Optimal Inflation Targets, "Conservative" Central Banks, and Linear Inflation Contracts: Comment*

Roel M. W. J. Beetsma

Dutch Ministry of Economic Affairs and CEPR[†]

Henrik Jensen

University of Copenhagen and EPRU[‡]

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Abstract

Recently, Svensson (1997) has shown that a combination of state-contingent inflation targeting and central banker "conservatism" produces optimal monetary policy if employment is persistent. We argue that the state-contingent nature of the scheme may undermine its credibility. We then show that the optimal policy in Svensson's model can nevertheless be attained through state-independent delegation.

Keywords: Monetary policy; state-independent delegation; credibility problems, output persistence.

JEL Codes: E42, E52, E58.

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[†]Mailing address: Ministry of Economic Affairs, Economic Policy Directorate (AEP), P.O. Box 20101, 2500 EC The Hague, The Netherlands (phone: +31.70.3796078; fax: +31.70.3797018; e-mail: R.Beetsma@minez.nl).

[‡]Mailing address: Institute of Economics, University of Copenhagen, Studiestraede 6, DK-1455 Copenhagen K, Denmark (phone: +45 35 32 30 43; fax: +45 35 32 30 00; e-mail: Henrik.Jensen@econ.ku.dk).

In a recent article, Lars E. O. Svensson (1997) shows that state-contingent inflation targeting induces an independent and "conservative" (in the sense of Kenneth Rogoff, 1985) central bank to follow the optimal monetary policy rule when employment is persistent.[1] In this comment we argue that the state-contingent nature of the delegation scheme may undermine its credibility. Subsequently we show that it nevertheless is possible to attain the optimal rule in Svensson's model through state-*in*dependent delegation.

In his forceful critique, Bennett T. McCallum (1995) argues that the literature on monetary policy delegation does not address the basic credibility problem underlying monetary policy, but that delegation merely relocates the problem from the central bank to its political principal, for example, the government. This is because the principal always has the incentive to change the delegation arrangement once inflation expectations have been formed. McCallum's critique may be particularly relevant for state-contingent delegation mechanisms, because if it is possible to adjust, for example, an inflation target before inflation expectations are formed (as is the case in Svensson's model), it should also be possible to change the target when expectations have already been determined.

Credibility of a delegation arrangement typically relies upon some costs that the principal incurs when the arrangement is altered (this is often implicitly assumed in the literature, as noted by Stanley Fischer, 1995; see Susanne Lohmann, 1992, on the nature of such costs). Hence, a state-contingent delegation mechanism is *paradoxical*, because on the one hand its credibility requires that altering the arrangement be sufficiently costly, while on the other hand such costs may preclude adjustment of the arrangement when actually justified by a change in the state of the economy. Moreover, one way of making these costs sufficiently large would be to include the arrangement in the central bank's charter. But as noted by Torsten Persson and Guido Tabellini (1993), it may only be possible to write simple, i.e. state-independent, arrangements in the charter.[2] Hence, only state-independent delegation may be credible and feasible.

As mentioned, we show that it is possible to achieve the optimal monetary policy rule in Svensson's model through state-independent delegation. More specifically, the central bank is required to make an appropriate trade-off between achieving a constant nominal income growth target and attaining the socially optimal (constant) target values of output and inflation.[3] Such an arrangement should be more robust to McCallum's critique, because one could specify the constant target values in the central bank's charter.

[1]Similarly, Ben Lockwood (1997) finds that the optimal linear inflation contract under output persistence is state-contingent.

[2]Svensson himself does acknowledge that "*In practice, only constant targets may be feasible*" (p. 110).

[3]Nominal income (growth) targeting is the subject of an ongoing debate among policymakers and researchers (see Robert E. Hall and N. Gregory Mankiw, 1994, and the references therein). Although

Even if state-contingent inflation targeting is possible, our arrangement may be preferable in other respects. For example, adjusting the inflation target to reflect new economic realities would require negotiations and compromises among the various parties that are involved. Such a process runs the risk of political pressure or interference.[4,5] Furthermore, our arrangement may be relatively more acceptable to the public, because Svensson's requires that, in order to provide the right incentives for the central bank, the inflation target needs to be set lower when unemployment is higher. Such an arrangement might be hard to sell to the public.[6] In contrast, our arrangement keeps targets constant, irrespective of whether employment is low or high. Finally, under our arrangement there is no need to impose target values for the primary policy variables (inflation and output) which differ from their socially optimal values. This contrasts with Svensson's arrangement where the inflation target is on average below the socially optimal inflation rate, which could cause problems with the public's understanding of the arrangement (as is also argued by Persson and Tabellini, 1998).

The remainder of this comment is organized as follows. Section I presents the model and shows that by imposing a constant nominal income growth target on the central bank, the optimal monetary policy rule is achieved. Section II concludes with a discussion of the robustness of our results. The appendix contains the technical details.

I. State-Independent Delegation and Optimal Monetary Policy

The model and the notation follow Svensson (1997). However, because we will focus on nominal income growth as an intermediate target, we will present the model (as in Svensson, 1995) in terms of output rather than employment. It is formulated over an infinite horizon. There is a private sector, a government and a central bank. Each period, inflation expectations π_t^e are formed first. They are assumed to be rational, i.e., $\pi_t^e =$

explicit nominal income growth targeting is rarely adopted by central banks, their policies may implicitly be guided by nominal income growth, because data on nominal income are often more precise and more readily available than real income and inflation data. This comment, however, addresses nominal income growth targeting from a normative perspective.

[4] A case in point is the fact that the New Zealand Reserve Bank Act of 1989 leaves open the possibility of resetting the inflation target (range) under some circumstances. In 1996 this caused heated public debates which lead some observers to conclude that the country was "back to politicized monetary policy." See Frederic S. Mishkin and Adam S. Posen (1997).

[5] A detailed analysis of the political economy of delegation under output persistence is beyond the scope of this comment. See Gunnar Jonsson (1997) for an explicit analysis of political processes within the current framework.

[6] There is casual evidence that incentive schemes which promote low inflation in times of high unemployment are considered provocative to the public. For example, a proposal to induce low inflation through pecuniary rewards to the central banker was not included in the New Zealand Reserve Bank Act of 1989. See Charles Goodhart and José Viñals (1994).

$\mathrm{E}_{t-1}[\pi_t]$, where $\mathrm{E}_{t-1}[.]$ is the expectations operator conditional on all information available in period $t-1$. Then, a supply shock ε_t hits the economy. Next, the policymaker selects the inflation rate, π_t (over which it has perfect control). Finally, output is determined.

The (log of) output is given by a Phillips-curve relationship:

$$y_t = \rho y_{t-1} + \alpha\left(\pi_t - \pi_t^e\right) + \varepsilon_t, \quad \alpha > 0, \quad 0 < \rho < 1, \tag{1}$$

where ε_t is an i.i.d. mean-zero shock with finite variance. Output is persistent with the degree of persistence given by ρ. The long-run natural output level, $\lim_{s\to\infty}\mathrm{E}_{t-1}[y_s]$, is normalized to zero. Society's (and the government's) preferences are represented by $\mathrm{E}_0\left[\sum_{t=1}^{\infty}\beta^{t-1}L_t\right]$, where $0 < \beta < 1$ is a discount factor and $L_t = \frac{1}{2}\left[\left(\pi_t - \pi^*\right)^2 + \lambda(y_t - y^*)^2\right]$ is the per-period loss function. Here, π^* is the socially optimal inflation rate and $y^* > 0$ is the socially optimal output level. The latter is assumed to exceed the long-run natural output level, which may be too low, for example, because of distortions in the labor market. Parameter $\lambda > 0$ is the relative weight attached to output versus inflation stabilization.

The socially optimal inflation rule is (see Svensson, 1997)

$$\pi_t = \pi^* - s\varepsilon_t, \quad s \equiv \frac{\lambda\alpha}{1 + \lambda\alpha^2 - \beta\rho^2} > 0, \tag{2}$$

which shows that inflation should on average match its socially optimal rate and that the degree of output stabilization in response to supply shocks is increasing in λ (because society cares relatively more about output), in ρ (because current shocks have a stronger effect on future output) and in β (as the effect on future output is considered to be more important now).

In reality, commitment to an inflation rule is usually not feasible. In the following, we will therefore consider the delegation of monetary policy to a central bank acting in a discretionary fashion. Delegation involves the assignment of an objective function to the central bank, which is then free to select the inflation rate. Specifically, the central bank minimizes $\mathrm{E}_0\left[\sum_{t=1}^{\infty}\beta^{t-1}L_t^b\right]$ where L_t^b, the per-period loss function assigned to the central bank, is given by

$$L_t^b = \frac{1}{2}\left[(1+f)\left(\pi_t - \pi^*\right)^2 + \lambda(y_t - y^*)^2 + \psi(g_t - g)^2\right], \tag{3}$$

where

$$g_t \equiv \pi_t + y_t - y_{t-1}, \tag{4}$$

is the nominal income growth rate. Hence, the central bank shares society's preferences, but is assigned a constant nominal income growth target equal to g. Parameter ψ captures the relative weight attached to this target. In addition, the central bank may be required to attach relatively more ($f > 0$) or less ($f < 0$) emphasis than society on stabilizing inflation around its social optimum. Such an objective function could be imposed by including the policy targets in the central bank's charter and imposing an appropriate "system of rewards and punishments" (cf. Rogoff, 1985) based on deviations of realized policy variables from their targets.[7] Following Persson and Tabellini (1993, 1998) or Carl E. Walsh (1995) one could also broadly interpret such a system as a "performance contract."

To derive the central bank's policy, let $V^b(y_{t-1})$ be the discounted sum of the central bank's per-period loss functions from period t and onwards under delegation. Then, the Bellman equation associated with its decision problem is given by

$$V^b(y_{t-1}) = \mathrm{E}_{t-1} \min_{\pi_t} \left\{ \frac{1}{2} \left[(1+f)(\pi_t - \pi^*)^2 + \lambda(y_t - y^*)^2 + \psi(g_t - g)^2 \right] + \beta V^b(y_t) \right\}, \quad (5)$$

where the minimization is performed subject to (1) and (4), taking π_t^e as given. For later use we state the relevant first-order condition:

$$(1+f)(\pi_t - \pi^*) + \lambda\alpha(y_t - y^*) + \psi(1+\alpha)(g_t - g) + \beta\alpha V_y^b(y_t) = 0, \quad (6)$$

where V_y^b denotes the first-order derivative of V^b. It turns out that the central bank selects an inflation rate of the following form:

$$\pi_t = a - b\varepsilon_t - cy_{t-1}, \quad (7)$$

where a, b and c are functions of the parameters of the model and of the delegation parameters.[8] If $f = \psi = 0$, then $L_t^b = L_t$, which corresponds to the case in which monetary policy is under direct control of the government. For this purely discretionary case, the expressions for a (with y^* replacing ℓ^*), b and c are given in equations (20) and (21) in Svensson (1997). One has that $a > \pi^*$, $c > 0$ and $b > s$ which compared with the socially optimal inflation rule, (2), reveals three biases. First, $a > \pi^*$ reflects a

[7] Such a system could, for example, involve making the reappointment of the central banker dependent upon the deviations of policy variables from targets. Also, the arrangement could require the central bank to justify in public, through official reports or before parliament, any deviations from the nominal income growth target. The parameter ψ could then be interpreted as quantifying the extent to which deviations are tolerated, e.g., it could be a proxy for the width of a target band for nominal income growth.

[8] The solution methodology parallels that in Svensson (1997). In particular, one conjectures V^b to be linear-quadratic, whereby (6) yields policy as a function of the unknown parameters of V^b. By inserting the policy function and the implied expression for expected inflation back into (5), the conjecture is verified. The unknown parameters — and thus the policy function — can subsequently be identified.

constant *average* inflation bias. It arises because the output target y^* is above the long-run natural rate. This creates a permanent incentive for monetary expansion which in a rational expectations equilibrium only leads to higher inflation. Secondly, $c > 0$ implies a *state-contingent* inflation bias which arises because the incentive for monetary expansion depends on past period's output due to output persistence (see Lockwood and Apostolis Philippopoulos, 1994). Finally, because $b > s$, there is a *stabilization bias* (see Lockwood, 1997, and Svensson, 1997). The source of this bias is the anticipation of a state-contingent inflation bias in the future, which provides an additional incentive to stabilize current output against supply-shocks. As a result, inflation will be too variable.

For (7) to reproduce (2), the delegation parameters thus need to be selected in such a way that $a = \pi^*$, $c = 0$ and $b = s$. The following proposition, which is proven in Appendix A, shows that this is indeed possible:

PROPOSITION. *The optimal inflation rule (2) is attained by setting* $(\psi, f, g) = (\psi^*, f^*, g^*)$ *where* ψ^*, f^*, *and* g^* *are constants given by, respectively, (8), (9) and (10) below.*

The remainder of this section explains this result.

It is convenient to consider the elimination of the state-contingent inflation bias first, so that $c = 0$. This is done by setting ψ equal to:

$$\psi^* \equiv \frac{\lambda\alpha\rho}{(1-\rho)\left[1-\beta\rho^2+\alpha\left(1-\beta\rho\right)\right]} > 0. \tag{8}$$

To offset the incentive for higher inflation if y_{t-1} is below the long-run natural level [see (7)], ψ should be positive. To gain some intuition for expression (8), suppose first that the central bank were to act as if the future carries no weight. That is, as if $\beta = 0$. Consider a one-unit reduction in y_{t-1}, which lowers y_t by ρ units. This raises the marginal benefit of inflation by $\lambda\alpha\rho$, cf. the first-order condition (6). In addition, nominal income growth is increased by $1-\rho$, which increases the marginal cost of inflation by $\psi(1+\alpha)(1-\rho)$. At $\psi = \psi^*$ the changes in the marginal benefit and cost of inflation should be equal, hence $\lambda\alpha\rho = \psi^*(1+\alpha)(1-\rho)$, which gives (8) for $\beta = 0$. However, because the central bank *does* care about the future, i.e., $\beta > 0$, the incentive for higher inflation if y_{t-1} is lower is even stronger. The reason is that an inflation surprise will reduce the state-contingent inflation bias in future periods also, because of the persistence in output. As a result, deviations from the nominal income growth target should be punished more severely. Indeed, (8) shows that ψ^* is increasing in β. Moreover, ψ^* is increasing in λ, because a stronger concern for output raises the marginal benefit of inflation if y_{t-1} is reduced. Correspondingly, the marginal cost of inflation needs to be raised. A higher ρ implies that a reduction in y_{t-1} has a larger effect on current output, thereby raising both the current and future marginal benefit of inflation. Hence, ψ^* also depends positively on ρ. Finally, ψ^* is increasing in α, because a higher α makes inflation more effective at raising output.

Given that $\psi = \psi^*$, the stabilization bias is eliminated (i.e., $b = s$) if f equals:

$$f^* \equiv \psi^* \frac{(1+\alpha)(1-\beta\rho^2-\lambda\alpha\rho)}{\lambda\alpha\rho} - 1 \gtreqless 0. \tag{9}$$

Although the stabilization bias arises from the presence of a state-contingent inflation bias, removing the latter does not ensure efficient shock stabilization. Introducing a nominal income growth target distorts stabilization such that the socially optimal balance between output and inflation variability will in general not be attained. To restore this balance, the central bank generally has to attach a different relative weight to inflation stabilization than society, as (9) shows. For example, as we saw from (2), higher values of λ, β or ρ require a more active stabilization policy from a social perspective. Therefore, f^* needs to be reduced. In fact, f^* may even be negative, implying that the central bank should pay relatively more attention than society to output rather than inflation stabilization. Hence, in contrast to Svensson's state-contingent inflation targeting arrangement, our arrangement does not provide an unambiguous case for central bank conservatism.[9]

Finally, the average inflation bias is eliminated (i.e., $a = \pi^*$) if the central bank targets nominal income growth at a constant rate of:

$$g^* \equiv \pi^* - \frac{\lambda\alpha y^*}{\psi^*\left[1-\beta\rho+\alpha(1-\beta)\right]} < \pi^*. \tag{10}$$

The nominal income growth target should be lower than the socially optimal inflation rate. To see this, note first that if $y^* = 0$, the average inflation bias is absent. The optimal nominal income growth target would then simply be given by π^*, cf. (10). However, if $y^* > 0$, the nominal income growth target should be set below π^* in order to offset the, in equilibrium futile, incentive to raise output above its long-run natural level. Given ψ^*, this incentive is stronger if λ, α, β and ρ are higher, which requires a corresponding reduction in g^*.

[9] Conservatism is needed under Svensson's arrangement because the state-contingent inflation target introduces additional inflation variability. This is corrected by having the central banker attach a higher relative weight to inflation stabilization.

II. Discussion

Above we have analyzed constant nominal income growth targeting as a way to attain the socially optimal inflation rule in Svensson's (1997) model. In order to induce the central bank to perform policy according to this rule, the state-contingent inflation bias has to be eliminated. Because both the state-contingent inflation bias and nominal income growth depend on lagged output, introducing a nominal income growth target eliminates the effect of lagged output on inflation if the target is given an appropriate relative weight. Not surprisingly, this relative weight is increasing in the parameters that in the absence of nominal income growth targeting would worsen the state-contingent inflation bias. However, the introduction of a nominal income growth target distorts the balance between inflation and output variability, which requires that the central bank be made more or less conservative. Finally, with an appropriate choice of the rate at which nominal income growth is targeted, the average inflation bias is eliminated.

The result that our arrangement can induce the central bank to choose the optimal inflation rule depends on the specific Phillips-curve that we employ. For more elaborate specifications of the Phillips-curve, involving more state variables, our result will not hold in general. In such cases, state-contingent arrangements would be needed to induce the central bank to follow a policy according to the optimal inflation rule. However, as we argued above, such arrangements may not be credible, and it is therefore of interest to compare the performance of our arrangement with state-*in*dependent inflation targeting. As an example, consider an extension of the model in which the private sector receives some signal θ_{t-1} about the period-t supply shock prior to the formation of period-t expectations (as in Berthold Herrendorf and Lockwood, 1997). In the current context, the (log) of output then becomes:

$$y_t = \rho y_{t-1} + \alpha \left(\pi_t - \pi_t^e\right) + \theta_{t-1} + \varepsilon_t, \tag{11}$$

where θ_{t-1} is an i.i.d. mean-zero, finite-variance shock, which is independent of ε_τ for any τ. Hence, θ_{t-1} introduces an additional state variable which creates a new and independent source of variation in the inflation bias. Even though our constant nominal income growth targeting arrangement is no longer able to reproduce the optimal inflation rule when (11) applies, we show in Appendix B that it outperforms the constant inflation targeting arrangement discussed in Svensson (1997, Section II.C). The reason is that nominal income growth targeting is superior in terms of addressing the state-contingent inflation bias associated with output persistence because nominal income growth depends on lagged output, as explained above.

Appendix

A. Proof of the proposition

In proving the proposition, we first have to find the solution to the central banker's minimization problem as given by equation (5). Then, we demonstrate that $\psi = \psi^*$, $f = f^*$ and $g = g^*$ secures that $a = \pi^*$, $b = s$, and $c = 0$, i.e., that the socially optimal inflation rule is attained.

Given the linear-quadratic structure of the model, V^b will be quadratic,

$$V^b\left(y\right) = \gamma_0 + \gamma_1 y + \frac{1}{2}\gamma_2 y^2, \tag{A.1}$$

with the coefficients γ_0, γ_1 and γ_2 to be determined below. Substituting $V_y^b(y_t) = \gamma_1 + \gamma_2 y_t$ into the necessary (and sufficient) first-order condition [equation (6)] yields

$$(1+f)(\pi_t - \pi^*) + \lambda\alpha\left(y_t - y^*\right) + \psi\left(1+\alpha\right)\left(g_t - g\right) + \beta\alpha\left(\gamma_1 + \gamma_2 y_t\right) = 0. \tag{A.2}$$

Substituting $y_t = \rho y_{t-1} + \alpha\left(\pi_t - \pi_t^e\right) + \varepsilon_t$ and $g_t = \pi_t + y_t - y_{t-1}$ into (A.2), and applying that expectations are formed rationally, it is straightforward to derive the inflation rate, *given* the parameters of the value function:

$$\pi_t = a - b\varepsilon_t - cy_{t-1},$$

[equation (7)] where

$$a = \frac{\left(1+f\right)\pi^* + \lambda\alpha y^* + \psi\left(1+\alpha\right)g - \beta\alpha\gamma_1}{\Omega}, \tag{A.3}$$

$$b = \frac{\alpha\left(\lambda + \beta\gamma_2\right) + \psi\left(1+\alpha\right)}{\alpha^2\left(\lambda + \beta\gamma_2\right) + 1 + f + \psi\left(1+\alpha\right)^2}, \tag{A.4}$$

$$c = \frac{\alpha\rho\left(\lambda + \beta\gamma_2\right) - \psi\left(1+\alpha\right)\left(1-\rho\right)}{\Omega}, \tag{A.5}$$

and where

$$\Omega \equiv 1 + f + \psi\left(1+\alpha\right). \tag{A.6}$$

The remaining step in solving for inflation as a function of the parameters of the model, is the identification of γ_1 and γ_2 (the parameter γ_0 is not of interest, as it does not affect a, b or c; cf. above). To do this, note that by definition of the Bellman equation and the conjectured form of the value function, (A.1), the following recursion must hold in equilibrium:

$$\gamma_0 + \gamma_1 y_{t-1} + \frac{\gamma_2}{2} y_{t-1}^2 = \mathrm{E}_{t-1} \left\{ \frac{1}{2} \left[(1+f)(\pi_t - \pi^*)^2 + \lambda (y_t - y^*)^2 + \psi (g_t - g)^2 \right] + \beta \left[\gamma_0 + \gamma_1 y_t + \frac{\gamma_2}{2} y_t^2 \right] \right\}. \tag{A.7}$$

Now substitute (7), $y_t = \rho y_{t-1} + \alpha (\pi_t - \pi_t^e) + \varepsilon_t$ and $g_t = \pi_t + y_t - y_{t-1}$ into (A.7), which can then be written as

$$\begin{aligned} \gamma_0 + \gamma_1 y_{t-1} + \frac{\gamma_2}{2} y_{t-1}^2 = \mathrm{E}_{t-1} \Bigg\{ & \frac{1+f}{2} \left[a - b\varepsilon_t - c y_{t-1} - \pi^* \right]^2 \\ & + \frac{\lambda}{2} \left[\rho y_{t-1} + (1 - \alpha b) \varepsilon_t - y^* \right]^2 \\ & + \frac{\psi}{2} \left[a + (1 - (1+\alpha) b) \varepsilon_t - (c + 1 - \rho) y_{t-1} - g \right]^2 \\ & + \beta \left[\gamma_0 + \gamma_1 (\rho y_{t-1} + (1-\alpha b)\varepsilon_t) + \frac{1}{2}\gamma_2 (\rho y_{t-1} + (1-\alpha b)\varepsilon_t)^2 \right] \Bigg\} \end{aligned} \tag{A.8}$$

We write out each of the terms on the right-hand side of (A.8) separately:

$$\begin{aligned} & \frac{1+f}{2} \mathrm{E}_{t-1} \left[a - b\varepsilon_t - c y_{t-1} - \pi^* \right]^2 = \\ & \frac{1+f}{2} \left[a^2 + b^2 \sigma_\epsilon^2 + (\pi^*)^2 - 2a\pi^* \right] + (1+f) c (\pi^* - a) y_{t-1} + \frac{1+f}{2} c^2 y_{t-1}^2, \end{aligned}$$

$$\frac{\lambda}{2} \mathrm{E}_{t-1} \left[\rho y_{t-1} + (1-\alpha b) \varepsilon_t - y^* \right]^2 = \frac{\lambda}{2} \left[\rho^2 y_{t-1}^2 + (1-\alpha b)^2 \sigma_\epsilon^2 + (y^*)^2 - 2\rho y_{t-1} y^* \right],$$

$$\begin{aligned} & \frac{\psi}{2} \mathrm{E}_{t-1} \left[a + (1 - (1+\alpha) b) \varepsilon_t - (c + 1 - \rho) y_{t-1} - g \right]^2 \\ = & \frac{\psi}{2} \left[a^2 + g (g - 2a) + (1 - (1+\alpha) b)^2 \sigma_\epsilon^2 \right] \\ & + \psi (c + 1 - \rho)(g - a) y_{t-1} + \frac{\psi}{2} (c + 1 - \rho)^2 y_{t-1}^2, \end{aligned}$$

$$\begin{aligned} & \beta \mathrm{E}_{t-1} \left[\gamma_0 + \gamma_1 (\rho y_{t-1} + (1-\alpha b)\varepsilon_t) + \frac{1}{2}\gamma_2 (\rho y_{t-1} + (1-\alpha b)\varepsilon_t)^2 \right] \\ = & \beta \left[\gamma_0 + \frac{1}{2}\gamma_2 (1-\alpha b)^2 \sigma_\epsilon^2 + \gamma_1 \rho y_{t-1} + \frac{1}{2}\gamma_2 \rho^2 y_{t-1}^2 \right], \end{aligned}$$

where $\sigma_\epsilon^2 \equiv \mathrm{E}_{t-1}\left[\varepsilon_t^2\right]$ is the variance of ε_t. Collecting terms on the right-hand sides of these four equations, we can write the right-hand side of (A.8) as:

$$\begin{aligned}
& \frac{\lambda}{2}\left[(1-\alpha b)^2\sigma_\epsilon^2+(y^*)^2\right]+\frac{1+f}{2}\left[a^2+b^2\sigma_\epsilon^2+\pi^*(\pi^*-2a)\right] \\
+ \quad & \frac{\psi}{2}\left[a^2+g(g-2a)+(1-(1+\alpha)b)^2\sigma_\epsilon^2\right]+\beta\gamma_0+\frac{1}{2}\beta\gamma_2(1-\alpha b)^2\sigma_\epsilon^2 \\
& +\left[-\lambda\rho y^*+(1+f)c(\pi^*-a)+\psi(c+1-\rho)(g-a)+\beta\gamma_1\rho\right]y_{t-1} \\
& +\frac{1}{2}\left[\lambda\rho^2+(1+f)c^2+\psi(c+1-\rho)^2+\beta\gamma_2\rho^2\right]y_{t-1}^2.
\end{aligned}$$

This verifies the conjectured form of the value function. Equating the coefficients of y_{t-1}^2 and y_{t-1} in this expression to the corresponding coefficients on the left-hand side of (A.8) yields two Riccati equations in, respectively, γ_2 and γ_1:

$$\gamma_2 = \lambda\rho^2+(1+f)c^2+\psi(c+1-\rho)^2+\beta\gamma_2\rho^2, \tag{A.9}$$

$$\gamma_1 = -\lambda\rho y^*+(1+f)c(\pi^*-a)+\psi(c+1-\rho)(g-a)+\beta\gamma_1\rho, \tag{A.10}$$

where a, given by (A.3), depends on γ_1, while c, given by (A.5), depends only on γ_2. Hence, the system (A.9) and (A.10) is recursive. Equation (A.9) determines γ_2, which can be substituted into (A.10) to determine γ_1. However, it is convenient to solve (A.9) directly in terms of c. For this purpose, use (A.5) to express γ_2 as a function of c, which, inserted back into (A.9), results in the following equation for c:

$$\begin{aligned}
0 = \; & \beta\alpha\rho(1+f+\psi)c^2-\left[\Omega\left(1-\beta\rho^2\right)-2\beta\alpha\rho\psi(1-\rho)\right]c \\
& +\lambda\alpha\rho-\psi(1-\rho)\left[1-\beta\rho^2+\alpha(1-\beta\rho)\right].
\end{aligned} \tag{A.11}$$

The existence of real solutions for c (and γ_2) requires that

$$\begin{aligned}
& \left[\Omega\left(1-\beta\rho^2\right)-2\beta\alpha\rho\psi(1-\rho)\right]^2 \\
& -4\beta\alpha\rho(1+f+\psi)\left[\lambda\alpha\rho-\psi(1-\rho)\left(1-\beta\rho^2+\alpha(1-\beta\rho)\right)\right]\geq 0.
\end{aligned} \tag{A.12}$$

If the left-hand side of (A.12) is positive, then (A.11) has two solutions. This implies the existence of two inflation equilibria, as in Lockwood and Philippopoulos (1994). However, the literature on monetary delegation has argued that only the equilibrium associated with the smaller root of (A.11) is relevant; see, for example, Svensson (1997). The solution for c is therefore given by

$$
\begin{aligned}
c &= \frac{\Omega\left(1-\beta\rho^2\right)-2\beta\alpha\rho\psi\left(1-\rho\right)}{2\beta\alpha\rho\left(1+f+\psi\right)} \\
&\quad -\frac{\sqrt{\left[\Omega\left(1-\beta\rho^2\right)-2\beta\alpha\rho\psi\left(1-\rho\right)\right]^2-4\beta\alpha\rho\left(1+f+\psi\right)\Phi}}{2\beta\alpha\rho\left(1+f+\psi\right)}, \qquad \text{(A.13)}
\end{aligned}
$$

where

$$
\Phi=\lambda\alpha\rho-\psi\left(1-\rho\right)\left[1-\beta\rho^2+\alpha\left(1-\beta\rho\right)\right]. \qquad \text{(A.14)}
$$

Because we now have a solution for c in terms of the parameters of the model, it suffices to solve a and b in terms of c. First we solve for a. We rewrite (A.3) to express γ_1 in terms of a:

$$
\gamma_1=\frac{\left(1+f\right)\pi^*+\lambda\alpha y^*+\psi\left(1+\alpha\right)g-a\Omega}{\beta\alpha}. \qquad \text{(A.15)}
$$

Then rewrite (A.10) as

$$
\gamma_1\left(1-\beta\rho\right)=-\lambda\rho y^*+(1+f)c\left(\pi^*-a\right)+\psi\left(c+1-\rho\right)\left(g-a\right), \qquad \text{(A.16)}
$$

and substitute (A.15) into (A.16) and multiply both sides by $\beta\alpha$ to give:

$$
\begin{aligned}
&\left(1+f\right)\pi^*\left(1-\beta\rho\right)+\left(1-\beta\rho\right)\lambda\alpha y^*+\left(1-\beta\rho\right)\psi\left(1+\alpha\right)g-a\Omega\left(1-\beta\rho\right) \\
= &\ -\beta\alpha\lambda\rho y^*+\beta\alpha(1+f)c\left(\pi^*-a\right)+\beta\alpha\psi\left(c+1-\rho\right)\left(g-a\right).
\end{aligned}
$$

This yields the explicit solution for a:

$$
a=\frac{\lambda\alpha y^*+\left(1+f\right)\left[1-\beta\left(\rho+\alpha c\right)\right]\pi^*+\psi\left[1-\beta\left(\rho+\alpha c\right)+\alpha\left(1-\beta\right)\right]g}{\Omega\left(1-\beta\rho\right)-\left(1+f\right)\alpha\beta c-\psi\alpha\beta\left(c+1-\rho\right)}. \qquad \text{(A.17)}
$$

Note this solution for a is unbounded if $\Omega\left(1-\beta\rho\right)-\left(1+f\right)\alpha\beta c-\psi\alpha\beta\left(c+1-\rho\right)=0$ (and so is the solution for γ_1). Therefore, the value function fails to exist for this particular parameter configuration and the equilibrium exhibits a discontinuity. To rule this out, the following restriction on the parameters must be imposed (Svensson, 1997, makes a similar assumption):

$$
\Omega\left(1-\beta\rho\right)-\left(1+f\right)\alpha\beta c-\psi\alpha\beta\left(c+1-\rho\right)>0. \qquad \text{(A.18)}
$$

Finally, to find b, we first rewrite (A.5) as

$$
\alpha\left(\lambda+\beta\gamma_2\right)+\psi\left(1+\alpha\right)=\frac{c\Omega+\psi\left(1+\alpha\right)}{\rho}, \qquad \text{(A.19)}
$$

hence,

$$\alpha^2(\lambda+\beta\gamma_2)+(1+f)+\psi(1+\alpha)^2=\frac{\alpha c\Omega+\alpha\psi(1+\alpha)(1-\rho)+(1+f)\rho+\psi(1+\alpha)^2\rho}{\rho},$$

which [using the definition of Ω, (A.6)] can be written as:

$$\alpha^2(\lambda+\beta\gamma_2)+(1+f)+\psi(1+\alpha)^2=\frac{(\alpha c+\rho)\Omega+\psi(1+\alpha)\alpha}{\rho}. \tag{A.20}$$

Substitute (A.19) for the numerator of (A.4) and (A.20) for the denominator of (A.4). This yields the solution for b as a function of c:

$$b=\frac{c\Omega+\psi(1+\alpha)}{(\alpha c+\rho)\Omega+\psi(1+\alpha)\alpha}. \tag{A.21}$$

To sum up, the central bank conducts policy according to (7), with a, b and c, respectively, given by (A.17), (A.21) and (A.13). Now we consider how to choose the delegation parameters ψ, f, and g so as to obtain $a=\pi^*$, $b=s$, and $c=0$.

From (A.13), $c=0$ requires that $\Phi=0$. From the definition of Φ, (A.6), it follows that $\Phi=0$ if we set ψ equal to

$$\psi^*\equiv\frac{\lambda\alpha\rho}{(1-\rho)[1-\beta\rho^2+\alpha(1-\beta\rho)]},$$

which is indeed the expression for ψ^* in equation (8).

Given that $\psi=\psi^*$, hence that $\Phi=c=0$, expression (A.21) reduces to

$$b=\frac{\psi^*(1+\alpha)}{\rho\Omega^*+\psi^*(1+\alpha)\alpha}, \text{ where } \Omega^*\equiv 1+f+\psi^*(1+\alpha).$$

Hence, in order to secure $b=s$, we must have [using the definition of s, see equation (2)]:

$$\frac{\psi^*(1+\alpha)}{\rho\Omega^*+\psi^*(1+\alpha)\alpha}=\frac{\lambda\alpha}{1+\lambda\alpha^2-\beta\rho^2}.$$

By the definition of Ω^* this requires us to set f equal to

$$f^*\equiv\psi^*\frac{(1+\alpha)(1-\beta\rho^2-\lambda\alpha\rho)}{\lambda\alpha\rho}-1,$$

which is indeed the expression for f^* in equation (9).[10]

[10] As mentioned in the main text, f^* can be negative. This yields a proper solution, because the second-order condition for a minimum is satisfied. To see this, differentiate (A.2) with respect to π_t, which yields

Finally, to ensure that $a = \pi^*$, we see from (A.17) that the following condition must hold:

$$\frac{\lambda\alpha y^* + (1+f^*)(1-\beta\rho)\pi^* + \psi^*[1-\beta\rho+\alpha(1-\beta)]g}{\Omega^{**}(1-\beta\rho) - \psi^*\alpha\beta(1-\rho)} = \pi^*,$$

where $\Omega^{**} \equiv 1 + f^* + \psi^*(1+\alpha)$ and where we use that $c = 0$ if $\psi = \psi^*$. This condition requires that g be set equal to

$$g^* \equiv \pi^* - \frac{\lambda\alpha y^*}{\psi^*[1-\beta\rho+\alpha(1-\beta)]},$$

which is indeed the expression for g^* in equation (10).

This completes the proof that by setting $\psi = \psi^*$, $f = f^*$ and $g = g^*$ one has that $a = \pi^*$, $b = s$, and $c = 0$ as required. It may be interesting to note that in contrast to the state-contingent inflation targeting approach, our approach does not require restrictions on the parameter space when the delegation parameters are set optimally. To see this, substitute ψ^* into (A.12) which reveals that the existence condition is always fulfilled.[11] Moreover, the continuity condition, (A.18), turns out to be irrelevant. This is seen by substituting $a = \pi^*$, $c = 0$ and $g = g^*$ into (A.10), which reveals that the solution for γ_1 is always finite.

B. Proof that constant nominal income growth targeting outperforms constant inflation targeting when output is given by (11)

First, note that if equation (11) applies, the socially-optimal inflation rule is again given by equation (2). It is easy to see this, because inflation expectations are formed after the relevant θ-shock has materialized. Hence, output cannot be stabilized in the face of these shocks and, therefore, it is optimal for society not to have inflation react to the θ-shock.

We prove our claim as follows. First, we introduce another targeting regime (hybrid targeting), of which constant inflation targeting is a special case. We show that there exists a constant nominal income growth targeting arrangement which produces exactly the same inflation rule as hybrid targeting. Then, we show that it is suboptimal to restrict the hybrid targeting regime to the special case of constant inflation targeting.

$\alpha^2(\lambda+\beta\gamma_2) + (1+f) + \psi(1+\alpha)^2$ [where we have used equations (1) and (4)]. Use $c = 0$ in (A.5) to substitute $\psi(1+\alpha)(1-\rho)/\rho$ for $\alpha(\lambda+\beta\gamma_2)$. Then, substituting (9) for f and using that $\psi > 0$ by (8), it is clear that the second-order derivative is positive.

[11] The reason why existence may fail under inflation targeting (cf. Svensson, 1997, Appendix B) is that the optimal state-contingent inflation target is a function of c, implying that the state-contingent incentive for surprise inflation is offset. Thereby, the term cy_{t-1} does not appear in the equilibrium inflation rule. The target, however, is only well-defined if c is a real number. Our approach, on the other hand, involves an arrangement which sets c equal to zero for all parameter combinations.

The hybrid arrangement (HYAR) involves assigning the following per-period loss to the central bank:

$$L_t^h = \frac{1}{2}\left[\left(1+\tilde{f}\right)\left(\pi_t - \tilde{\pi}\right)^2 + \lambda(y_t - y^*)^2 + \tilde{\psi} g_t^2\right]. \qquad \text{(B.1)}$$

Hence, under HYAR inflation is targeted at the constant rate $\tilde{\pi}$ and nominal income growth is targeted at a rate of zero. Parameters $\tilde{\psi}$ and $\tilde{f}$ are the other delegation parameters. Constant inflation targeting as considered by Svensson (1997, Section II.C) corresponds to the special case with $\tilde{\psi} = 0$.

The proof now proceeds in two steps. In *Step 1* we show that there exists a mapping between the delegation parameters (f, ψ, g) under constant nominal income growth targeting and $(\tilde{f}, \tilde{\psi}, \tilde{\pi})$ under HYAR which ensures that constant nominal income growth targeting can produce exactly the same inflation rule as HYAR whenever $\tilde{\psi} \neq 0$. In *Step 2*, we then show that a HYAR with $\tilde{\psi} = 0$ cannot be optimal. Combining this with the result of *Step 1* we have that constant nominal income growth targeting outperforms constant inflation targeting.

Step 1:

First we solve the model for the constant nominal income growth targeting arrangement. The relevant Bellman equation is given by:

$$V^b\left(y_{t-1}\right) = \mathrm{E}_{t-1} \min_{\pi_t}\left\{\frac{1}{2}\left[\left(1+f\right)\left(\pi_t - \pi^*\right)^2 + \lambda(y_t - y^*)^2 + \psi(g_t - g)^2\right] + \beta V^b\left(y_t\right)\right\},$$

where the minimization is performed subject to

$$y_t = \rho y_{t-1} + \alpha\left(\pi_t - \pi_t^e\right) + \theta_{t-1} + \varepsilon_t, \qquad \text{(B.2)}$$

[equation (11)] and $g_t = \pi_t + y_t - y_{t-1}$, taking π_t^e as given. The relevant first-order condition is:

$$\left(1+f\right)\left(\pi_t - \pi^*\right) + \lambda\alpha(y_t - y^*) + \psi(1+\alpha)(g_t - g) + \beta\alpha \mathrm{E}\left[V_y^b\left(y_t\right) \mid I_t\right] = 0,$$

where I_t is the information set that includes all variables dated t and earlier, except for θ_t. Hence, I_t is the information set that is available at the moment that π_t is selected.

Conjecture that the value function is the following quadratic function:

$$V^b\left(y_t\right) = \gamma_0 + \gamma_1 y_t + \frac{1}{2}\gamma_2 y_t^2 + \gamma_3 \theta_t + \frac{1}{2}\gamma_4 \theta_t^2 + \gamma_5 \theta_t y_t.$$

Hence, the first-order condition can be written as:

$$(1+f)(\pi_t - \pi^*) + \lambda\alpha\left(y_t - y^*\right) + \psi\left(1+\alpha\right)\left(g_t - g\right) + \beta\alpha\left(\gamma_1 + \gamma_2 y_t\right) = 0.$$

Substituting $y_t = \rho y_{t-1} + \alpha\left(\pi_t - \pi_t^e\right) + \theta_{t-1} + \varepsilon_t$ and $g_t = \pi_t + y_t - y_{t-1}$ into this equation and applying that expectations are formed rationally, it is straightforward to derive the inflation rate, *given* the parameters of the value function:

$$\pi_t = a - b\varepsilon_t - cy_{t-1} - d\theta_{t-1}, \tag{B.3}$$

where

$$a = \frac{(1+f)\,\pi^* + \lambda\alpha y^* + \psi\left(1+\alpha\right)g - \beta\alpha\gamma_1}{\Omega}, \tag{B.4}$$

$$b = \frac{\alpha\left(\lambda + \beta\gamma_2\right) + \psi\left(1+\alpha\right)}{\alpha^2\left(\lambda + \beta\gamma_2\right) + 1 + f + \psi\left(1+\alpha\right)^2}, \tag{B.5}$$

$$c = \frac{\alpha\rho\left(\lambda + \beta\gamma_2\right) - \psi\left(1+\alpha\right)\left(1-\rho\right)}{\Omega}, \tag{B.6}$$

$$d = \frac{\alpha\left(\lambda + \beta\gamma_2\right) + \psi\left(1+\alpha\right)}{1 + f + \psi\left(1+\alpha\right)}. \tag{B.7}$$

Further,

$$y_t = \rho y_{t-1} + (1-\alpha b)\varepsilon_t + \theta_{t-1}, \tag{B.8}$$

$$g_t = a + \left[1 - (1+\alpha)b\right]\varepsilon_t - (c + 1 - \rho)y_{t-1} + (1-d)\theta_{t-1}. \tag{B.9}$$

The following recursion must hold in equilibrium:

$$\begin{aligned}
& \gamma_0 + \gamma_1 y_{t-1} + \frac{1}{2}\gamma_2 y_{t-1}^2 + \gamma_3\theta_{t-1} + \frac{1}{2}\gamma_4\theta_{t-1}^2 + \gamma_5\theta_{t-1}y_{t-1} \\
= \; & \mathrm{E}_{t-1}\left\{\frac{1}{2}\left[(1+f)\left(\pi_t - \pi^*\right)^2 + \lambda\left(y_t - y^*\right)^2 + \psi\left(g_t - g\right)^2\right]\right. \\
& \left. + \beta\left[\gamma_0 + \gamma_1 y_t + \frac{1}{2}\gamma_2 y_t^2 + \gamma_3\theta_t + \frac{1}{2}\gamma_4\theta_t^2 + \gamma_5\theta_t y_t\right]\right\}.
\end{aligned}$$

Substituting (B.3), (B.8) and (B.9) into the right-hand side of this expression, yields:

$$
\begin{aligned}
& \gamma_0+\gamma_1 y_{t-1}+\frac{1}{2}\gamma_2 y_{t-1}^2+\gamma_3\theta_{t-1}+\frac{1}{2}\gamma_4\theta_{t-1}^2+\gamma_5\theta_{t-1}y_{t-1} \qquad \text{(B.10)}\\
= \; & \mathrm{E}_{t-1}\left\{\frac{1+f}{2}\left[a-b\varepsilon_t-cy_{t-1}-d\theta_{t-1}-\pi^*\right]^2\right.\\
& +\frac{\lambda}{2}\left[\rho y_{t-1}+(1-\alpha b)\,\varepsilon_t+\theta_{t-1}-y^*\right]^2\\
& +\frac{\psi}{2}\left[a+[1-(1+\alpha)b]\varepsilon_t-(c+1-\rho)y_{t-1}+(1-d)\theta_{t-1}-g\right]^2\\
& \left.+\beta\left[\gamma_0+\gamma_1 y_t+\frac{1}{2}\gamma_2 y_t^2+\gamma_3\theta_t+\frac{1}{2}\gamma_4\theta_t^2+\gamma_5\theta_t y_t\right]\right\}.
\end{aligned}
$$

We write out each of the terms on the right-hand side of (B.10) separately:

$$
\begin{aligned}
& \frac{1+f}{2}\mathrm{E}_{t-1}\left[a-b\varepsilon_t-cy_{t-1}-d\theta_{t-1}-\pi^*\right]^2=\\
& \frac{1+f}{2}\left[a^2+b^2\sigma_\epsilon^2+(\pi^*)^2-2a\pi^*\right]+(1+f)\,c\,(\pi^*-a)\,y_{t-1}+\frac{1+f}{2}c^2y_{t-1}^2\\
& +(1+f)\,d\,(\pi^*-a)\,\theta_{t-1}+\frac{1+f}{2}d^2\theta_{t-1}^2+(1+f)\,cd\theta_{t-1}y_{t-1},
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\lambda}{2}\mathrm{E}_{t-1}\left[\rho y_{t-1}+(1-\alpha b)\,\varepsilon_t+\theta_{t-1}-y^*\right]^2\\
= \; & \frac{\lambda}{2}\left[\rho^2y_{t-1}^2+(1-\alpha b)^2\sigma_\epsilon^2+(y^*)^2-2\rho y_{t-1}y^*\right]\\
& -\lambda y^*\theta_{t-1}+\frac{1}{2}\lambda\theta_{t-1}^2+\lambda\rho\theta_{t-1}y_{t-1},
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\psi}{2}\mathrm{E}_{t-1}\left[a+(1-(1+\alpha)\,b)\,\varepsilon_t-(c+1-\rho)\,y_{t-1}+(1-d)\theta_{t-1}-g\right]^2\\
= \; & \frac{\psi}{2}\left[a^2+g\,(g-2a)+(1-(1+\alpha)\,b)^2\,\sigma_\epsilon^2\right]\\
& +\psi\,(c+1-\rho)\,(g-a)\,y_{t-1}+\frac{\psi}{2}\,(c+1-\rho)^2\,y_{t-1}^2\\
& +\psi(a-g)(1-d)\theta_{t-1}+\frac{1}{2}\psi(1-d)^2\theta_{t-1}^2-\psi(c+1-\rho)(1-d)\theta_{t-1}y_{t-1},
\end{aligned}
$$

$$
\begin{aligned}
& \beta\mathrm{E}_{t-1}\left[\gamma_0+\gamma_1 y_t+\frac{1}{2}\gamma_2 y_t^2+\gamma_3\theta_t+\frac{1}{2}\gamma_4\theta_t^2+\gamma_5\theta_t y_t\right]\\
= \; & \beta\left[\gamma_0+\frac{1}{2}\gamma_2\,(1-\alpha b)^2\,\sigma_\epsilon^2+\frac{1}{2}\gamma_4\sigma_\theta^2\right]+\beta\rho\gamma_1 y_{t-1}+\frac{1}{2}\beta\rho^2\gamma_2 y_{t-1}^2\\
& +\beta\gamma_1\theta_{t-1}+\frac{1}{2}\beta\gamma_2\theta_{t-1}^2+\beta\rho\gamma_2\theta_{t-1}y_{t-1},
\end{aligned}
$$

where $\sigma_\theta^2 \equiv \mathrm{E}_{t-1}\left[\theta_t^2\right]$ is the variance of θ_t. Collecting terms on the right-hand sides of these four equations, we can write the right-hand side of (B.10) as:

$$
\begin{aligned}
&\frac{\lambda}{2}\left[(1-\alpha b)^2\sigma_\epsilon^2+(y^*)^2\right]+\frac{1+f}{2}\left[a^2+b^2\sigma_\epsilon^2+\pi^*\left(\pi^*-2a\right)\right]\\
&+\frac{\psi}{2}\left[a^2+g\left(g-2a\right)+\left(1-\left(1+\alpha\right)b\right)^2\sigma_\epsilon^2\right]\\
&+\beta\gamma_0+\frac{1}{2}\beta\gamma_2\left(1-\alpha b\right)^2\sigma_\epsilon^2+\frac{1}{2}\beta\gamma_4\sigma_\theta^2\\
&+\left[-\lambda\rho y^*+\left(1+f\right)c\left(\pi^*-a\right)+\psi\left(c+1-\rho\right)\left(g-a\right)+\beta\gamma_1\rho\right]y_{t-1}\\
&+\frac{1}{2}\left[\lambda\rho^2+\left(1+f\right)c^2+\psi\left(c+1-\rho\right)^2+\beta\gamma_2\rho^2\right]y_{t-1}^2\\
&+\left[-\lambda y^*+\left(1+f\right)d\left(\pi^*-a\right)+\psi(a-g)(1-d)+\beta\gamma_1\right]\theta_{t-1}\\
&+\frac{1}{2}\left[\lambda+\left(1+f\right)d^2+\psi(1-d)^2+\beta\gamma_2\right]\theta_{t-1}^2\\
&+\left[\lambda\rho+\left(1+f\right)cd-\psi(1-d)(c+1-\rho)+\beta\rho\gamma_2\right]\theta_{t-1}y_{t-1}.
\end{aligned}
$$

This confirms the conjectured form of the value function. To obtain the outcomes for π_t, y_t and g_t we only need to solve for γ_2 and γ_1. The relevant Riccati equations are therefore given by (A.9) and (A.10) just as in the case when output is given by equation (1).

Now, we solve the model for HYAR. The Bellman equation is:

$$
V^h\left(y_{t-1}\right)=\mathrm{E}_{t-1}\min_{\pi_t}\left\{\frac{1}{2}\left[\left(1+\tilde{f}\right)\left(\pi_t-\tilde{\pi}\right)^2+\lambda(y_t-y^*)^2+\tilde{\psi}g_t^2\right]+\beta V^h\left(y_t\right)\right\}.
$$

Proceeding in the same way as under constant nominal income growth targeting, we find that inflation is given by:

$$
\pi_t=\tilde{a}-\tilde{b}\varepsilon_t-\tilde{c}y_{t-1}-\tilde{d}\theta_{t-1}, \tag{B.11}
$$

where

$$
\tilde{a}=\frac{\left(1+\tilde{f}\right)\tilde{\pi}+\lambda\alpha y^*-\beta\alpha\tilde{\gamma}_1}{\tilde{\Omega}}, \tag{B.12}
$$

$$
\tilde{b}=\frac{\alpha\left(\lambda+\beta\tilde{\gamma}_2\right)+\tilde{\psi}\left(1+\alpha\right)}{\alpha^2\left(\lambda+\beta\tilde{\gamma}_2\right)+1+\tilde{f}+\tilde{\psi}\left(1+\alpha\right)^2}, \tag{B.13}
$$

$$
\tilde{c}=\frac{\alpha\rho\left(\lambda+\beta\tilde{\gamma}_2\right)-\tilde{\psi}\left(1+\alpha\right)\left(1-\rho\right)}{\tilde{\Omega}}, \tag{B.14}
$$

$$
\tilde{d}=\frac{\alpha\left(\lambda+\beta\tilde{\gamma}_2\right)+\tilde{\psi}\left(1+\alpha\right)}{\tilde{\Omega}}, \tag{B.15}
$$

with

$$
\tilde{\Omega}\equiv 1+\tilde{f}+\tilde{\psi}\left(1+\alpha\right),
$$

and where we have conjectured that the value function is of the following format:

$$V^h(y_t) = \tilde{\gamma}_0 + \tilde{\gamma}_1 y_t + \frac{1}{2}\tilde{\gamma}_2 y_t^2 + \tilde{\gamma}_3\theta_t + \frac{1}{2}\tilde{\gamma}_4\theta_t^2 + \tilde{\gamma}_5\theta_t y_t.$$

In equilibrium the following recursion must hold

$$\begin{aligned}
& \tilde{\gamma}_0 + \tilde{\gamma}_1 y_{t-1} + \tfrac{1}{2}\tilde{\gamma}_2 y_{t-1}^2 + \tilde{\gamma}_3\theta_{t-1} + \tfrac{1}{2}\tilde{\gamma}_4\theta_{t-1}^2 + \tilde{\gamma}_5\theta_{t-1}y_{t-1} \\
= \; & \mathrm{E}_{t-1}\left\{\frac{1+\tilde{f}}{2}\left[\tilde{a} - \tilde{b}\varepsilon_t - \tilde{c}y_{t-1} - \tilde{d}\theta_{t-1} - \tilde{\pi}\right]^2\right. \\
& + \frac{\lambda}{2}\left[\rho y_{t-1} + \left(1 - \alpha\tilde{b}\right)\varepsilon_t + \theta_{t-1} - y^*\right]^2 \\
& + \frac{\tilde{\psi}}{2}\left[\tilde{a} + [1-(1+\alpha)\tilde{b}]\varepsilon_t - (\tilde{c}+1-\rho)y_{t-1} + (1-\tilde{d})\theta_{t-1}\right]^2 \\
& \left. + \beta\left[\tilde{\gamma}_0 + \tilde{\gamma}_1 y_t + \frac{1}{2}\tilde{\gamma}_2 y_t^2 + \tilde{\gamma}_3\theta_t + \frac{1}{2}\tilde{\gamma}_4\theta_t^2 + \tilde{\gamma}_5\theta_t y_t\right]\right\}
\end{aligned} \tag{B.16}$$

We write out each of the terms on the right-hand side of (B.16) separately:

$$\begin{aligned}
& \frac{1+\tilde{f}}{2}\mathrm{E}_{t-1}\left[\tilde{a} - \tilde{b}\varepsilon_t - \tilde{c}y_{t-1} - \tilde{d}\theta_{t-1} - \tilde{\pi}\right]^2 = \\
& \frac{1+\tilde{f}}{2}\left[\tilde{a}^2 + \tilde{b}^2\sigma_\epsilon^2 + \tilde{\pi}^2 - 2\tilde{a}\tilde{\pi}\right] + \left(1+\tilde{f}\right)\tilde{c}\left(\tilde{\pi} - \tilde{a}\right)y_{t-1} + \frac{1+\tilde{f}}{2}\tilde{c}^2 y_{t-1}^2 \\
& + \left(1+\tilde{f}\right)\tilde{d}\left(\tilde{\pi} - \tilde{a}\right)\theta_{t-1} + \frac{1+\tilde{f}}{2}\tilde{d}^2\theta_{t-1}^2 + \left(1+\tilde{f}\right)\tilde{c}\tilde{d}\theta_{t-1}y_{t-1},
\end{aligned}$$

$$\begin{aligned}
& \frac{\lambda}{2}\mathrm{E}_{t-1}\left[\rho y_{t-1} + \left(1-\alpha\tilde{b}\right)\varepsilon_t + \theta_{t-1} - y^*\right]^2 \\
= \; & \frac{\lambda}{2}\left[\rho^2 y_{t-1}^2 + \left(1-\alpha\tilde{b}\right)^2\sigma_\epsilon^2 + (y^*)^2 - 2\rho y_{t-1}y^*\right] \\
& -\lambda y^*\theta_{t-1} + \frac{1}{2}\lambda\theta_{t-1}^2 + \lambda\rho\theta_{t-1}y_{t-1},
\end{aligned}$$

$$\begin{aligned}
& \frac{\tilde{\psi}}{2}\mathrm{E}_{t-1}\left[\tilde{a} + \left(1-(1+\alpha)\tilde{b}\right)\varepsilon_t - \left(\tilde{c}+1-\rho\right)y_{t-1} + (1-\tilde{d})\theta_{t-1}\right]^2 \\
= \; & \frac{\tilde{\psi}}{2}\left[\tilde{a}^2 + \left(1-(1+\alpha)\tilde{b}\right)^2\sigma_\epsilon^2\right] - \tilde{\psi}\tilde{a}\left(\tilde{c}+1-\rho\right)y_{t-1} + \frac{\tilde{\psi}}{2}\left(\tilde{c}+1-\rho\right)^2 y_{t-1}^2 \\
& + \tilde{\psi}\tilde{a}(1-\tilde{d})\theta_{t-1} + \frac{1}{2}\tilde{\psi}(1-\tilde{d})^2\theta_{t-1}^2 - \tilde{\psi}(\tilde{c}+1-\rho)(1-\tilde{d})\theta_{t-1}y_{t-1},
\end{aligned}$$

$$\begin{aligned}
& \beta \mathrm{E}_{t-1}\left[\tilde{\gamma}_0+\tilde{\gamma}_1 y_t+\frac{1}{2} \tilde{\gamma}_2 y_t^2+\tilde{\gamma}_3 \theta_t+\frac{1}{2} \tilde{\gamma}_4 \theta_t^2+\tilde{\gamma}_5 \theta_t y_t\right] \\
= & \beta\left[\tilde{\gamma}_0+\frac{1}{2} \tilde{\gamma}_2\left(1-\alpha \tilde{b}\right)^2 \sigma_\epsilon^2+\frac{1}{2} \tilde{\gamma}_4 \sigma_\theta^2\right]+\beta \rho \tilde{\gamma}_1 y_{t-1}+\frac{1}{2} \beta \rho^2 \tilde{\gamma}_2 y_{t-1}^2 \\
& +\beta \tilde{\gamma}_1 \theta_{t-1}+\frac{1}{2} \beta \tilde{\gamma}_2 \theta_{t-1}^2+\beta \rho \tilde{\gamma}_2 \theta_{t-1} y_{t-1} .
\end{aligned}$$

Collecting terms on the right-hand sides of these four equations, we can write the right-hand side of (B.16) as:

$$\begin{aligned}
& \frac{\lambda}{2}\left[\left(1-\alpha \tilde{b}\right)^2 \sigma_\epsilon^2+\left(y^*\right)^2\right]+\frac{1+\tilde{f}}{2}\left[(\tilde{a}-\tilde{\pi})^2+\tilde{b}^2 \sigma_\epsilon^2\right]+\frac{\tilde{\psi}}{2}\left[\tilde{a}^2+\left(1-(1+\alpha) \tilde{b}\right)^2 \sigma_\epsilon^2\right] \\
& +\beta \tilde{\gamma}_0+\frac{1}{2} \beta \tilde{\gamma}_2\left(1-\alpha \tilde{b}\right)^2 \sigma_\epsilon^2+\beta \tilde{\gamma}_4 \sigma_\theta^2 \\
& +\left[-\lambda \rho y^*+\left(1+\tilde{f}\right) \tilde{c}(\tilde{\pi}-\tilde{a})-\tilde{\psi}(\tilde{c}+1-\rho) \tilde{a}+\beta \rho \tilde{\gamma}_1\right] y_{t-1} \\
& +\frac{1}{2}\left[\lambda \rho^2+\left(1+\tilde{f}\right) \tilde{c}^2+\tilde{\psi}(\tilde{c}+1-\rho)^2+\beta \rho^2 \tilde{\gamma}_2\right] y_{t-1}^2 \\
& +\left[-\lambda y^*+\left(1+\tilde{f}\right) \tilde{d}(\tilde{\pi}-\tilde{a})+\tilde{\psi} \tilde{a}(1-\tilde{d})+\beta \tilde{\gamma}_1\right] \theta_{t-1} \\
& +\frac{1}{2}\left[\lambda+\left(1+\tilde{f}\right) \tilde{d}^2+\tilde{\psi}(1-\tilde{d})^2+\beta \tilde{\gamma}_2\right] \theta_{t-1}^2 \\
& +\left[\lambda \rho+\left(1+\tilde{f}\right) \tilde{c} \tilde{d}-\tilde{\psi}(1-\tilde{d})(\tilde{c}+1-\rho)+\beta \rho \tilde{\gamma}_2\right] \theta_{t-1} y_{t-1} .
\end{aligned}$$

Hence, the relevant Riccati equations are given by:

$$\tilde{\gamma}_2 = \lambda \rho^2+\left(1+\tilde{f}\right) \tilde{c}^2+\tilde{\psi}(\tilde{c}+1-\rho)^2+\beta \rho^2 \tilde{\gamma}_2, \tag{B.17}$$

$$\tilde{\gamma}_1 = -\lambda \rho y^*+(1+\tilde{f}) \tilde{c}(\tilde{\pi}-\tilde{a})-\tilde{\psi}(\tilde{c}+1-\rho) \tilde{a}+\beta \tilde{\gamma}_1 \rho. \tag{B.18}$$

Now we demonstrate that there exists a mapping of delegation parameters which induces the same inflation rule under constant nominal income growth targeting and under HYAR, i.e., such that $a=\tilde{a}$, $b=\tilde{b}$, $c=\tilde{c}$, and $d=\tilde{d}$. First note that parameters b, c, d and γ_2 are determined by the system (B.5), (B.6), (B.7) and (A.9). Then note that parameters $\tilde{b}, \tilde{c}, \tilde{d}$ and $\tilde{\gamma}_2$ are determined by the system (B.13), (B.14), (B.15) and (B.17). But if we set $f=\tilde{f}$ and $\psi=\tilde{\psi}$, these two systems, and hence their solutions, are *exactly the same.*

To achieve $a=\tilde{a}$, first note that the explicit solution for a can, along the same lines as in the proof of the proposition, be found as

$$a=\frac{\lambda \alpha y^*+(1+f)[1-\beta(\rho+\alpha c)] \pi^*+\psi[1-\beta(\rho+\alpha c)+\alpha(1-\beta)] g}{\Omega(1-\beta \rho)-(1+f) \alpha \beta c-\psi \alpha \beta(c+1-\rho)}, \tag{B.19}$$

which in fact is identical to (A.17). Furthermore, note that we can rewrite (B.12) in terms of $\tilde{\gamma}_1$ as

$$\tilde{\gamma}_1 = \frac{\left(1+\tilde{f}\right)\tilde{\pi} + \lambda\alpha y^* - \tilde{a}\tilde{\Omega}}{\alpha\beta}, \tag{B.20}$$

and rewrite (B.18) as

$$\tilde{\gamma}_1 (1-\beta\rho) = -\lambda\rho y^* + (1+\tilde{f})\tilde{c}(\tilde{\pi}-\tilde{a}) - \tilde{\psi}(c+1-\rho)\tilde{a}. \tag{B.21}$$

Substitute (B.20) into (B.21) and multiply both sides by $\alpha\beta$ to give:

$$\begin{aligned} & \left(1+\tilde{f}\right)\tilde{\pi}(1-\beta\rho) + \lambda\alpha y^*(1-\beta\rho) - \tilde{a}\tilde{\Omega}(1-\beta\rho) \\ = \; & -\alpha\beta\lambda\rho y^* + \alpha\beta(1+\tilde{f})\tilde{c}(\tilde{\pi}-\tilde{a}) - \alpha\beta\tilde{\psi}(\tilde{c}+1-\rho)\tilde{a}, \end{aligned}$$

from which we obtain the explicit solution of $\tilde{a}$:

$$\tilde{a} = \frac{\lambda\alpha y^* + \left(1+\tilde{f}\right)[1-\beta(\rho+\alpha\tilde{c})]\tilde{\pi}}{\tilde{\Omega}(1-\beta\rho) - \left(1+\tilde{f}\right)\alpha\beta\tilde{c} - \tilde{\psi}\alpha\beta(\tilde{c}+1-\rho)}. \tag{B.22}$$

Comparing (B.19) with (B.22), it then follows — assuming again that continuity condition (A.18) holds — that $a = \tilde{a}$ if g is set such that

$$(1+f)[1-\beta(\rho+\alpha c)](\tilde{\pi}-\pi^*) = \psi[1-\beta(\rho+\alpha c)+\alpha(1-\beta)]g, \tag{B.23}$$

where we have used $f = \tilde{f}$ and $\psi = \tilde{\psi}$ (and, hence, that $\Omega = \tilde{\Omega}$ and $c = \tilde{c}$). Clearly, this is feasible (note that c does not depend on g) if $\tilde{\psi} \neq 0$ and $1-\beta(\rho+\alpha c) > 0$. The latter is just the continuity condition under inflation targeting as shown by Svensson [1997, equation (A9) in Appendix A], which, of course, must also be satisfied here.

Hence, if $\tilde{\psi} \neq 0$, choosing $f = \tilde{f}$, $\psi = \tilde{\psi}$ and g so as to satisfy (B.23), ensures that constant nominal income growth targeting produces the same inflation rule as HYAR.

Step 2:

Denote the optimal combination of delegation parameters $(\tilde{f}, \tilde{\psi}, \tilde{\pi})$ under constant inflation targeting by $(\tilde{f}^*, 0, \tilde{\pi}^*)$ (remember that constant inflation targeting is the special case of HYAR in which $\tilde{\psi} = 0$). Furthermore, denote the parameters of the inflation rule under optimal constant inflation targeting by $(\tilde{a}^*, \tilde{b}^*, \tilde{c}^*, \tilde{d}^*)$.

First, note that $\tilde{\psi}^* = 0$ implies that $\tilde{c}^* \neq 0$.[12] We now show that it is possible to choose a combination $(\tilde{f}, \tilde{\psi}, \tilde{\pi})$ under HYAR with $\tilde{\psi} \neq 0$ which produces $\tilde{c} = 0$, while keeping $\tilde{a}$, $\tilde{b}$ and $\tilde{d}$ at their values under optimal constant inflation targeting, $\tilde{a}^*$, $\tilde{b}^*$ and $\tilde{d}^*$, respectively. In other words, we can eliminate the state-contingent inflation bias associated with output persistence, without changing the other coefficients of the solution for inflation under constant inflation targeting. Clearly, this reduces society's equilibrium welfare loss. Hence, the optimal arrangement under HYAR is characterized by $\tilde{\psi} \neq 0$.

Because the solution for $\tilde{c}$ is given by expression (A.13) with f and ψ replaced by $\tilde{f}$ and $\tilde{\psi}$, respectively, we can set $\tilde{c} = 0$ by setting

$$\tilde{\psi} = \frac{\alpha\left(\lambda + \beta\tilde{\gamma}_2\right)}{\left(1+\alpha\right)\left(1-\rho\right)}. \tag{B.24}$$

Note that, in fact, $\tilde{\psi} = \psi^* \neq 0$ as given by (8), because the system (A.5) and (A.9) determining c and γ_2 is the same as the system (B.14) and (B.17) that determines $\tilde{c}$ and $\tilde{\gamma}_2$, if we replace f by $\tilde{f}$ and ψ by $\tilde{\psi}$.

Substitute (B.24) into the expression (B.13) for $\tilde{b}$, which then becomes:

$$\tilde{b} = \frac{\alpha\left(\lambda + \beta\tilde{\gamma}_2\right)}{\alpha\left(\alpha+\rho\right)\left(\lambda+\beta\tilde{\gamma}_2\right) + \left(1-\rho\right)\left(1+\tilde{f}\right)}. \tag{B.25}$$

We can now choose $\tilde{f}$ so as to make sure that $\tilde{b} = \tilde{b}^*$. Denote this value of $\tilde{f}$ by $\tilde{f}\left(\tilde{b}^*\right)$.[13] Then notice from (B.13) and (B.15) that, *irrespective* of the specific choice of the delegation parameters, the following relation applies:

$$\tilde{d} = \frac{\tilde{b}}{1-\alpha\tilde{b}}.$$

Hence, if we set $\tilde{f}$ at $\tilde{f}\left(\tilde{b}^*\right)$, we not only ensure that $\tilde{b} = \tilde{b}^*$ but also that $\tilde{d} = \tilde{d}^*$.

Finally, through an appropriate choice of $\tilde{\pi}$ in (B.22), after having substituted $\tilde{c} = 0$, $\tilde{\psi} = \psi^*$ and $\tilde{f}\left(\tilde{b}^*\right)$, we can ensure that $\tilde{a}$ equals $\tilde{a}^*$.

We have thus shown that the optimal constant inflation targeting arrangement ($\tilde{\psi} = 0$) is outperformed by some HYAR with $\tilde{\psi} \neq 0$. By the arguments of *Step 1* it is therefore

[12] Remember that the solution for $\tilde{c}$ is given by expression (A.13) with f and ψ replaced by $\tilde{f}$ and $\tilde{\psi}$, respectively. With constant inflation targeting ($\tilde{\psi} = 0$) we always have that $\tilde{c} \neq 0$: imposing $\tilde{\psi} = 0$, the only possibility to have $\tilde{c} = 0$ would be if $\tilde{f} \to -1$. However, the denominator of the expression for $\tilde{c}$ goes to zero at a faster rate than its numerator. Hence, $\tilde{c} = 0$ is ruled out if $\tilde{\psi} = 0$.

[13] Note in solving for $\tilde{f}(\tilde{b}^*)$ that $\tilde{\gamma}_2$ does not depend on $\tilde{f}$ [this follows from setting $\tilde{c} = 0$ in (B.17)]. $\tilde{\gamma}_2$ only depends on the basic model parameters and $\tilde{\psi} = \psi^*$, which itself is a function of the basic model parameters.

also outperformed by some appropriately chosen constant nominal income growth targeting arrangement. Note that the arguments of this proof stress that having the central bank focus on nominal income growth, i.e., $\tilde{\psi} \neq 0$, makes it possible to address the state-contingent inflation bias associated with output persistence in a way that is superior to constant inflation targeting (as claimed in Section II).

References

Fischer, Stanley, 1995, Modern Approaches to Central Banking, in Forrest Capie, Stanley Fischer, Charles Goodhart and Norbert Schnadt, eds., The Future of Central Banking (Cambridge: Cambridge University Press), 262-308.

Goodhart, Charles and José Viñals, 1994, Strategy and Tactics of Monetary Policy: Examples from Europe and the Antipodes, in Jeffrey C. Fuhrer, ed., Goals, Guidelines, and Constraints facing Monetary Policymakers (Federal Reserve Bank of Boston, Boston), 139-187.

Hall, Robert E. and N. Gregory Mankiw, 1994, Nominal Income Targeting, in N. Gregory Mankiw, ed., Monetary Policy (University of Chicago Press, Chicago).

Herrendorf, Berthold and Ben Lockwood, 1997, Rogoff's "Conservative" Central Banker Restored, Journal of Money, Credit, and Banking 29, 476-495.

Jonsson, Gunnar, 1997, Monetary Politics and Unemployment Persistence, Journal of Monetary Economics 39, 303-326.

Lockwood, Ben, 1997, State-Contingent Inflation Contracts and Unemployment Persistence, Journal of Money, Credit, and Banking 29, 286-299.

Lockwood, Ben and Apostolis Philippopoulos, 1994, Insider Power, Unemployment Dynamics and Multiple Inflation Equilibria, Economica 61, 59-77.

Lohmann, Susanne, 1992, Optimal Commitment in Monetary Policy: Credibility versus Flexibility, American Economic Review 82, 273-286.

McCallum, Bennett T., 1995, Two Fallacies Concerning Central-Bank Independence, American Economic Review, Papers and Proceedings 85, 207-211.

Mishkin, Frederic S. and Adam S. Posen, 1997, Inflation Targeting: Lessons From Four Countries, Federal Reserve Bank of New York Economic Policy Review 3, 9-110.

Persson, Torsten and Guido Tabellini, 1993, Designing Institutions for Monetary Stability, Carnegie-Rochester Conference Series on Public Policy 39, 53-84.

Persson, Torsten and Guido Tabellini, 1998, Political Economics and Macroeconomic Policy, forthcoming in John B. Taylor and Michael Woodford, eds., Handbook of Macroeconomics.

Rogoff, Kenneth, 1985, The Optimal Degree of Commitment to an Intermediate Monetary Target, Quarterly Journal of Economics 100, 1169-1189.

Svensson, Lars E. O., 1995, Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts, NBER Working Paper No. 5251.

Svensson, Lars E. O., 1997, Optimal Inflation Targets, 'Conservative' Central Banks, and Linear Inflation Contracts, American Economic Review 87, 98-114.

Walsh, Carl E., 1995, Optimal Contracts for Independent Central Bankers, American Economic Review 85, 150-167.