Supplementary Appendix

to

Structural Convergence under Reversible and Irreversible Monetary Unification

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D. Benefits of unification through efficient shock stabilization

In this appendix we present a model where the benefits from monetary unification arise from efficiency gains in terms of common shock stabilization. The output schedules, (1), are replaced with

$$\begin{aligned} y_t^H &= \pi_t^H - \mathrm{E}_{t-1}\left[\pi_t^H\right] - \alpha\left(\pi_t^L - \mathrm{E}_{t-1}\left[\pi_t^L\right]\right) - \overline{y}_t - \epsilon_t, \quad 0 < \alpha < 1, \\ y_t^L &= \pi_t^L - \mathrm{E}_{t-1}\left[\pi_t^L\right] - \alpha\left(\pi_t^H - \mathrm{E}_{t-1}\left[\pi_t^H\right]\right) - \epsilon_t, \end{aligned} \qquad \text{(D.1)}$$

where ϵ_t is an i.i.d. supply shock with $\mathrm{E}_{t-1}[\epsilon_t] = 0$ and $\mathrm{E}_{t-1}[\epsilon_t^2] = \sigma_\epsilon^2$. There are international spillovers, because an unexpected increase in inflation in country i lowers i's real wage rate relative to j's real wage ($j \neq i$) and, hence, causes a diversion of economic activity from j ($j \neq i$) to i (cf. Martin, 1995). These spillovers are ignored in the main text, as they would, qualitatively speaking, have no effect on the results. They are important, however, in the presence of supply shocks, as will become clear. Both countries are hit by a common supply shock ϵ_t. We could also allow for country-specific shocks, but the results would be qualitatively unaffected if their variance is not too large compared with the variance of ϵ_t. As in the main text, we assume that L and H are not two arbitrary countries, but two countries that could, in principle, form a monetary union were it not for differences in structural distortions.

The utilities of GL and GH are now given by the expectations of (4) and (5), respectively, conditional on period $t-1$ information and with $b = 0$, while the utilities of the central banks are now given by the expectation of the right-hand side of (6), conditional on period $t-1$ information.

The outcomes for inflation and output under monetary policy independence are

$$\begin{aligned} \pi_t^H &= \psi \overline{y}_t + \mu^I \epsilon_t, & \pi_t^L &= \mu^I \epsilon_t, \\ y_t^H &= -\overline{y}_t - \left[1 - (1-\alpha)\mu^I\right]\epsilon_t, & y_t^L &= -\left[1 - (1-\alpha)\mu^I\right]\epsilon_t, \end{aligned}$$

where $\mu^I \equiv \psi \left[1 + \psi (1+\alpha)\right] / \Lambda$, $\Lambda \equiv (1+\psi)^2 - \psi^2\alpha^2 > 0$, and GL's and GH's per-period pay-offs consequently become, respectively,

$$u^{GL,I} = -\Omega^I \sigma_\epsilon^2, \qquad u_t^{GH,I}(\overline{y}_t) = -\psi(1+\psi)(\overline{y}_t)^2 - \Omega^I\sigma_\epsilon^2 - \delta(\overline{y}_t - \varphi_t)^2,$$

with $\Omega^I \equiv \psi(1+\psi)\left[1+\psi(1+\alpha)\right]^2/\Lambda^2 > 0$.

Under monetary unification, the outcomes for inflation and output are

$$\begin{aligned} \pi_t &= \frac{\psi\lambda(1-\alpha)}{1+\lambda}\overline{y}_t + \mu^U\epsilon_t, \\ y_t^H &= -\overline{y}_t - \left[1-(1-\alpha)\mu^U\right]\epsilon_t, \qquad y_t^L = -\left[1-(1-\alpha)\mu^U\right]\epsilon_t, \end{aligned}$$

where $\mu^U \equiv \psi(1-\alpha)/\left[1+\psi(1-\alpha)^2\right]$, leading to the following per-period pay-offs:

$$\begin{aligned} u^{GL,U}(\overline{y}_t) &= -\left[\frac{\psi\lambda(1-\alpha)}{1+\lambda}\overline{y}_t\right]^2 - \Omega^U\sigma_\epsilon^2, \\ u_t^{GH,U}(\overline{y}_t) &= -\psi\left[1+\frac{\psi\lambda^2(1-\alpha)^2}{(1+\lambda)^2}\right](\overline{y}_t)^2 - \Omega^U\sigma_\epsilon^2 - \delta(\overline{y}_t-\varphi_t)^2, \end{aligned}$$

with $\Omega^U \equiv \psi/\left(1+\psi(1-\alpha)^2\right)$. Under independent monetary policymaking, each of the central banks neglects the negative externality on the other country of its own response to the common shock. As a result, the central banks' responses to disturbances are too active. These externalities, however, are internalized under monetary unification, thereby leading to an efficient trade-off between output and inflation variability. More specifically, it is easy to verify that $\Omega^U < \Omega^I$, which demonstrates that the utility loss from supply shock variability is smallest under monetary unification.

E. Proof of Proposition 2

First, suppose that for all $t < T^{ru}$, $\overline{y}_t = \overline{y}_t^I$. Then, because $\overline{y}_{T^{ru}-1}^I \geq \overline{y}^*$ and because $\overline{y}_t^I$ is decreasing over time, it is suboptimal for GL to admit country H to join the union if $t < T^{ru}$. Now, for any $t < T^{ru}$, consider GH's incentives to deviate from $\overline{y}_t = \overline{y}_t^I$. A deviation $\overline{y}_t^d > \overline{y}_t^I$ is suboptimal as it will not induce a different monetary regime, while under monetary independence, $\overline{y}_t^I$ is optimal, cf. (14). A deviation $\overline{y}^* < \overline{y}_t^d < \overline{y}_t^I$ is ruled out by a similar argument. Finally, consider a deviation $\overline{y}_t^d \leq \overline{y}^*$. Although this induces monetary unification, it is suboptimal because $u_t^{GH,I}\left(\overline{y}_t^I\right) > u_t^{GH,U}\left(\overline{y}^*\right)$ [by (18)] and $u_t^{GH,U}\left(\overline{y}^*\right) \geq u_t^{GH,U}\left(\overline{y}_t^d\right)$. The latter follows because $u_t^{GH,U}$ is strictly concave with a unique maximum at $\overline{y}_t^U$, $\overline{y}_t^U > \overline{y}_t^I \geq \overline{y}^*$, for all $t < T^{ru}$.

F. Proof that $W_{I,\widetilde{T}-j+1}^{GL} \geq W_{U,\widetilde{T}-j+1}^{GL}$

If $I_{\widetilde{T}-j}$, the equilibrium outcome for $\overline{y}_{\widetilde{T}-j+1}$ equals $\overline{y}_{\widetilde{T}-j+1}^*$ or $\overline{y}_{\widetilde{T}-j+1}^I$. In either case, it is lower than the equilibrium outcome $\overline{y}_{\widetilde{T}-j+1}^U$ if $U_{\widetilde{T}-j}$. Hence, under $I_{\widetilde{T}-j}$, GL's utility in period $\widetilde{T}-j+1$ is at least as high as under $U_{\widetilde{T}-j}$. Further, GL always has the *option* to invite H to form a union on the basis of the outcome for $\overline{y}_{\widetilde{T}-j+1}$ if $I_{\widetilde{T}-j}$. If it does so, then the discounted sum of utility flows from $\widetilde{T}-j+2$ and onwards equals that under $U_{\widetilde{T}-j}$. If it does not do so, a comparison of utilities similar to the one above yields that, under independence, GL's utility in period $\widetilde{T}-j+2$ is at least as high as under a union. Etcetera.

G. Proof of Lemma 3

The optimality of the central banks' strategies is trivial. For GH, deviating from $\overline{y}_{\widetilde{T}-j}^I$, while still inducing $I_{\widetilde{T}-j}$, is suboptimal. Similarly, deviating from $\overline{y}_{\widetilde{T}-j}^*$, while still inducing $U_{\widetilde{T}-j}$, is suboptimal (remember that $\overline{y}_{\widetilde{T}-j}^* \leq \overline{y}^* < \overline{y}_{\widetilde{T}-j}^U$) as $u_{\widetilde{T}-j}^{GH,U}(y)$ is a strictly concave function of y reaching a maximum at $y = \overline{y}_{\widetilde{T}-j}^U$. If $U_{\widetilde{T}-j-1}$, GL has no alternative, but to set $U_{\widetilde{T}-j}$ (because unification is irreversible), while GH's dominant strategy is to set $\overline{y}_{\widetilde{T}-j} = \overline{y}_{\widetilde{T}-j}^U$. If $I_{\widetilde{T}-j-1}$, then, by the definition of $\overline{y}_{\widetilde{T}-j}^*$, (19), GL has no incentive to deviate from the prescribed choice.

H. Proof of Proposition 5

Suppose that irreversible unification takes place in period $t-1$. Hence, $\overline{y}_{t-1} = \overline{y}_{t-1}^*$ and $\overline{y}_t = \overline{y}_t^U$. First, suppose that $\overline{y}_t^* > 0$. Then, by Lemma 4, $\overline{y}_{t-1}^* < \overline{y}_t^*$. Moreover, under the assumptions of Proposition 5, $t \leq \widetilde{T}-1$, so that by (17), $\overline{y}_t^U > \overline{y}^* \geq \overline{y}_t^*$ (for this last comparison, see the paragraph preceding Lemma 3). Combining all this, one has that $\overline{y}_{t-1} < \overline{y}_t$. Now, suppose that $\overline{y}_t^* = 0$. Because the discounted sum of GL's utility flows as of period t under unification is lower than under monetary independence (remember that $\overline{y}_\tau^U > \overline{y}_\tau^I$, for $\tau < T$, by Lemma 2), the assumption that GL admits H into the union in period $t-1$, requires that GL's utility flow in period $t-1$ must be higher under unification than under independence. By the definition of $\overline{y}^*$, this, in turn, requires that $\overline{y}_{t-1} < \overline{y}^*$. Again, because $t \leq \widetilde{T} - 1$, by (17), $\overline{y}_t^U > \overline{y}^*$. Hence, $\overline{y}_{t-1} < \overline{y}_t$.

I. Proof of GL's indifference between reversibility and irreversibility

In any period before unification takes place, GL receives a utility flow of $u^{GL,I} = 0$, cf. (8). Further, remember that from period $\widetilde{T}$ onward, the utility flows under reversibility and irreversibility are the same. Now, consider the time from the period of unification up to and including period $\widetilde{T} - 1$. Under reversibility, in each of the periods $T^{ru}, ..., \widetilde{T} - 1$, GL receives a constant utility flow of $u^{GL,U}(\overline{y}^*) = u^{GL,I} = 0$. For the case of irreversibility, combine (19) and (20), so that we can write $W_{I,T^{iu}}^{GL} = u^{GL,I} + \beta_L W_{I,T^{iu}+1}^{GL}$. We can repeatedly expand the right-hand side of this expression, to see that $W_{I,T^{iu}}^{GL}$ equals the discounted sum of the per-period utility flow $u^{GL,I}$ during $T^{iu}, ..., \widetilde{T} - 1$.