# Written Exam at the Department of Economics winter 2018-19 

# Corporate Finance and Incentives 

Final Exam

February 11, 2019
(3-hour closed book exam)

Answers only in English.

## This exam question consists of 4 pages in total

NB: If you fall ill during an examination at Peter Bangs Vej, you must contact an invigilator who will show you how to register and submit a blank exam paper. Then you leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

## Be careful not to cheat at exams!

- You cheat at an exam, if during the exam, you:
- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub-questions that do not necessarily have equal weight. Please provide intermediate calculations.

## Problem 1 (APT 25\%)

Consider a financial market with four assets whose returns are given by

$$
\left[\begin{array}{l}
R_{1} \\
R_{2} \\
R_{3} \\
R_{4}
\end{array}\right]=\left[\begin{array}{l}
0.165 \\
0.049 \\
0.125 \\
0.052
\end{array}\right]+\left[\begin{array}{ccc}
0.7 & 0.5 & 1.2 \\
-0.3 & 0.9 & 0.4 \\
0.8 & -0.2 & 1.1 \\
0.1 & 0.6 & -0.4
\end{array}\right]\left[\begin{array}{l}
F_{1} \\
F_{2} \\
F_{3}
\end{array}\right]+\left[\begin{array}{l}
\varepsilon_{1} \\
\varepsilon_{2} \\
\varepsilon_{3} \\
\varepsilon_{4}
\end{array}\right],
$$

where $\varepsilon_{1}, \varepsilon_{2}, \varepsilon_{3}, \varepsilon_{4}$ have mean zero and are uncorrelated with each other as well as uncorrelated with $F_{1}, F_{2}, F_{3}$. Also, $E\left[F_{1}\right]=E\left[F_{2}\right]=E\left[F_{3}\right]=0$.

1) Find a risk-free portfolio and compute the risk-free interest rate $r_{f}$.
2) Find the three pure factor portfolios.
3) Compute the expected return and risk premiums of the three factor portfolios.
4) Finally introduce a fifth asset with factor equation $R_{5}=0.21+F_{1}+F_{2}+F_{3}+\epsilon_{5}$. Does this give rise to an arbitrage opportunity, and if so, how can you exploit this?

## Problem 2 (Corporate Governance 25\%)

A Founder owns $60 \%$ of a firm and makes decisions. Outside investors own the rest.
The Founder must choose among project 1 and project 2. Both projects result in either successful present value 100, or failing present value -40 . The projects differ in their probabilities: Project 1 succeeds with chance $40 \%$, while project 2 succeeds with chance $70 \%$. The Founder receives private benefit 27 if project 1 is chosen.

1) Show project 2 is efficient, i.e., $.7(100)-.3(40)>.4(100)-.6(40)+27$, but the Founder will choose project 1, i.e., $.6(.7(100)-.3(40))<.6(.4(100)-.6(40))+27$.
2) Compute the expected payoff to outside investors.

Assume next that the Founder purchases $10 \%$ more of the firm from the outside investors, bringing the Founder's ownership share to $70 \%$.
3) Show the Founder will now choose the efficient project 2.
4) Show that the Founder is willing to pay more for the $10 \%$ ownership than it was previously worth to outside investors.
5) Show that the Founder is unwilling to pay more for the $10 \%$ ownership than such an ownership share is worth to outside investors after the change in project choice.

## Problem 3 (Options 25\%)

Some options are written on an underlying stock. The assets are traded today, and all options expire one year from now. There are then five possible states of the world: Very Bad, Bad, Neutral, Good, and Very Good.

All asset prices are given by risk-neutral pricing (i.e., the arbitrage-free pricing model). Some other assets are also traded, and it has been concluded that the risk-free interest rate is $3 \%$, with risk-neutral probabilities as given in the table below.

At the same day as the options expire, the stock will pay a dividend. The dividend and the post-dividend value of the stock is a function of the state. The following table summarizes this and the risk-neutral probability distribution:

| State | Very Bad | Bad | Neutral | Good | Very Good |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Probability | $15 \%$ | $25 \%$ | $35 \%$ | $20 \%$ | $5 \%$ |
| Dividend | 0 | 0 | 5 | 8 | 12 |
| Post-dividend stock value | 10 | 30 | 50 | 80 | 120 |

The options are American. It is irrelevant to exercise any of them before the expiry date. However, on that date, after the state has been realized, the option owner can choose whether to exercise before or after the dividend is paid.

1) Explain that on the expiry date, the call option owner would rather exercise before than after the dividend is paid.
2) On the expiry date, will a put option owner prefer to exercise before or after the dividend is paid?
3) Consider a call option with strike price 70 . Write down its future value in the five states. Compute its option premium in today's market.
4) Consider a put option with strike price 70 . Write down its future value in the five states. Compute its option premium in today's market.
5) Do the premiums from 3) and 4) satisfy the Put-Call parity? Briefly comment on this.

## Problem 4 (Various Themes 25\%)

1) Explain that Modigliani-Miller's second proposition has the following implication. Let $\beta_{U}$ denote the CAPM beta of the unlevered assets in a firm. Let $\beta_{E}$ denote the CAPM beta of its equity, and let $E$ denote the value of equity. Likewise, let $\beta_{D}$ denote the CAPM beta of its debt, valued at $D$. Then the proposition implies

$$
\beta_{E}=\beta_{U}+\frac{D}{E}\left(\beta_{U}-\beta_{D}\right) .
$$

2) Discuss the following connection between financial markets and the real economy from the perspective of asset pricing and investor behaviour. The Economist magazine printed this on January 5, 2019 (part of it quoted here): "The S\&P 500 tumbled in value by $15 \%$ between November 30th and December 24th. That investors have become more risk-averse can also be seen in the bond markets. The high-yield spread, or excess interest rate over government debt, paid by companies with a poor credit rating has been rising. Meanwhile the yield on the ten-year Treasury bond has dropped from $2.98 \%$ to $2.63 \%$ over the past month, as investors have rushed to the safety of government paper. Perhaps the main concerns, though, are provoked by the Fed itself. On December 19th its Open Markets Committee (FOMC) delivered its fourth interest-rate increase of 2018, even though financial conditions have tightened to become less supportive of growth. That added to fears that monetary policy will tighten beyond what the economy can bear. The reason monetary policymakers seem so blasé compared with investors is that, setting aside the financial indicators flashing amber, America's economy appears to be doing well. The labour market went from strength to strength in 2018, and most indicators of consumer confidence remain at ten-year highs. There are several ways this disconnect between market jitters and robust economic indicators could disappear. (i) The direct effect of stock-owners feeling poorer could cut spending. The plunging stockmarket could hit consumer and business confidence, crimping spending and investment. (ii) Judging by the past couple of decades, if stockmarket turmoil persists the Fed will respond by lowering its forecasts for growth. That would feed into a looser policy stance. (iii) There is a chance that the stockmarket will rouse itself from its slump in coming weeks."
3) Consider a European call option written on a stock that pays out no dividends before the options expire. Assume that the risk-free interest rate is always positive. Let $K$ denote the strike price, let $S$ denote the stock price, and let $C$ denote the call premium. Explain that the put-call parity implies $C>S-K$.
