Written Exam at the Department of Economics winter 2017-18

Contract Theory

Final Exam

January 11, 2018

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam paper consists of five pages in total, including this one

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Adverse selection and optimal procurement with specific functional forms

This adverse selection model with two types is based on the one that we studied in the course, but it assumes a linear surplus function and a quadratic cost function.

A firm (the agent, A) interacts with a government procurement agency (the principal, P). A produces office material that P wants to purchase. A's cost of producing q units of office material is given by the quadratic function

$$C(q, \theta) = \frac{\theta q^2}{2}, \tag{1}$$

where θ is an efficiency parameter. The value for P of receiving q units of office material is given by the linear function

$$S(q) = q. \tag{2}$$

The efficiency parameter θ can take two values: $\theta \in \{\underline{\theta}, \overline{\theta}\}$, with $0 < \underline{\theta} < \overline{\theta}$. A knows the value of θ perfectly. However, P knows only that $\Pr[\theta = \underline{\theta}] = \nu$ and $\Pr[\theta = \overline{\theta}] = 1 - \nu$, with $0 < \nu < 1$. The procurement agency has all the bargaining power and makes a take-it-or-leave-it offer to the firm. A contract can specify the quantity q that A must produce and deliver and the payment t that A will receive. Suppose that P wants to offer different contracts to the two types of firms and wants both types to produce a positive quantity. P is risk neutral and its payoff, given a quantity q and a payment t, equals $V = S(q) - t$. A is also risk neutral and its payoff, given a quantity q and a payment t, equals $U = t - C(q, \theta)$. A's outside option (the same for the two types) would yield the payoff zero.

P offers a menu of two distinct contracts to A. As in the course, the contract variables are indicated either with "upper-bars" or with "lower-bars", depending on which type the contract is aimed at. P's problem is to choose $\left(\underline{t}, \underline{q}, \overline{t}, \overline{q}\right)$ so as to maximize

$$\nu \left[S\left(\underline{q}\right) - \underline{t}\right] + (1 - \nu)\left[S(\overline{q}) - \overline{t}\right] \tag{3}$$

subject to the following four constraints:

$$\overline{t} - C(\overline{q}, \overline{\theta}) \geq 0, \tag{IR-bad}$$

$$\underline{t} - C\left(\underline{q}, \underline{\theta}\right) \geq 0, \tag{IR-good}$$

$$\overline{t} - C(\overline{q}, \overline{\theta}) \geq \underline{t} - C\left(\underline{q}, \overline{\theta}\right), \tag{IC-bad}$$

$$\underline{t} - C\left(\underline{q}, \underline{\theta}\right) \geq \overline{t} - C(\overline{q}, \underline{\theta}). \tag{IC-good}$$

(a) Explain in words what each one of the four constraints says and why it must be satisfied. Also explain the nature of the trade-off that the principal faces.

(b) The first-best optimal quantities are defined by $S'\left(\underline{q}^{FB}\right) = C_q\left(\underline{q}^{FB}, \underline{\theta}\right)$ and $S'\left(\overline{q}^{FB}\right) = C_q\left(\overline{q}^{FB}, \overline{\theta}\right)$, respectively. Assume that the constraints (IR-good) and (IC-bad) are lax at the second-best optimum (so that they can be disregarded). Show that, at the second-best optimum, the good type's quantity is *not* distorted relative to the first best ($\underline{q}^{SB} = \underline{q}^{FB}$) and that the bad type's quantity is distorted downwards ($\overline{q}^{SB} < \overline{q}^{FB}$).

Question 2: Moral hazard in a regulatory problem

Consider the following model of the interaction between a benevolent government (the principal) and a regulator (the agent). The regulator's task is to decide whether to award the monopoly rights in a particular market to Firm A or to Firm B. Whether it is, from a social welfare point of view, most desirable to choose Firm A or Firm B depends on a state of nature $\theta \in \{A, B\}$, which initially is unknown to all parties. In particular, if $\theta = i$ and Firm i is chosen, a surplus $S > 0$ is generated; otherwise, the surplus equals zero. The prior probability that Firm A is the socially desirable firm equals $\Pr[\theta = A] \stackrel{\text{def}}{=} \pi \in \left(0, \frac{1}{2}\right)$. That is, given the information available at the outset, Firm B is the most likely one to yield a positive surplus. Before making the decision which firm to choose, the regulator has the opportunity to "investigate". Doing this means that the regulator (i) incurs a personal cost $\psi > 0$ and (ii) learns θ perfectly.

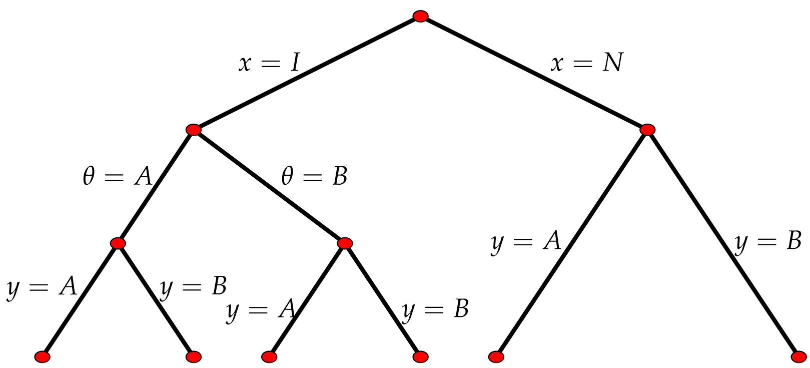

Figure 1: An illustration of the regulator's choices. First she chooses $x = I$ or $x = N$ (investigate or not). Thereafter she chooses $y = A$ or $y = B$ (which firm to award the monopoly). If having chosen $x = I$ (investigate), she knows the true state θ when choosing y.

In summary, the regulator makes the following choices. First she chooses $x \in \{I, N\}$, where I means investigate and N means not investigate. If $x = I$, the regulator incurs the cost $\psi > 0$ and learns the true state. Then she chooses which firm to award the monopoly, $y \in \{A, B\}$. A decision tree showing the regulator's choices is shown in Figure 1.

The government can observe the regulator's choice of firm, y, but not her choice whether to investigate, x. Moreover, the government cannot observe whether or not, after the regulator's decision, a positive surplus is created. However, if the regulator chooses B and if this is the "wrong" decision (i.e., if $\theta = A$), then, with probability $\gamma \in (0, 1)$, (i) *the government learns about this and (ii) the regulator incurs a* *personal (additional and exogenous) cost $L \in (0, \psi)$.*[1]

The government can commit to making a monetary payment t to the regulator that is contingent on the observed outcome. In particular, the assumptions made above mean that the payment can be contingent on three different events:

- $y = A$ (denote the payment after this event by t_A);
- $y = B$ and there is no report from Firm A (denote this payment by t_B);
- $y = B$ and there *is* a report from Firm A (denote this payment by t_0).

Both the government and the regulator are risk neutral, but the regulator is protected by limited liability: $t_0 \geq 0$, $t_A \geq 0$, and $t_B \geq 0$. The regulator's outside option yields a payoff of zero. The timing of events is as follows. (i) The government chooses (t_0, t_A, t_B), trying to maximize the expected surplus that is generated, net of the expected payments to the regulator. (ii) The regulator decides whether or not to accept the contract offer and then, if accepting, chooses x and y as described above.

Outcome 1

- The principal's payoff: $V = S - \pi t_A - (1-\pi) t_B$.
- Constraints:

$$t_A \geq (1-\gamma) t_B + \gamma (t_0 - L), \tag{4}$$

$$t_B \geq t_A, \tag{5}$$

$$\pi t_A + (1-\pi) t_B - \psi \geq t_A, \tag{6}$$

$$\pi t_A + (1-\pi) t_B - \psi \geq (1-\gamma\pi) t_B + \gamma\pi (t_0 - L). \tag{7}$$

$$\pi t_A + (1-\pi) t_B - \psi \geq 0, \tag{8}$$

Outcome 2

- The principal's payoff: $V = (1-\pi) S - (1-\gamma\pi) t_B - \gamma\pi t_0$.
- Constraints:

$$(1-\gamma\pi) t_B + \gamma\pi (t_0 - L) \geq t_A, \tag{9}$$

$$(1-\gamma\pi) t_B + \gamma\pi (t_0 - L) \geq \pi t_A + (1-\pi) t_B - \psi, \tag{10}$$

$$(1-\gamma\pi) t_B + \gamma\pi (t_0 - L) \geq 0, \tag{11}$$

Consider the two boxes above, with the headings Outcome 1 and Outcome 2. In the box for outcome i (for $i = 1, 2$), a payoff expression for the government and a number of constraints are listed. If the government wants to induce outcome i, then it should maximize this payoff function subject to the constraints listed in the box and the three limited liability constraints indicated above in the text.

Answer the following questions.

(a) Explain in words what behavior outcome 1 and outcome 2, respectively, involves (i.e., explain what the regulator's induced actions x and y are, for each of the two outcomes).

[1] We can interpret this assumption as capturing the idea that Firm A (but not Firm B) is able to exert pressure on the regulator and thereby try to influence her decision. In particular, if the regulator's decision goes against A in an "unfair" way (in the sense that $y = B$ in spite of $\theta = A$), then, with positive probability, A can (i) report the true state to the government and (ii) penalize the regulator personally (perhaps by hurting her career opportunities by speaking ill of her).

(b) Solve formally the problem of inducing outcome 1 (i.e., derive the optimal values of t_0, t_A, and t_B, given that outcome 1 should be achieved). Graphical arguments are allowed.

(c) Solve formally the problem of inducing outcome 2 (i.e., derive the optimal values of t_0, t_A, and t_B, given that outcome 2 should be achieved). Graphical arguments are allowed.

By solving for the overall optimal contract (i.e., by first computing the optimal way of inducing each possible outcome and then investigate which outcome that yields the highest payoff for the government), one can show the following. Suppose that π satisfies $\pi \in \left(0, \frac{1}{2}\right)$ but is sufficiently close to one-half and that S is not too large. Also suppose that the overall optimal contract is offered. Then, as we gradually increase L from a value close to zero to a value close to ψ, the ex ante probability that $x = B$ goes from (i) one, to (ii) zero, to (iii) π. That is, the outcome for Firm B (assuming that it wants to be awarded the monopoly) is (i) very good if the regulator's cost L is low; (ii) as L grows larger, the outcome becomes very bad for Firm B; and (iii) as L increases even more, the outcome for Firm B again becomes fairly good.

(d) Explain (in words) the economic reasons for the result described immediately above. Why is there a lack of monotonicity? Why does Firm B prefer either a low or a high value of L to an intermediate value?

End of Exam