# Optimal Degrees of Transparency in Monetary Policymaking

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## Appendices not for publication

### C. Solution for an arbitrary terminal condition in case of full information

Instead of assuming (9), assume that $\mathrm{E}\left[\pi_3 | I_2^P\right] = T$. In period 2 the central bank then solves $\min_{x_2} \left[\lambda \left(x_2 - x_1^*\right)^2 + \pi_2^2\right]$ subject to (2) and $\mathrm{E}\left[\pi_3 | I_2^P\right] = T$. The relevant first-order condition is $\lambda \left(x_2 - x_1^*\right) + \kappa \left(T + \kappa x_2\right) = 0$ from which the period 2 solution emerges as

$$x_2 = \frac{\lambda x_1^* - \kappa T}{\lambda + \kappa^2}, \quad \pi_2 = \frac{\lambda \left(\kappa x_1^* + T\right)}{\lambda + \kappa^2}. \tag{C.1}$$

In finding the solution for period 1, assume that $\theta = \eta = 0$, and thus $x_1^* = x_0^*$. This is inconsequential, as the main issue is to demonstrate when a value of $x_0^*$ lower than $x^*$ is beneficial. Allowing for a $\theta$-shock only adds an independent source of fluctuations. Likewise, the way the control error affects the outcomes is orthogonal to the issue at hand. Hence, period 1 equilibrium is found by solving $\min_{x_1} \left[\lambda \left(x_1 - x_0^*\right)^2 + \pi_1^2\right]$ subject to (1) and $\mathrm{E}\left[\pi_2 | I_1^P\right] = \lambda \left(\kappa x_0^* + T\right) / \left(\lambda + \kappa^2\right)$. The first-order condition is $\lambda \left(x_1 - x_0^*\right) + \kappa[\lambda \left(\kappa x_0^* + T\right) / \left(\lambda + \kappa^2\right) + \kappa x_1 + \varepsilon] = 0$, implying

$$x_1 = \frac{\lambda \left(\lambda x_0^* - \kappa T\right)}{\left(\lambda + \kappa^2\right)^2} - \frac{\kappa}{\lambda + \kappa^2}\varepsilon, \quad \pi_1 = \frac{\lambda \left[\kappa \left[2\lambda + \kappa^2\right] x_0^* + \lambda T\right]}{\left(\lambda + \kappa^2\right)^2} + \frac{\lambda}{\lambda + \kappa^2}\varepsilon. \tag{C.2}$$

As the response to the inflation shock is invariant to $x_0^*$, we can ignore it when assessing the set of desirable values of $x_0^*$. Society's loss from a policy leading to (C.1)-(C.2) is then simply given by

$$\begin{aligned} L^S &= \lambda\left(\frac{\lambda\left(\lambda x_0^* - \kappa T\right)}{\left(\lambda+\kappa^2\right)^2} - x^*\right)^2 + \left(\frac{\lambda\left[\kappa\left[2\lambda+\kappa^2\right]x_0^* + \lambda T\right]}{\left(\lambda+\kappa^2\right)^2}\right)^2 \\ &+\lambda\left(\frac{\lambda x_0^* - \kappa T}{\lambda+\kappa^2} - x^*\right)^2 + \left(\frac{\lambda\left(\kappa x_0^* + T\right)}{\lambda+\kappa^2}\right)^2. \end{aligned}$$

One then finds that

$$\begin{aligned} \frac{\partial L^S}{\partial x_0^*} &= 2\lambda\left(\frac{\lambda\left(\lambda x_0^* - \kappa T\right)}{\left(\lambda+\kappa^2\right)^2} - x^*\right)\frac{\lambda^2}{\left(\lambda+\kappa^2\right)^2} + 2\left(\frac{\lambda\left[\kappa\left[2\lambda+\kappa^2\right]x_0^* + \lambda T\right]}{\left(\lambda+\kappa^2\right)^2}\right)\frac{\lambda\kappa\left[2\lambda+\kappa^2\right]}{\left(\lambda+\kappa^2\right)^2} \\ &+2\lambda\left(\frac{\lambda x_0^* - \kappa T}{\lambda+\kappa^2} - x^*\right)\frac{\lambda}{\lambda+\kappa^2} + 2\left(\frac{\lambda\left(\kappa x_0^* + T\right)}{\lambda+\kappa^2}\right)\frac{\lambda\kappa}{\lambda+\kappa^2}, \end{aligned} \tag{C.3}$$

and as $\partial^2 L^S/\partial x_0^{*2} > 0$ it follows that $\partial L^S/\partial x_0^* = 0$ characterizes the value of $x_0^*$ securing the lowest social loss. We find that this must satisfy

$$\begin{aligned} &\lambda\left(\frac{\lambda\left(\lambda x_0^* - \kappa T\right)}{\left(\lambda+\kappa^2\right)^2} - x^*\right)\frac{\lambda}{\lambda+\kappa^2} + \left(\frac{\lambda\left[\kappa\left[2\lambda+\kappa^2\right]x_0^* + \lambda T\right]}{\left(\lambda+\kappa^2\right)^2}\right)\frac{\kappa\left[2\lambda+\kappa^2\right]}{\lambda+\kappa^2} \\ &+\lambda\left(\frac{\lambda x_0^* - \kappa T}{\lambda+\kappa^2} - x^*\right) + \frac{\lambda\kappa\left(\kappa x_0^* + T\right)}{\lambda+\kappa^2} \\ =&\ 0. \end{aligned}$$

If the left hand side evaluated at $x_0^* = x^*$ is positive, then it follows that a value $x_0^* < x^*$ is preferable to society. This is the case if

$$\begin{aligned} &\lambda\left(\frac{\lambda\left(\lambda x^* - \kappa T\right)}{\left(\lambda+\kappa^2\right)^2} - x^*\right)\frac{\lambda}{\lambda+\kappa^2} + \left(\frac{\lambda\left[\kappa\left[2\lambda+\kappa^2\right]x^* + \lambda T\right]}{\left(\lambda+\kappa^2\right)^2}\right)\frac{\kappa\left[2\lambda+\kappa^2\right]}{\lambda+\kappa^2} \\ &+\lambda\left(\frac{\lambda x^* - \kappa T}{\lambda+\kappa^2} - x^*\right) + \frac{\lambda\kappa\left(\kappa x^* + T\right)}{\lambda+\kappa^2} \\ >&\ 0, \end{aligned}$$

which after tedious manipulation reduces to

$$x^*\kappa\left(2\lambda+\kappa^2\right) + T\lambda > 0.$$

This always holds for $T \geq 0$.

## D. Numerical simulations: deviations from Table 2's baseline

The following tables report the optimal degrees of transparency for 6 deviations from the baseline parameter configuration presented in Table 2. The deviations are the following (where the other parameters are kept at their baseline value). Higher and lower relative concern for output fluctuations (Tables A1 and A2, respectively). Higher and lower elasticity of inflation with respect to output (Tables A3 and A4, respectively). Higher and lower variance of the control error (Tables A5 and A6, respectively). For a discussion of these sensitivity analyses, see Section 3 of the main text.

Table A1: Optimal degrees of transparency
Deviation from baseline: Higher relative concern for output fluctuations

| $\sigma_\theta^2 = 0.05$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.000 | 0.842 | 0.900 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.827 | 0.000 | 0.838 | 0.899 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.485 | 0.000 | 0.834 | 0.898 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.002 | 0.000 | 0.827 | 0.896 |

| $\sigma_\theta^2 = 0.50$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.000 | 0.000 | 0.074 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.980 | 0.000 | 0.000 | 0.063 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.906 | 0.000 | 0.000 | 0.049 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.662 | 0.000 | 0.000 | 0.037 |

| $\sigma_\theta^2 = 1.00$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.995 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.957 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.812 | 0.000 | 0.000 | 0.000 |

The optimal values of $\tau$ for the case of $\lambda = 1.50$, $x^* = 1.00$, $\kappa = 2/3$ and $\sigma_\eta^2 = 1.00$.

Table A2: Optimal degrees of transparency
Deviation from baseline: Lower relative concern for output fluctuations

| $\sigma_\theta^2 = 0.05$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.601 | 0.979 | 0.989 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.924 | 0.489 | 0.977 | 0.988 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.819 | 0.364 | 0.974 | 0.988 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.601 | 0.141 | 0.968 | 0.986 |

| $\sigma_\theta^2 = 0.50$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.796 | 0.866 | 0.909 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.983 | 0.710 | 0.850 | 0.902 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.936 | 0.628 | 0.828 | 0.895 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.796 | 0.441 | 0.785 | 0.881 |

| $\sigma_\theta^2 = 1.00$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.871 | 0.821 | 0.848 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.996 | 0.807 | 0.790 | 0.835 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.966 | 0.737 | 0.761 | 0.824 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.871 | 0.593 | 0.706 | 0.797 |

The optimal values of $\tau$ for the case of $\lambda = 0.50$, $x^* = 1.00$, $\kappa = 2/3$ and $\sigma_\eta^2 = 1.00$.

Table A3: Optimal degrees of transparency

Deviation from baseline: Higher elasticity of inflation w.r.t. output

| $\sigma_\theta^2 = 0.05$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.708 | 0.985 | 0.992 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.980 | 0.668 | 0.983 | 0.992 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.947 | 0.623 | 0.982 | 0.992 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.870 | 0.534 | 0.980 | 0.991 |

| $\sigma_\theta^2 = 0.50$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.855 | 0.905 | 0.934 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.999 | 0.825 | 0.896 | 0.932 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.989 | 0.800 | 0.888 | 0.929 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.954 | 0.732 | 0.874 | 0.923 |

| $\sigma_\theta^2 = 1.00$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.908 | 0.872 | 0.890 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.999 | 0.885 | 0.863 | 0.886 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.998 | 0.863 | 0.851 | 0.883 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.978 | 0.819 | 0.830 | 0.873 |

The optimal values of $\tau$ for the case of $\lambda = 1.00$, $x^* = 1.00$, $\kappa = 1.00$ and $\sigma_\eta^2 = 1.00$.

Table A4: Optimal degrees of transparency

Deviation from baseline: Lower elasticity of inflation w.r.t. output

| $\sigma_\theta^2 = 0.05$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.000 | 0.354 | 0.554 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.000 | 0.000 | 0.345 | 0.551 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.000 | 0.000 | 0.334 | 0.548 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.000 | 0.000 | 0.318 | 0.542 |

| $\sigma_\theta^2 = 0.50$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.758 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.007 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.000 | 0.000 | 0.000 | 0.000 |

| $\sigma_\theta^2 = 1.00$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.885 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.521 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.001 | 0.000 | 0.000 | 0.000 |

The optimal values of $\tau$ for the case of $\lambda = 1.00$, $x^* = 1.00$, $\kappa = 1/3$ and $\sigma_\eta^2 = 1.00$.

TABLE A5: OPTIMAL DEGREES OF TRANSPARENCY
DEVIATION FROM BASELINE: HIGHER VARIANCE OF THE CONTROL ERROR

| $\sigma_\theta^2 = 0.05$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.001 | 0.948 | 0.969 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.912 | 0.001 | 0.945 | 0.968 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.768 | 0.001 | 0.943 | 0.967 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.454 | 0.001 | 0.938 | 0.966 |

| $\sigma_\theta^2 = 0.50$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.181 | 0.603 | 0.719 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.986 | 0.053 | 0.583 | 0.712 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.941 | 0.000 | 0.563 | 0.705 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.794 | 0.000 | 0.526 | 0.692 |

| $\sigma_\theta^2 = 1.00$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.449 | 0.391 | 0.499 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.996 | 0.353 | 0.358 | 0.485 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.972 | 0.252 | 0.337 | 0.477 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.881 | 0.027 | 0.270 | 0.449 |

The optimal values of $\tau$ for the case of $\lambda = 1.00$, $x^* = 1.00$, $\kappa = 2/3$ and $\sigma_\eta^2 = 1.50$.

Table A6: Optimal degrees of transparency

Deviation from baseline: Lower Variance of the Control Error

| $\sigma_\theta^2 = 0.05$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.000 | 0.842 | 0.906 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.736 | 0.000 | 0.836 | 0.904 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.304 | 0.000 | 0.828 | 0.902 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.000 | 0.000 | 0.813 | 0.897 |

| $\sigma_\theta^2 = 0.50$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.000 | 0.000 | 0.156 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.956 | 0.000 | 0.000 | 0.134 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.818 | 0.000 | 0.000 | 0.115 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.382 | 0.000 | 0.000 | 0.076 |

| $\sigma_\theta^2 = 1.00$ | $x_0^* = 0.00$ | $x_0^* = 1.00$ | $x_0^* = 2.00$ | $x_0^* = 3.00$ |
|---|---|---|---|---|
| $\sigma_\varepsilon^2 = 0.00$ | 0.999 | 0.001 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 0.50$ | 0.990 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 1.00$ | 0.912 | 0.000 | 0.000 | 0.000 |
| $\sigma_\varepsilon^2 = 2.00$ | 0.633 | 0.000 | 0.000 | 0.000 |

The optimal values of $\tau$ for the case of $\lambda = 1.00$, $x^* = 1.00$, $\kappa = 2/3$ and $\sigma_\eta^2 = 0.50$.