vectors, and $\mathbf{X}_t$ is the column vector of the seven predetermined variables

$$\mathbf{X}_t \equiv [\widetilde{g}_t \quad y_t^n \quad \varepsilon_t \quad x_t \quad \pi_t \quad \mathrm{E}_t\pi_{t+1} \quad i_{t-1}]',$$

$\boldsymbol{\chi}_t$ is the column vector of the three forward-looking variables

$$\boldsymbol{\chi}_t \equiv [\mathrm{E}_t x_{t+1} \quad \mathrm{E}_t x_{t+2} \quad \mathrm{E}_t\pi_{t+2}]',$$

and

$$\boldsymbol{\xi}_{t+1} = \left[\xi_{t+1}^g \quad \xi_{t+1}^y \quad \xi_{t+1} \quad \xi_{t+1}^g - \xi_{t+1}^y \quad \xi_{t+1} \quad (\phi+\rho)\,\xi_{t+1} \quad 0\right]',$$

is the column vector of innovations. Details on the solution procedures of the model (and matrix $\mathbf{A}$ and vectors $\mathbf{B}^2$, $\mathbf{B}^1$, and $\mathbf{B}$), are available in a supplementary appendix upon request.[18]

The model is, as mentioned, solved under the assumption of either discretionary or commitment policymaking. From each case, one recovers expressions for the dynamic evolution of the economy of the form

$$\begin{bmatrix} \mathbf{X}_{t+1} \\ \mathrm{E}_t\boldsymbol{\chi}_{t+1} \end{bmatrix} = \boldsymbol{\Omega} \begin{bmatrix} \mathbf{X}_t \\ \boldsymbol{\chi}_t \end{bmatrix} + \begin{bmatrix} \boldsymbol{\xi}_{t+1} \\ \mathbf{0}_{3\times 1} \end{bmatrix}, \quad t > 1, \tag{27}$$

with $\mathbf{X}_1$ given and $\boldsymbol{\chi}_1 = \mathbf{H}\mathbf{X}_1$, where $\boldsymbol{\Omega}$ and $\mathbf{H}$ are matrices found by the solution algorithms (differing, of course, between discretion and commitment). It is important to note that, by implication of the solution algorithms, (27) represents a case where *only* fundamentals matter for the dynamics of the economy (endogenous variables are a function of predetermined variables only). Hence, the TRE under discretion and commitment are determinate by construction. Under both forms of policy, the model is then used to provide 600 periods of data by applying (27) in a stochastic simulation. From these simulations, time series for i_t, x_t, $\mathrm{E}_t\pi_{t+1}$, $\mathrm{E}_t\pi_{t+2}$ are extracted and used for subsequent estimations of interest rate response functions.

As these estimations merely are meant to serve as rough illustrations of what can, and what cannot, be concluded from these, the model as given by (23)-(24) has not been calibrated so as to match business cycle properties accurately. The model is interpreted as quarterly, with inflation and the nominal interest rate measured at annual rates. The adopted parameter values have been chosen so as not to be grossly inconsistent with

[18] This appendix also shows how to amend the solution algorithms in order to handle the presence of the expected future controls in (25); this procedure follows Svensson (2000) closely.

$\theta = 0.5,$	$\phi = 0.3,$	$\sigma = 0.25,$	$\kappa = 0.1,$
$\lambda = 0.5,$	$\upsilon = 0.05,$	$\varsigma = 0$	$\beta = 0.99,$
$\sigma_g = 0.02,$	$\sigma_y = 0.007,$	$\sigma_\varepsilon = 0.02,$	
$\gamma_g = 0.25,$	$\gamma_y = 0.98,$	$\rho = 0.00.$	

Table 1: Parameter configuration for simulations

existing estimates of demand and inflation equations,[19] and so as not to give unreasonable unconditional standard deviations of the output gap and inflation under either form of policy. The parameter values, which to a large extent are similar to those in Jensen (2001), are summarized in Table 1, where σ_g, σ_y, and σ_ε, respectively, represent the standard deviations of the innovations ξ^g_{t+1}, ξ^y_{t+1}, and ξ_{t+1} (under this parameterization, the standard deviations of inflation and the output gap are between 2 and 2.5 percent). Note that concerning the loss function, convergence of the solution algorithm is secured by putting a small weight on interest rate variability *per se*, while *not* attaching any loss to interest rate changes. If one does this, i.e., sets $\varsigma > 0$, the result is an implausible high degree of interest rate smoothing under discretion even for values of ς around 0.005.[20]

The estimated interest rate equations are of the general form

$$i_t = b_0\pi_t + b_1\mathrm{E}_t\pi_{t+1} + b_2\mathrm{E}_t\pi_{t+2} + a_0x_t + \alpha i_{t-1},$$

but each estimation only uses a subset of the right-hand side variables.[21] Table 2 shows estimation results on data when the central bank is optimizing under discretion. It is seen that a simple Taylor-type rule of the form $i_t = b_0\pi_t + a_0x_t$ is identified in the data, although the central bank is not adhering to one. Also, note that the table shows that the estimated coefficients are such that an associated IRE would be indeterminate. Hence, if one infers that the central bank has been following a mechanical Taylor rule, one would (knowing the underlying economic structure) conclude that the economy is vulnerable to sunspot fluctuations. Nevertheless, the economy has actually been in a determinate TRE in 600 periods. Estimating a Taylor rule depending upon the one-period ahead inflation

[19] Note that in the empirical literature there is in particular disagreement about the degree of persistence in the inflation adjustment equation.

[20] In this case, the solution for the interest rate, when expressed as a function of the predetermined variables, exhibits an equilibrium response coefficient of i_t towards i_{t-1} of around 0.3. This is surprisingly high given that interest rate changes give only a negligible loss.

[21] All estimations presented here are simple OLS regressions. In all the cases not involving $\mathrm{E}_t\pi_{t+2}$ the right-hand side variables are predetermined and independent. OLS is problematic when estimations involve $\mathrm{E}_t\pi_{t+2}$, which is endogenous. Future work will address this problem.

Variables in estimated interest rate functions, $i_t = ...$[a]					Stability of associated IRE[b]
π_t	$E_t\pi_{t+1}$	$E_t\pi_{t+2}$	x_t	i_{t-1}	
0.26*	—	—	2.23*	—	I
0.28*	—	—	2.34*	-0.10*	I
—	0.63*	—	2.20*	—	I
—	0.89*	—	2.34*	-0.13*	I
—	—	1.81*	2.06*	—	D
—	—	2.24*	2.17*	-0.12*	D

[a] "*" denotes significance at the 5% level
[b] "I" denotes indeterminacy of the IRE under the interest rate rule; "D" denotes determinacy

Table 2: Coefficients in estimated interest rate functions. Discretion data

Variables in estimated interest rate functions, $i_t = ...$[a]					Stability of associated IRE[b]
π_t	$E_t\pi_{t+1}$	$E_t\pi_{t+2}$	x_t	i_{t-1}	
-0.00	—	—	0.79*	—	I
0.05	—	—	0.70*	0.42*	I
—	-0.45*	—	0.79*	—	I
—	-0.00	—	0.71*	0.41*	I
—	—	-3.52*	0.69*	—	I
—	—	-1.92*	0.67*	0.33*	I

[a] "*" denotes significance at the 5% level
[b] "I" denotes indeterminacy of the IRE under the interest rate rule; "D" denotes determinacy

Table 3: Coefficients in estimated interest rate functions. Commitment data

expectations gives the same result, although the estimated coefficients are "closer" to render the associated IRE determinate. In the case of a Taylor rule depending upon the two-period ahead inflation expectations (the time horizon at which policy can affect inflation), then one gets estimates which suggest that policy has been conducted according to an "active" Taylor rule (even though it has not).[22]

Turning to the case where policy is conducted under commitment, Table 3 shows estimation results which never portray a Taylor-type rule behavior. In fact, the signs on inflation measures are often negative (consistent with what was shown to be possible in Subsection 2.4 for the simple model). The associated IRE are always indeterminate, and an economist believing that policy has been conducted within an instrument rule framework would in all likelihood call for a revision in policymaking advising that the central bank should respond more aggressively towards inflation. This would be unwarranted,

[22] In all cases, the lagged interest rate is significant but of small magnitude.

as the economy is in the optimal, and determinate, equilibrium.[23] This illustrates that identification of interest rate relationships in data reflecting a behavior not in conformity with the Taylor principle, need not be a reflection of disastrous monetary policymaking. In fact, it might as well reflect the performance of a central bank acting optimal and with an ability to commit.

4. Concluding remarks

In this paper, I have investigated the issue of real equilibrium determinacy within a simple model of the "New Neo-Classical Synthesis" style. It turns out that when the central bank is adhering to a targeting rule of the kind associated with minimization of the social loss function, there is always real determinacy. This may, as is well known, not be the case if the bank is adhering to an instrument rule, where restrictions on the response coefficients are needed.

Moreover, the analysis emphasized that the equilibrium relationships between the nominal interest rate and macroeconomic variables arising under a targeting rule reveal little about the economy's stability properties.

Appendix

A. Proof of Lemma 2

When $i_t = \{[\kappa(1-\rho)+\sigma\lambda\rho] / (\sigma[\kappa^2+\lambda(1-\beta\rho)])\}\,\varepsilon_t + (1/\sigma)\,\mu_t$, the system (1)-(2) can be written in matrix form as

$$\begin{bmatrix} \varepsilon_{t+1} \\ \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 \\ \sigma\beta^{-1} + \dfrac{\kappa(1-\rho)+\sigma\lambda\rho}{\kappa^2+\lambda(1-\beta\rho)} & 1+\sigma\beta^{-1}\kappa & -\sigma\beta^{-1} \\ -\beta^{-1} & -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ 0 \\ 0 \end{bmatrix}.$$

The real and positive roots of the system are ρ and

$$\frac{1+\beta\left(1+\sigma\beta^{-1}\kappa\right) \pm \sqrt{\left(1+\beta^{-1}\left(1+\sigma\beta^{-1}\kappa\right)\right)^2 - 4\beta}}{2\beta}.$$

[23] Note that the coefficient on the lagged interest rate is now of considerably magnitude; not because the central bank has an objective to "smooth" interest rates *per se*, but because of the history dependent policy it follows under commitment.

Tedious algebra shows that one root is greater than 1, and two are below 1. As there are two jump variables, this means there is one unstable root too few in order to ensure determinacy; cf. Blanchard and Kahn (1980).

B. IRE and determinacy when $i_t = b_1 \mathbf{E}_t \pi_{t+1} + (1/\sigma)\,\mu_t$

Using $i_t = b_1 \mathrm{E}_t \pi_{t+1} + (1/\sigma)\,\mu_t$, the system (1)-(2) can be written in matrix form as

$$\begin{bmatrix} \varepsilon_{t+1} \\ \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 \\ \sigma\beta^{-1} & 1 - \sigma\beta^{-1}\kappa\,(b_1 - 1) & -\sigma\beta^{-1}\,(b_1 - 1) \\ -\beta^{-1} & -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ 0 \\ 0 \end{bmatrix}.$$

The roots of the system are ρ and

$$\nu = \frac{1 + \beta - \sigma\kappa\,(b_1 - 1) \pm \sqrt{(1 + \beta - \sigma\kappa\,(b_1 - 1))^2 - 4\beta}}{2\beta}. \tag{B.1}$$

As $0 < \rho < 1$ and there are two jump variables, both values of ν as given by (B.1) must have modulus greater than one to ensure determinacy of the IRE. In the case the roots are real, a sufficient condition is that the lower value of (B.1) is numerically greater than 1. This is equivalent to the condition

$$\beta - 1 + \sqrt{(1 + \beta - \sigma\kappa\,(b_1 - 1))^2 - 4\beta} < 0,$$

which is satisfied whenever

$$0 < b_1 - 1 < \frac{2\,(1+\beta)}{\sigma\kappa}. \tag{B.2}$$

One can show that whenever (B.2) fails, the roots as given by (B.1) are real. Moreover, one can show that whenever the roots are complex, then they have modulus strictly greater than 1. Hence, (B.2) is a necessary and sufficient condition on b_1 for determinacy of the IRE.

C. IRE and indeterminacy when $i_t = -a_1 \mathbf{E}_t x_{t+1} + (1/\sigma)\,\mu_t$

To prove the assertion made in the main text that an IRE under $i_t = -\left[\lambda/\kappa + (1-\rho)\,/\,(\sigma\rho)\right]$ $\mathrm{E}_t x_{t+1} + (1/\sigma)\,\mu_t$ is always indeterminate, it obviously suffices to prove it for any $i_t = -a_1 \mathrm{E}_t x_{t+1} + (1/\sigma)\,\mu_t$ with $a_1 > 0$. In that case, the system (1)-(2) can be written in matrix form as

$$\begin{bmatrix} \varepsilon_{t+1} \\ \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 \\ \sigma\beta^{-1} & \frac{1+\sigma\beta^{-1}\kappa}{1+\sigma a_1} & -\frac{\sigma\beta^{-1}}{1+\sigma a_1} \\ -\beta^{-1} & -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ 0 \\ 0 \end{bmatrix}.$$

The real and positive roots of the system are ρ and

$$\frac{1+\beta+\sigma(\kappa+a_1) \pm \sqrt{(1+\beta+\sigma(\kappa+a_1))^2 - 4\beta(1+\sigma b)}}{2\beta(1+\sigma b)}.$$

Tedious algebra shows that one root is greater than 1, and two are below 1. As there are two jump variables, this means there is one unstable root too few in order to ensure determinacy.

D. Proof of Lemma 6

When i_t is given by (22), the system the system (1)-(2) can be written in matrix form as

$$\begin{bmatrix} \varepsilon_{t+1} \\ x_t \\ \mathrm{E}_t x_{t+1} \\ \mathrm{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sigma\left(\overline{c}+\beta^{-1}\right) & \sigma d & 1+\sigma\beta^{-1}\kappa & -\sigma\beta^{-1} \\ -\beta^{-1} & 0 & -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ x_{t-1} \\ x_t \\ \pi_t \end{bmatrix} + \begin{bmatrix} \xi_{t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

where

$$\begin{aligned} \overline{c} &\equiv \frac{\varphi}{\sigma}(1-\chi-\rho)\left(1-\frac{\lambda\sigma}{\kappa}\right), \\ d &\equiv -\frac{\chi}{\sigma}(1-\chi)\left(1-\frac{\lambda\sigma}{\kappa}\right). \end{aligned}$$

The associated IRE is indeterminate if the matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ \sigma d & 1+\sigma\beta^{-1}\kappa & -\sigma\beta^{-1} \\ 0 & -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix}$$

has more than one characteristic root with modulus less than one. This is not generally the case, but note that when d is zero, the system becomes the one examined in Appendix A, and the IRE is indeterminate. By continuity, the IRE is indeterminate when the parameters imply a value of d in the neighborhood of zero. Numerical investigations, however, demonstrate that indeterminacy of the IRE prevails for a wide range of plausible parameter values. Keeping $\beta = 0.99$ fixed and varying σ from 0.1 to 5 with a grid of 0.1, varying κ from 0.1 to 1.0 with a grid of 0.1, and varying λ from 0.025 to 2.0 with a grid of 0.025, yield $76,000$ parameter combinations. Of these, 88.45% rendered the IRE indeterminate. It is noteworthy that in the remaining cases where the IRE was determinate, $d > 0$ is always the case.[24] This reflects that parameter configurations implying $d < 0$ can never imply determinacy, because the Taylor principle (in its general form stating that expansionary pressures in the economy either in form of inflation or a high output gap will be met by a sufficient increase in the nominal interest rate) is always violated when a higher output gap in last period is met by a decrease in the nominal interest rate.

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[24] Thus, a necessary (but not sufficient) condition for determinacy is $\lambda\sigma/\kappa > 1$.

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