Written Exam for the B.Sc. or M.Sc. in Economics autumn 2014-2015

Corporate Finance and Incentives

Final Exam/ Elective Course/ Master's Course

18th December 2014

(3-hour closed book exam – access to Excel)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title, which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The exam consists of 5 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub questions that do not necessarily have equal weight. Please provide intermediate calculations. Good luck ☺

Problem 1 (CAPM, 20%)

Consider the following covariance matrix and mean returns for the four securities A, B, C, and D:

$$\Omega = \begin{pmatrix} 0.020 & 0.010 & 0.005 & 0.000 \\ 0.010 & 0.015 & 0.005 & 0.000 \\ 0.005 & 0.005 & 0.010 & 0.000 \\ 0.000 & 0.000 & 0.000 & 0.000 \end{pmatrix} \qquad E(R_i) = \begin{pmatrix} 20\% \\ 15\% \\ 12\% \\ 4\% \end{pmatrix}$$

1. What are the assumptions of CAPM? Which of these assumptions do not apply for APT?
2. Find the minimum-variance portfolio and the tangency portfolio and draw the efficient frontier.
3. Find the Capital Market Line and express it as an equation.
4. Find the four beta values and the Securities Market Line and express it as an equation.

Problem 2 (Fixed Income, 20%)

Consider a market with four bonds:

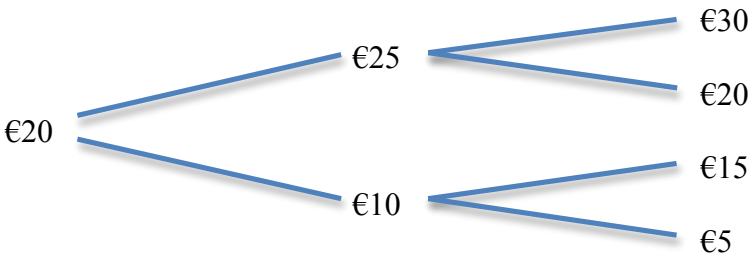
Type of bond	**Coupon**	**Price**
1-year bullet bond	2.0%	100.000
2-year serial bond	2.0%	99.525
3-year annuity bond	2.5%	99.700
4-year annuity bond	3.0%	99.832

The bonds pay exactly one, two, three and four years from today.

1. Find the four discount factors, the four spot rates and the four forward rates.
2. Find the YTM for the four bonds.
3. Explain why the spot rates and the YTM for the bonds are not the same.
4. Find the price and YTM for a newly introduced 4-year bullet bond with a coupon of 3%.
5. Find the Macaulay duration and the modified duration for the new 4-year bullet bond. Explain what these durations tell us.

Problem 3 (Options, 20%)

A non-dividend paying stock currently cost €20. In the next two periods it can either increase in price or decrease in price as shown in the binary tree. The risk free rate is 5% per period. An American call option on this stock has an exercise price of €20.



1) Assign risk free probabilities to the binary tree.
2) Use the risk free probabilities to price the call option.
3) Explain why an American put option can be hard to price using the put-call parity.
4) Calculate the value of the American put option.

Problem 4 (Capital structure 1, 20%)

A firm has the following perpetual expected cash flow:

Turnover	2000
Operating costs	500
EBITDA	1500
Depreciation & Amortization	500
EBIT	1000
Interest payments	600
Earnings before taxes (EBT)	400
Tax (25%)	100
Profit after tax	300

The debt rate is 5%, which is also the risk free rate, and return on equity at the current debt level is 15%. Average market return is 10%.

1) What is beta of equity for this firm?
2) Find the value of Debt, Equity, the tax shield, the un-levered firm, and the levered firm. Show how these five terms relate to each other in a balance sheet.
3) Calculate the WACC and show how WACC relates to NOPAT[1] and the value of the levered firm.
4) Calculate the beta of the unlevered firm (asset beta) by un-levering the equity beta (see appendix 1). Then calculate the corresponding return by the use of CAPM. Finally show how the return relates to NOPAT and the value of the unlevered firm.

[1] Net Operating Profit After Tax, and is calculated as profit after tax for the unlevered firm.

Problem 5 (Capital structure 2, 20%)

A firm has the following perpetual expected cash flow:

EBIT	1000
Interest payments	X
Earnings before taxes (EBT)	1000-X
Tax (25%)	(1000-X)*0.25
Profit after tax	(1000-X)*(1-0.25)

"X" is the interest payment, calculated as $D*r_D$ where "D" is the nominal Debt and "r_D" is the appropriate debt rate.

Table 1: Debt rates with risky debt

The debt rate is a function of the leverage in the following way:

$0 < D/(VU-D) \leq 2$ ➔ Debt rate (r_D) = 5%
$2 < D/(VU-D) \leq 4$ ➔ Debt rate (r_D) = 6%
$4 < D/(VU-D) \leq 6$ ➔ Debt rate (r_D) = 7%
$6 < D/(VU-D) \leq 8$ ➔ Debt rate (r_D) = 8%
$8 < D/(VU-D)$ ➔ Debt rate (r_D) = 10%

VU is Value of the Un-levered assets and is 10,000. Beta of the unlevered assets is 0.5. The risk free rate is 5%

1) Calculate the optimal level of debt in each of the 5 leverage brackets?
2) Set up 6 P&L's, one for each of the optimal debt levels and one for the un-levered firm. For now assume that the firm pays the risk free rate no matter the leverage (That is use 5% as the debt rate, r_D, and the optimal level of debt found in the previous question). Calculate the value of Equity, the tax shield, the un-levered firm, and the levered firm in each of the six scenarios. Further calculate the WACC and return on Equity.
3) Now assume that the equity holders require the same return as in the previous question but that debt is risky (That is use the debt rate, r_D, specified in table 1). Set up 6 new P&L's, one for each debt level, where the firm pays the interest rate specified in the table. Calculate the value of Equity, the tax shield, the un-levered firm, and the levered firm. Further calculate the WACC and find the optimal level of debt.
4) Express the lost value due to financial distress in a formula.
5) Find beta of debt for each of the five leverage brackets and explain what cost of financial distress is.

Appendix 1: Un-levering and re-levering

The (Asset) return of the unlevered firm (de-levering / un-levering):

$$r_A = \left(\frac{E}{E + D(1-T_C)}\right) r_E + \left(\frac{D(1-T_C)}{E + D(1-T_C)}\right) r_D$$

The (Asset) beta of the unlevered firm (de-levering / un-levering):

$$\beta_A = \left(\frac{E}{E + D(1-T_C)}\right) \beta_E + \left(\frac{D(1-T_C)}{E + D(1-T_C)}\right) \beta_D$$

Re-levering:

$$\beta_E = \left(\frac{E + D(1-T_C)}{E}\right) \beta_A - \left(\frac{D(1-T_C)}{E}\right) \beta_D = \beta_A + \left(\frac{D(1-T_C)}{E}\right) (\beta_A - \beta_D)$$

$$r_E = \left(\frac{E + D(1-T_C)}{E}\right) r_A - \left(\frac{D(1-T_C)}{E}\right) r_D = r_A + \left(\frac{D(1-T_C)}{E}\right) (r_A - r_D)$$

Where subscript "A" refer to (unlevered) Assets, "E" Equity, "D" Debt, and "T_C" is the corporate tax rate