Written Exam for the M.Sc. in Economics winter 2013-14-R
Advanced Development Economics – Macro aspects
Master's Course
Solution manual

Solution guide A.1.

This draws on the paper by Acemoglu et al (2001), Albouy (2012) and Glaeser et al (2004). The student should be able to account for the theory developed by the authors, which links settler mortality rates, to local settlements of Europeans, to early institutions and late institutions. Then it should be explained how the authors employ the settler mortality rate as an instrument for contemporary institutions. The exclusion restriction needs to be stated explicitly, and comments on its plausibility (or lack thereof) should be made. Finally, the students should discuss the empirical critique of the instrument voiced by Albouy (2012) as well as Glaeser et al (2004).

A2. This draws especially on Li and Zhang (2007).

The student should clarify the basic problem faced by a researcher when trying to gauge the impact of fertility on economic growth. That is, reverse causality and omitted variables. Reference can and should be made to relevant theories (especially on the quantity quality trade-off) where growth in income plays a role in determining family size (cf Galor, 2011).

Next it should be explained that a critical feature of the reform was that it did not apply to minority groups. The forms the basis of for a potential instrument for fertility, namely the fraction of the province population that are non-Han Chinese (the ethnic majority). The exclusion restriction should be clarified and discussed. In particular one might worry that ethnic minorities have other effects on growth that via fertility (e.g. provinces dominated by ethnic minorities might receive less infrastructure investments). Careful attention should be paid to the fact that other determinants of growth (education and physical capital accumulation) are controlled. Since the authors have access to panel data they can try to identify these determinants by GMM.

The results suggest a very large impact on growth, conditional on physical and even human capital. Stricktly speaking, therefore, it amounts to a TFP effect. It is not a requirement that the student remembers the point estimate (1 % point reduction in population growth raises long-run income by 140%) but it should be remarked that the impact is too large to be accounted for by the capital dillusion effect seen in a Solow model.

SOLUTIONS: EXERCISE B

Solution Q1:

$$\max \log(n_t) + (w - pn)$$
$$\frac{1}{n} - p = 0 \Rightarrow n = 1/p.$$

inserted into the budget constraint

$$w = (1/n)\,n + m = 1 + m \Leftrightarrow m = w - 1.$$

The first equation signifies that as the "price of provisions" increase people respond by lowering optimal family size; when the price of food increases the "price" of children goes up. The second equation shows us that as wages increase people respond by consuming a greater fraction of total income on m-goods. Hence Engel's law prevails.

Solution Q2: From equilbrium in m-market

$$mL = (w-1)\,L = (B-1)\,L = Y_m = BL_m$$

hence

$$L_m/L = \frac{B-1}{B}$$

and since $L_m/L + L_a/L = 1$

$$L_a/L = 1 - L_m/L = 1/B$$

From the no-arbitrage condition

$$w_a = pY_a/L_a = w_m = B$$

so

$$\begin{aligned} p &= \frac{B}{Y_a/L_a} = \frac{B}{AL_{at}^{-\alpha}X^{\alpha}} = \frac{B}{A\,(1/B)^{-\alpha}\,L^{-\alpha}X^{\alpha}} \\ &= \frac{B^{1-\alpha}L^{\alpha}}{AX^{\alpha}} \end{aligned}$$

(ii) When A increases the return to being in the a-sector increases, which induces an inflow of labor. Hence, due to the increase in A and the increase in L_a the relative supply of the a-good increases, which implies the price has to fall in equilibrium. An increase in L implies more labor in both sectors. But since labor is subject to diminishing return in the a-sector it also implies a lower average product which raises the price of a-goods in equilibrium. An increase in B makes it more attractive to move out of the a-sector, which implies lower relative supply of a-goods and therefore a higher price p.

Solution Q3. The key issue is the behavior of $\Psi(L)$.

We have $\frac{AX^\alpha}{B^{1-\alpha}}$

$$\Psi' = (1-\alpha)\frac{AX^\alpha}{B^{1-\alpha}}L_t^{-\alpha} > 0 \text{ for all } L$$

$$\Psi'' = -\alpha(1-\alpha)(1-\alpha)\frac{AX^\alpha}{B^{1-\alpha}}L_t^{-\alpha-1} < 0 \text{ for all } L$$

we also have

$$\Psi(0) = 0$$

and

$$\begin{aligned}\lim_{L\to 0}(1-\alpha)\frac{AX^\alpha}{B^{1-\alpha}}L_t^{-\alpha} &= \infty \\ \lim_{L\to\infty}(1-\alpha)\frac{AX^\alpha}{B^{1-\alpha}}L_t^{-\alpha} &= 0\end{aligned}$$

Hence, Ψ starts at zero, with a slope greater than one which tends to zero monotonically since $\Psi' > 0$ and $\Psi'' < 0$ for all L. This establishes a unique (non-trivial) steady state. Phasediagram illustrated in Figure 1.

Question 4. Using the phasediagram one can convince oneself that the steady state is globally stable. When $L_0 < L^*$ the population is evidently growing, as seen from figure 1. This will continue until the steady state is attained. Similarly, the population will be shrinking if $L_0 > L^*$.

Figure 2 shows the time paths. Initially $L_0 < L^*$. As a consequence $p < p^* = 1$ as seen from the equilibrium price ("1" has to be the steady state price, as the steady state requires $n = 1$ and since $n = 1/p$ by the household optimization). The reason is diminishing returns to labor input in agriculture. As the population grows productivity in the a-sector declines and the price increases. Gradually, therefore population growth abades towards steady state, where $n = 1$.

Question 5.

GDP:

$$Y = pAL_a^{1-\alpha}X^\alpha + BL_m$$

Now at all points in time the following three things are true. Population in m-sector

$$L_m = ((B-1)/B)L$$

and by labor market clearing in the a-sector:

$$L_a = (1/B)\, L$$

and, finally, the equilibrium price:

$$p_t = \frac{B^{1-\alpha}}{AL_t^{-\alpha} X^{\alpha}}.$$

Insert into the definition of Y:

$$\begin{aligned}
Y &= \frac{B^{1-\alpha}}{AL_t^{-\alpha} X^{\alpha}} A \left[L\,(1/B)\right]^{1-\alpha} X^{\alpha} + B\left[(B-1)/B\right] L \\
\frac{Y}{L} &= \frac{B^{1-\alpha}}{AL_t^{-\alpha} X^{\alpha} B^{1-\alpha}} A\,(X/L)^{\alpha} + B\left[(B-1)/B\right] \\
\frac{Y}{L} &= \frac{B^{1-\alpha} A\,(X/L)^{\alpha}}{AL_t^{-\alpha} X^{\alpha} B^{1-\alpha}} + B\left[(B-1)/B\right] \\
&= 1 + (B-1) \\
&= B.
\end{aligned}$$

Hence $y = B$ at all points in time. Steady state density (use $L_{t+1} = L_t = L^*$ in the Law of motion)

$$(L/X)^* = \left(\frac{A}{B^{1-\alpha}}\right)^{1/\alpha}.$$

Question 6. Figure 3 details the time paths. When B increases it draws in people from the a-sector to the m-sector. Hence, the relative supply of a-goods decline, for which reason p increases as illustrated. This leads to smaller families as the price of provisions increases; n drops below replacement. In the next generation there are fewer people around, including in the a-sector. Due to diminishing returns this increases productivity in the a-sector prompting a drop in p and an increase in fertility towards its replacement level. Gradually the economy adjusts towards the steady state featuring lower density, higher prosperity and a price which is back at 1.

Figure 4 details the results when we consider an increase in a-sector productivity. The consequence is an upward shift in the law of motion for L, which implies a higher long-run population density. The immediate impact of A is to lower the price of provisions p, which elevates fertility. There is no impact on y; the initial increase in the a-sector is counteracted by the declining price of a-goods which ensures that $pY_a/L_a = B$ holds througout. Over time the population rises, and therefore, the price p declines due to diminishing returns. Eventually, the economy settles down in a new steady state where density is higher, but living standards are unaffected.

Question 7. Free style. Worth bringing out: lack of growth until recently. The model explains how episodes of technological change - in agriculture - did not lead to greater income, but "only" more densely settle communities.

One might also wish to discuss Ashraf-Galor in light of the model. The find that time-since-Neolithic to be a strong determinant of density, but not income. Suggests the neolithic mainly unleashed innovative activties in the a-sector. Or, that maybe the income data is of too poor quality to say otherwise.

Also suggests that a process of technological change outside agriculture could unleash fertility transition and an onset of growth. Hence technological change (directed towards m-sector) would be important in forwarding the take-off. Delays in the transition could be caused e.g. by policies and institutions that limit the movements of people across sectors.

Figure 1: Phase diagram.

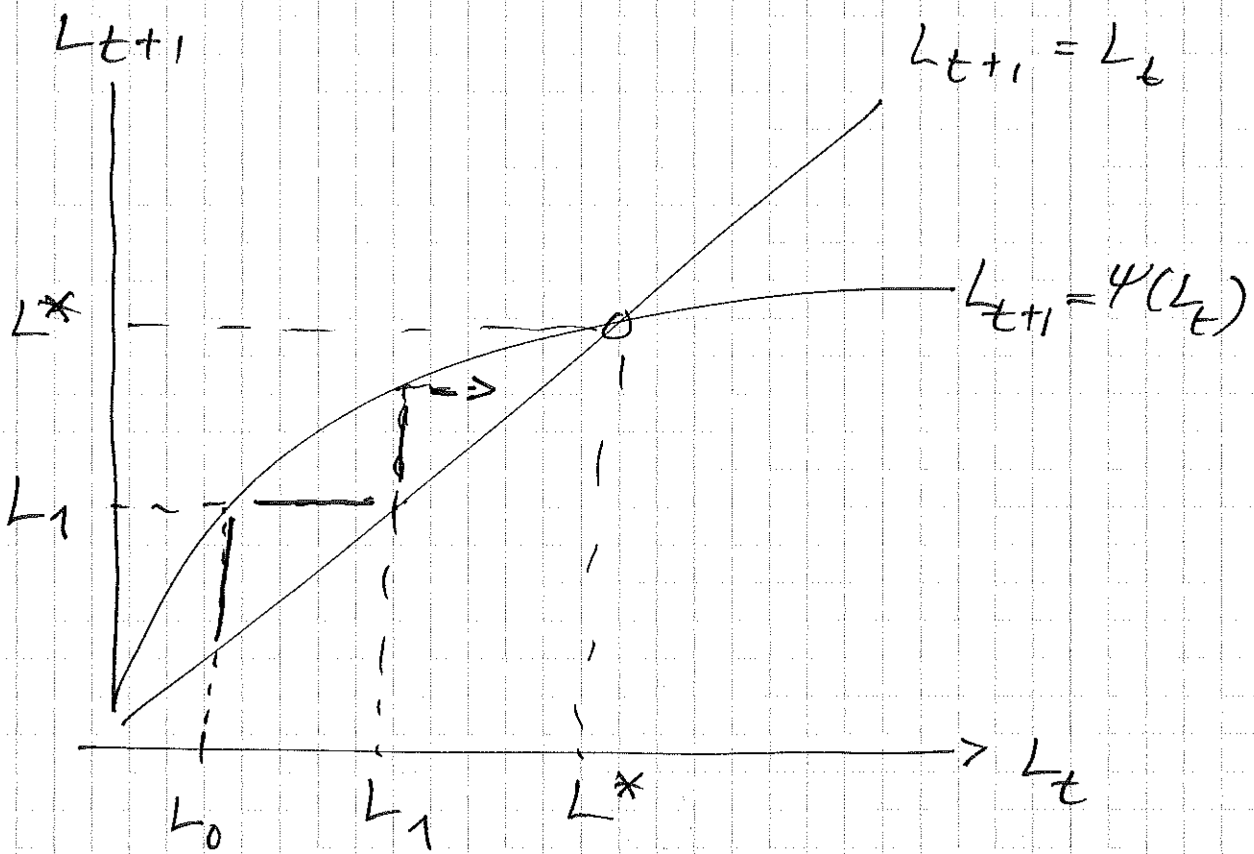


Figure 2: Time paths for p and n.

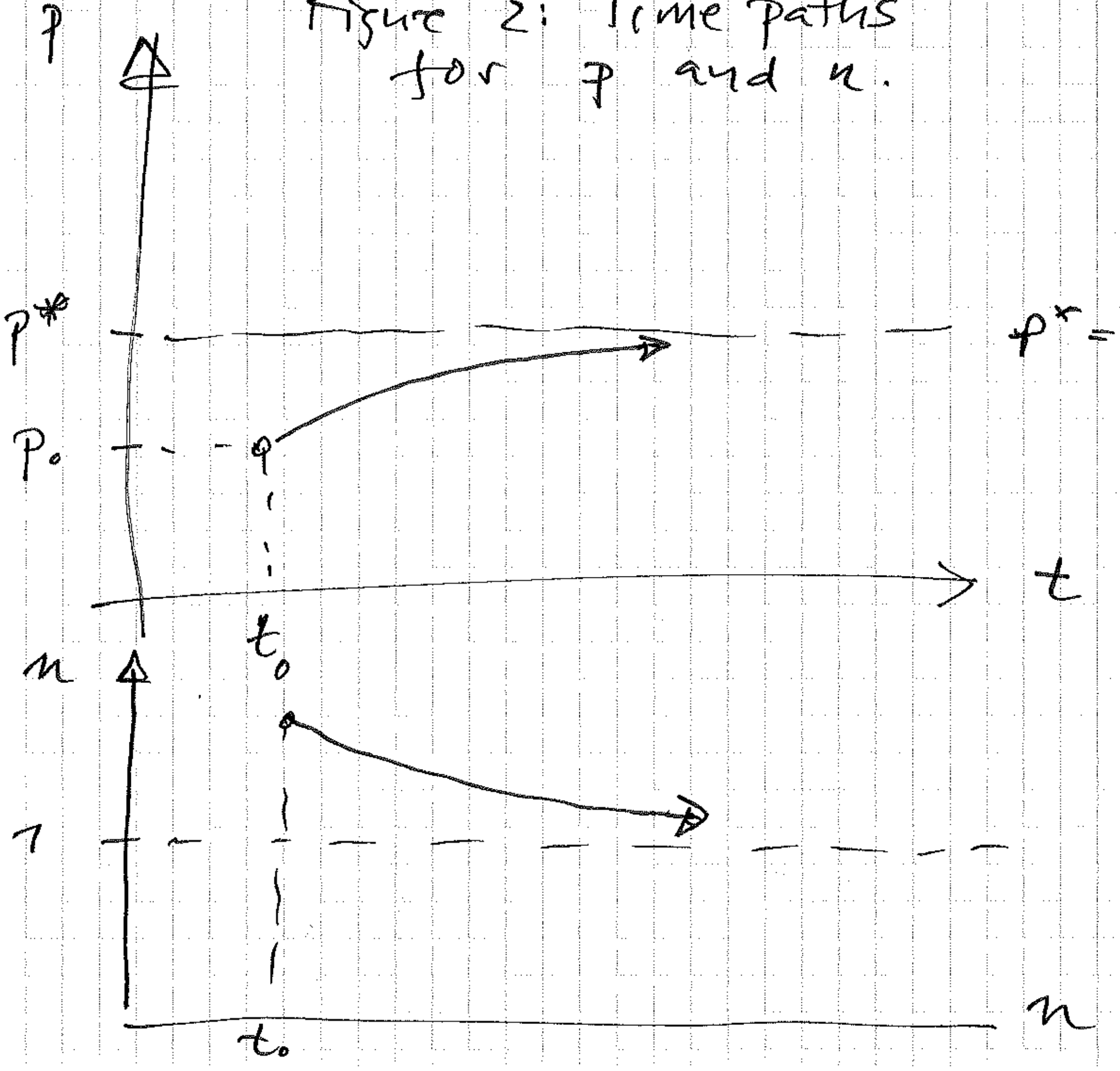
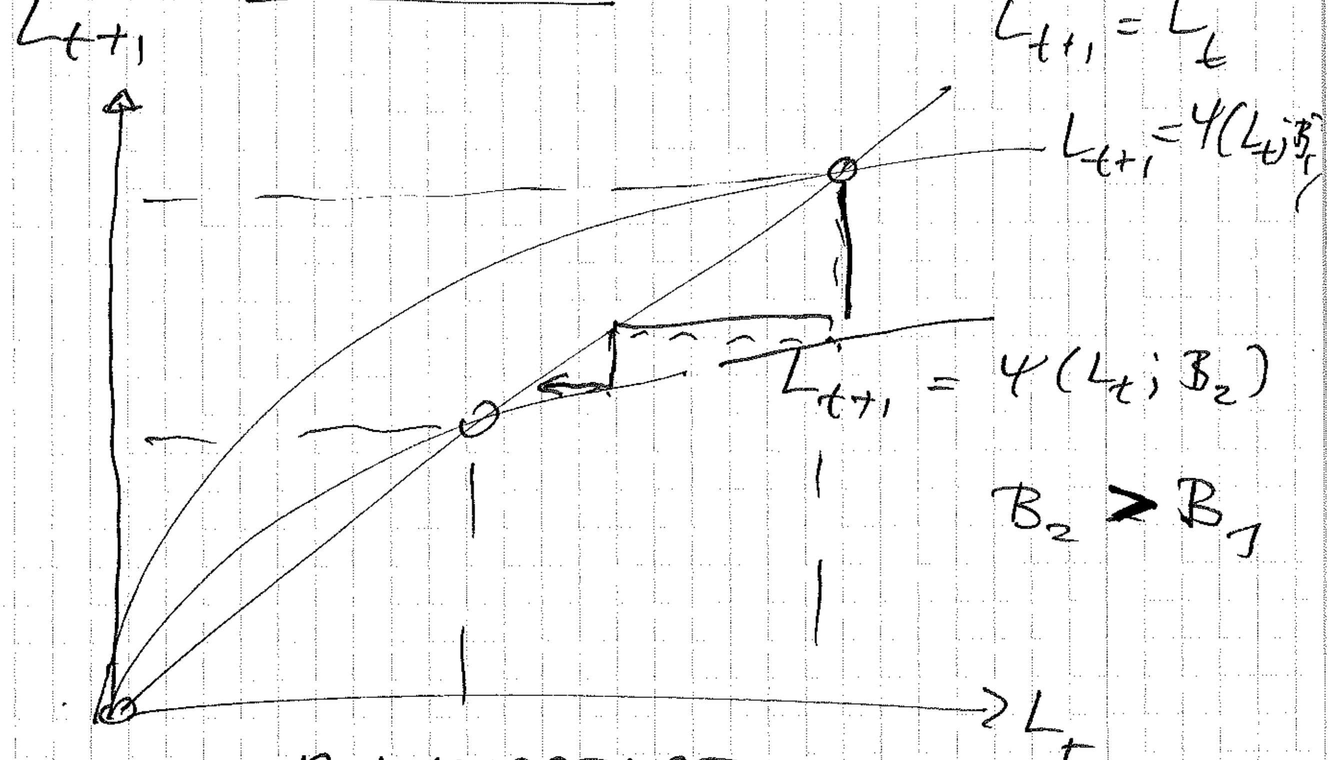
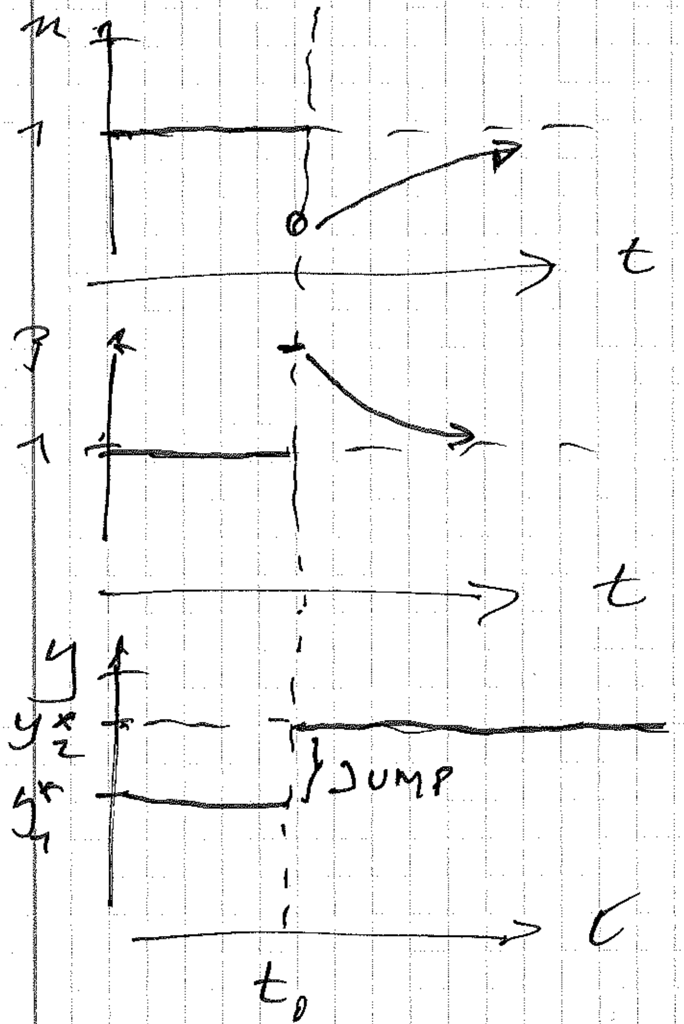


Figure 3



AN INCREASE IN B

TIME PATHS

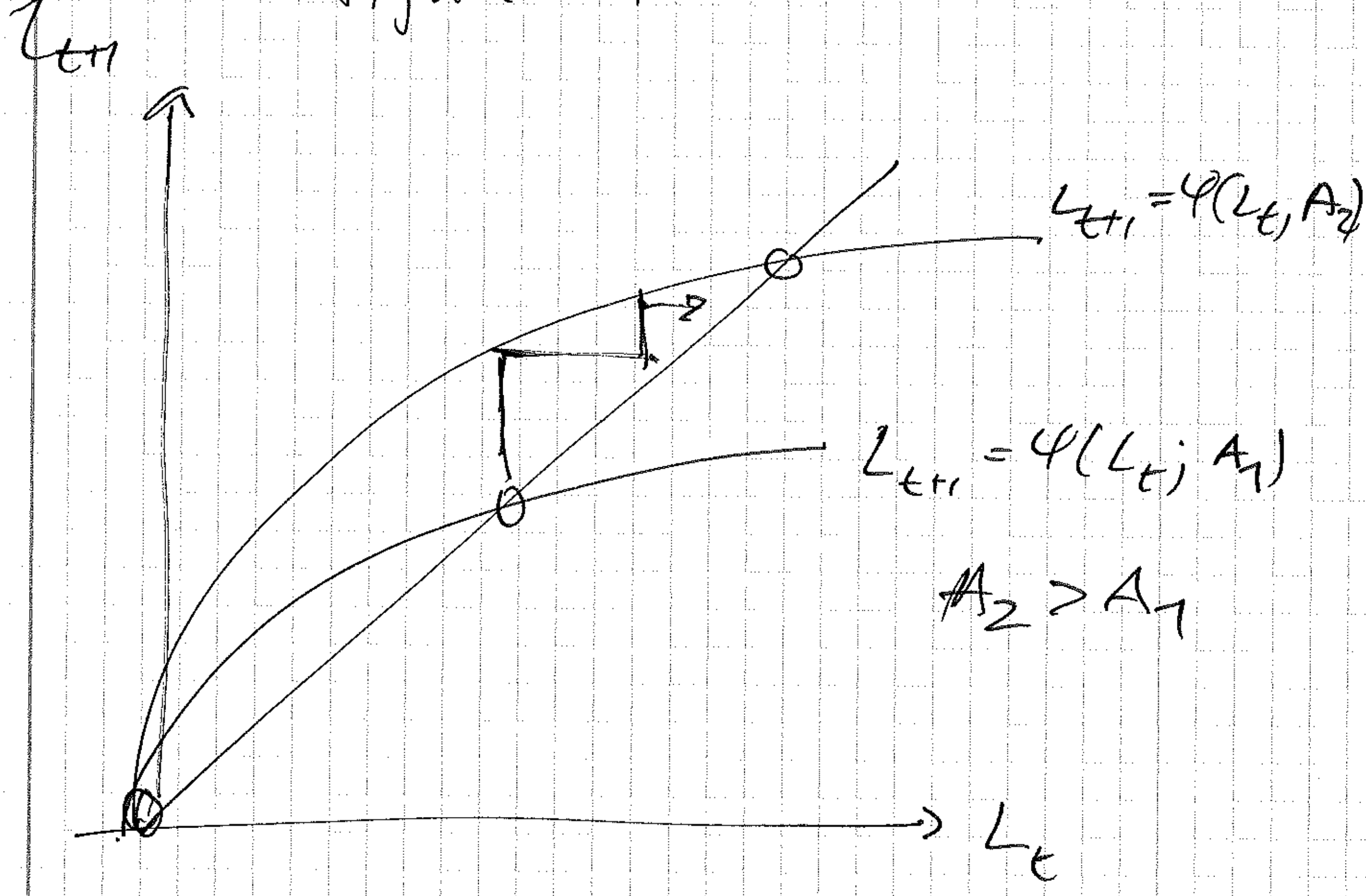


A DROP IN FERTILITY BELOW REPRODUCTION

DUE TO AN INCREASE IN P

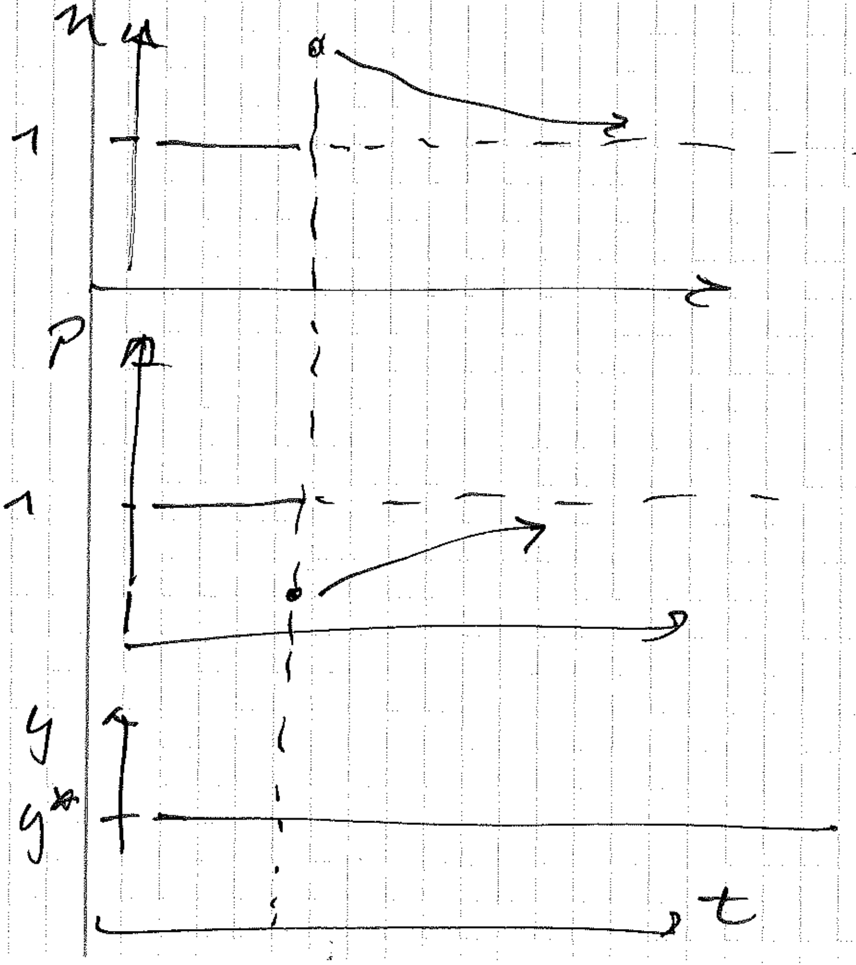
JUMP-WISE INCREASE IN y.

Figure 4



– AN INCREASE IN A –

TIME PATHS



AN INCREASE IN FERTILITY

PROMPTED BY A DECLINE IN P (DUE TO A↑)

NO IMPACT ON Y.