Written Exam at the Department of Economics
Winter 2018–19

Advanced Microeconometrics

Final Exam

February 7th, 2019
(3-hour closed book exam)

Answers only in English.

This exam question consists of 5 pages in total.

NB: If you fall ill during an examination at Peter Bangs Vej, you must contact an invigilator who will show you how to register and submit a blank exam paper. Then you leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Be careful not to cheat at exams!

You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed.
- Communicate with or otherwise receive help from other people.
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text.
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts.
- Or if you otherwise violate the rules that apply to the exam.

Note: The percentage weights indicated on each problem should only be regarded as indicative. The final grade will ultimately be based on an overall assessment of the quality of the answers to the exam questions in their totality.

Problem 1 (50%)

Consider the following nonlinear regression for the binary outcome $y_i \in \{0, 1\}$:

$$y_i = g(x_i, \beta) + u_i, \tag{1}$$

$$\mathrm{E}[y_i \mid x_i] = g(x_i, \beta) = \frac{\exp\{x_i'\beta\}}{1 + \exp\{x_i'\beta\}}, \tag{2}$$

for $i = 1, \ldots, N$, where x_i is a vector of observed characteristics, and β is the corresponding vector of regression coefficients.

Question 1.1: Within what range does this model restrict $\mathrm{E}[y_i \mid x_i]$ to lie? Justify your answer.

Question 1.2: Express the optimization problem that can be used to obtain the nonlinear least squares (NLS) estimator of β.

Question 1.3: Show that the first-order conditions for the NLS estimator of β can be expressed as

$$\sum_{i=1}^{N} w_i(y_i - g(x_i, \beta))x_i = 0, \qquad \text{with } w_i = \frac{\exp\{x_i'\beta\}}{(1 + \exp\{x_i'\beta\})^2}. \tag{3}$$

Question 1.4: Which condition is required for consistency of the NSL estimator of β? Is this condition fulfilled in this model? Justify your answer analytically.

Question 1.5: Show that $\mathrm{V}[y_i \mid x_i] = g(x_i, \beta)(1 - g(x_i, \beta))$. How can you use this result to improve on the NLS estimation of β? Describe briefly the alternative approach you suggest.

Question 1.6: Which alternative econometric model, assuming the same conditional expectation of the outcome as in Eq. (2), could you use to estimate β with maximum likelihood estimation (MLE)? Discuss briefly the differences between this MLE approach and the NLS approach.

Problem 2 (30%)

Consider a stock with price p_t at each time period $t = 0, \ldots, T$. You are interested in modeling the price fluctuations of this stock using the binary indicator $y_t = \mathbb{1}\{p_t - p_{t-1} > 0\}$, where $\mathbb{1}\{\cdot\}$ denotes the indicator function that is equal to 1 if the corresponding condition is fulfilled, to 0 otherwise.

For simplicity, assume that y_t is independent across time periods.

The goal of the analysis is to make inference on the parameter $\theta \equiv \Pr(y_t = 1)$, for $t = 1, \ldots, T$.

Question 2.1: Propose a model for this analysis and derive the corresponding likelihood function. Specify a prior distribution on θ that is a natural conjugate prior and derive the corresponding posterior distribution.

[Hint: You may use a distribution from Table 2.1, or a different one.]

Question 2.2: How would you choose the values of the prior parameters to obtain a flat prior? Show that with such a flat prior, the mean of the posterior distribution is asymptotically identical to the value of the maximum likelihood estimator.

Table 2.1: Some probability distributions.

Distribution	Density $f(\theta \mid a, b)$	Mean
Uniform	$\dfrac{1}{b-a}$	$\dfrac{a+b}{2}$
Beta	$\dfrac{\theta^{a-1}(1-\theta)^{b-1}}{B(a,b)}$	$\dfrac{a}{a+b}$
Gamma	$\dfrac{1}{\Gamma(a)b^a}\theta^{a-1}\exp\left\{-\dfrac{\theta}{b}\right\}$	ab
Inverse-Gamma	$\dfrac{b^a}{\Gamma(a)}\theta^{-a-1}\exp\left\{-\dfrac{b}{\theta}\right\}$	$\dfrac{b}{a-1}$ (for $a > 1$)

Problem 3 (20%)

You would like to compute the cumulative distribution function (CDF) of the standard normal distribution using an alternative to the `normcdf()` function provided in `MATLAB`.

One of your colleagues gives you the following piece of code, where you need to select one option for the two ingredients A and B, respectively:

```
function [cdf] = cdf_normal(x,M)

    rng(1);
    pdf_normal = @(t) exp(-(t.^2)/2)/sqrt(2*pi);

    % A: first ingredient
    z = rand(M,1);              % A1
    z = randn(M,1);             % A2
    z = x + randn(M,1);         % A3

    % B: second ingredient
    w = z.*x;                   % B1
    w = z.*pdf_normal(x);       % B2
    w = z < x;                  % B3
    w = pdf_normal(z) < x;      % B4

    % result
    cdf = mean(w);

end
```

Question 3.1: State your choice for the two ingredients A and B (for example, "A1 and B1"), and provide the corresponding mathematical expression of the formula used in this function.

Question 3.2: Describe precisely the theory behind this function. In particular, explain the role of the constant `M` and how it should be specified.

[Hint: see next page...]

[Hint: For any random variable Z with probability distribution function $f(\cdot)$ and cumulative distribution function $F(\cdot)$, remember that

$$E[Z] = \int z f(z) dz \quad \textit{and} \quad F(z) = \int_{-\infty}^{z} f(t) dt = E[\mathbb{1}\{Z \leq z\}],$$

where $\mathbb{1}\{\cdot\}$ is the indicator that is equal to 1 if the corresponding condition is fulfilled, to 0 otherwise.]