Written Exam for the B.Sc. or M.Sc. in Economics autumn 2014-2015

Corporate Finance and Incentives

Final Exam/ Elective Course/ Master's Course

20th February 2015

(3-hour closed book exam – access to Excel)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title, which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The exam consists of 4 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub questions that do not necessarily have equal weight. Please provide intermediate calculations. Good luck ☺

Problem 1 (Multi factor models and APT, 20%)

Assume that the following two-factor model can describe the return of the three stocks a, b and c:

$$r_a = .09 + F_1 + F_2 + e_a$$
$$r_b = .10 + 2F_1 + F_2 + e_b$$
$$r_c = .11 + F_1 + 2F_2 + e_c$$

With all the usual assumptions.

1) Find the pure factor portfolios.
2) If the risk free rate is 6%, what are then the pure factor equations and what are the risk premiums λ_1 and λ_2?
3) A new stock also follows the two-factor model: $r_x = 0.13 + 2F_1 + 2F_2 + e_x$. Show that this is an arbitrage and explain how you would exploit this arbitrage?
4) What are usually the expected values of F_1 and F_2? Explain why and give three examples of frequently used *macro economical* factors.

Problem 2 (Fixed Income, 30%)

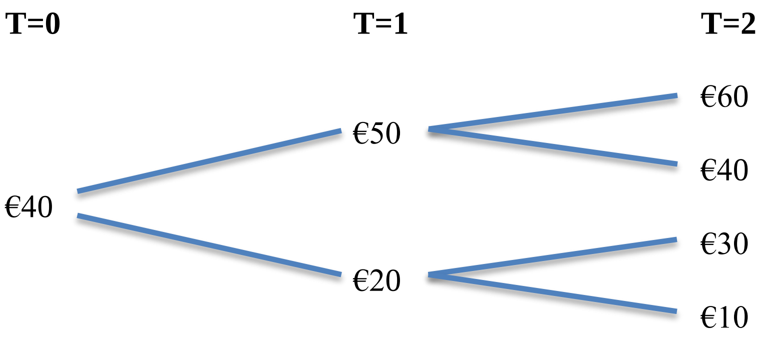
Explain what Fixed Income theory is. In your explanation you must include the below points (but should not feel limited to these). Answer by referring to the below numbers (so you need 12 number-bullets in your answer in order to answer all of them). A verbal answer gives 1 point. A verbal answer accompanied with a mathematical (formal) answer gives 2 points. A comprehensive verbal plus mathematical answer gives 2.5 points. Graphs are expected if relevant.

1. Bonds and valuation
2. Complete markets
3. Zero coupon bonds
4. Synthetically creating bonds
5. Bootstrapping
6. Yield to maturity of a bond (IRR)
7. Term structure
8. Spot rates, forward rates, discount factors
9. Types of bonds (the 3 usual suspects)
10. Duration (both Macaulay and Modified)
11. Convexity
12. What we use a Second order Taylor approximation* for

$$\frac{\Delta PV^*}{PV} = \frac{PV(c, y+\Delta y) - PV(c,y)}{PV(c,y)} \approx \frac{PV' \cdot \Delta y + 1/2\, PV'' \cdot \Delta y^2}{PV} = \underbrace{-D \cdot \frac{\Delta y}{(1+y)}}_{D_{Mod} \cdot \Delta y} + \underbrace{\frac{K+D}{2}\left(\frac{\Delta y}{1+y}\right)^2}_{\text{Curvature}}$$

Problem 3 (Options, 20%)

A non-dividend paying stock currently costs €40. In the next two periods it can either increase in price or decrease in price as shown in the binary tree. The risk free rate is 5% per period. Any option referred to has an exercise price of €40.



1) What is the price of a European call option and put option?
2) What is the price of an American call option and put option?

Make sure to comment on assumptions, and differences and similarities between the four types of options.

Problem 4 (Capital structure, 30%)

A firm has the following perpetual expected cash flow:

Turnover	3000
Operating costs	1000
EBITDA	2000
Depreciation & Amortization	800
EBIT	1200
Interest payments	800
Earnings before taxes (EBT)	400
Tax (50%)	200
Profit after tax	200

The debt rate is 5%, which is also the risk free rate, and return on equity at the current debt level is 20%. Average market return is 10%.

1) What is beta of equity for this firm? Explain your assumption(s).
2) Find the value of Debt, Equity, the tax shield, the un-levered firm, and the levered firm. Show how these five terms relate to each other in a balance sheet.
3) Calculate the WACC in two different ways. First way should be the usual way (weighted average) and the second way should be by relating NOPAT[1] to a balance sheet number.
4) Calculate the beta of the unlevered firm (asset beta) by un-levering the equity beta (see appendix 1). Then calculate the corresponding return by the use of CAPM. Show how this return relates to NOPAT and a balance sheet number.

[1] Net Operating Profit After Tax, and is calculated as profit after tax for the unlevered firm.

5) Find the value of Debt, Equity, the tax shield, the un-levered firm, and the levered firm, if the cash flow is expected to grow with 2.5% per year perpetually and that the D/E ratio is assumed to be constant over time.
6) Assume no growth. What is the optimal leverage if the debt rate is a function of debt in the following way: $r_D = r_f + D/100{,}000$ so a debt level of e.g. 2,000 will correspond to a debt rate of 7%. Debt is issued in blocks of 1000 which means that you have to consider the following possible debt levels: D = (0 ; 1000 ; 2000 ; 3000 ; etc). Explain your assumptions.

Appendix 1: Un-levering and re-levering

The (Asset) return of the unlevered firm (de-levering / un-levering):

$$r_A = \left(\frac{E}{E + D(1 - T_C)}\right) r_E + \left(\frac{D(1 - T_C)}{E + D(1 - T_C)}\right) r_D$$

The (Asset) beta of the unlevered firm (de-levering / un-levering):

$$\beta_A = \left(\frac{E}{E + D(1 - T_C)}\right) \beta_E + \left(\frac{D(1 - T_C)}{E + D(1 - T_C)}\right) \beta_D$$

Re-levering:

$$\beta_E = \left(\frac{E + D(1 - T_C)}{E}\right) \beta_A - \left(\frac{D(1 - T_C)}{E}\right) \beta_D = \beta_A + \left(\frac{D(1 - T_C)}{E}\right) (\beta_A - \beta_D)$$

$$r_E = \left(\frac{E + D(1 - T_C)}{E}\right) r_A - \left(\frac{D(1 - T_C)}{E}\right) r_D = r_A + \left(\frac{D(1 - T_C)}{E}\right) (r_A - r_D)$$

Where subscript “A” refer to (unlevered) Assets, “E” Equity, “D” Debt, and “T_C” is the corporate tax rate