Written Exam at the Department of Economics winter 2017-18

Contract Theory

Final (Resit) Exam

February 20, 2018

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language for which you registered during exam registration.

This exam paper consists of four pages in total, including this one

NB: If you fall ill during an examination at Peter Bangsvej, you must contact an invigilator in order to be registered as having fallen ill. In this connection, you must complete a form. Then you submit a blank exam paper and leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Adverse selection model with endogenous types

The following is an extended version of the adverse selection model that we studied in the course.

A firm (the agent) interacts with a government procurement agency (the principal). The firm produces office material that the procurement agency wants to buy. The firm's cost of producing q units of office material is given by the function $C(q, \theta)$, where θ is an efficiency parameter. This function satisfies

$$C(0, \theta) = 0, \qquad C_q > 0, \qquad C_{qq} \geq 0, \qquad C_\theta > 0, \qquad C_{q\theta} > 0, \qquad C_{qq\theta} \geq 0.$$

The value for the procurement agency of receiving q units of office material is given by the function $S(q)$, which satisfies

$$S'(q) > 0, \qquad S''(q) < 0, \qquad S(0) = 0.$$

The efficiency parameter θ can take two values: $\theta \in \{\underline{\theta}, \overline{\theta}\}$, with $0 < \underline{\theta} < \overline{\theta}$. Whether the firm is "good" (meaning $\theta = \underline{\theta}$) or "bad" (meaning $\theta = \overline{\theta}$) is stochastic. However, by exerting costly effort the firm can increase the probability of being a good firm (this is where the model differs from the one we studied in the course). In particular, if the firm makes an effort and thereby incurs the cost $\psi > 0$, it will be good with probability ν_1 and bad with probability $1 - \nu_1$. If the firm does *not* exert effort, it does not incur any effort cost and it will be good with probability ν_0 and bad with probability $1 - \nu_0$. We assume that $0 < \nu_0 < \nu_1 < 1$.

The timing of events is as follows.

1. The procurement agency chooses a menu of contracts. A contract can specify the quantity q that the firm must produce and deliver and the payment t that the firm will receive.

2. The firm decides whether to reject all contracts in the menu or to accept one of them.

3. If the firm accepted a contract at date 2, the firm first decides whether or not to exert effort and thus incur the cost ψ. Then the firm's efficiency parameter θ is drawn, either according to the ν_0 distribution or the ν_1 distribution (depending on whether the firm exerted effort). The realization of θ is observed by the firm itself. However, the procurement agency cannot observe (i) the firm's effort choice nor (ii) the realization of θ.

4. If the firm accepted a contract at date 2, production takes place and the procurement agency pays the firm the contractually specified payment t.

The procurement agency is risk neutral and its payoff, given a quantity q and a payment t, equals $V = S(q) - t$. The firm is also risk neutral and its payoff, given a quantity q and a payment t, equals $U = t - C(q, \theta) - \psi$ if it has exerted effort at date 3 and $U = t - C(q, \theta)$ otherwise. If the firm rejects all contracts at date 2, then its (outside option) payoff equals zero.

Suppose the procurement agency wants to induce the firm to exert effort. Also suppose that the parameters of the model are such that it is optimal to interact with both types and to offer them distinct contracts. Then we can write the procurement agency's problem as follows. The principal chooses $\left(\underline{q}, \overline{q}, \underline{t}, \overline{t}\right)$ so as to maximize its expected payoff,

$$V\left(\underline{t}, \underline{q}, \overline{t}, \overline{q}\right) = \nu_1 \left[S\left(\underline{q}\right) - \underline{t}\right] + (1 - \nu_1)\left[S\left(\overline{q}\right) - \overline{t}\right],$$

subject to six constraints:

$$\overline{t} - C\left(\overline{q}, \overline{\theta}\right) \geq 0, \tag{IR-bad}$$

$$\underline{t} - C\left(\underline{q}, \underline{\theta}\right) \geq 0, \tag{IR-good}$$

$$\overline{t} - C\left(\overline{q}, \overline{\theta}\right) \geq \underline{t} - C\left(\underline{q}, \overline{\theta}\right), \tag{IC-bad}$$

$$\underline{t} - C\left(\underline{q}, \underline{\theta}\right) \geq \overline{t} - C\left(\overline{q}, \underline{\theta}\right), \tag{IC-good}$$

$$\nu_1 \left[\underline{t} - C\left(\underline{q}, \underline{\theta}\right)\right] + (1 - \nu_1)\left[\overline{t} - C\left(\overline{q}, \overline{\theta}\right)\right] - \psi \geq \nu_0 \left[\underline{t} - C\left(\underline{q}, \underline{\theta}\right)\right] + (1 - \nu_0)\left[\overline{t} - C\left(\overline{q}, \overline{\theta}\right)\right], \tag{IC-effort}$$

$$\nu_1 \left[\underline{t} - C\left(\underline{q}, \underline{\theta}\right)\right] + (1 - \nu_1)\left[\overline{t} - C\left(\overline{q}, \overline{\theta}\right)\right] - \psi \geq 0. \tag{IR-ante}$$

The first-best levels of $\overline{q}$ and $\underline{q}$ are defined, in the usual way, by the following two equations:

$$S'\left(\overline{q}^{FB}\right) = C_q\left(\overline{q}^{FB}, \overline{\theta}\right), \qquad S'\left(\underline{q}^{FB}\right) = C_q\left(\underline{q}^{FB}, \underline{\theta}\right).$$

This means, in particular, that $\overline{q}^{FB} < \underline{q}^{FB}$. Now assume that the effort cost ψ takes a value in some intermediate range, so that it is neither very small nor very large. In particular, assume the following inequalities hold:

$$C\left(\overline{q}^{FB}, \overline{\theta}\right) - C\left(\overline{q}^{FB}, \underline{\theta}\right) < \frac{\psi}{\nu_1 - \nu_0} < C\left(\underline{q}^{FB}, \overline{\theta}\right) - C\left(\underline{q}^{FB}, \underline{\theta}\right). \tag{1}$$

Let $\overline{q}^{SB}$ and $\overline{q}^{SB}$ denote the (second-best) levels of $\overline{q}$ and $\underline{q}$, respectively—that is, the quantities that solve the problem shown above. Answer the following questions.

(a) How does $\underline{q}^{SB}$ relate to $\overline{q}^{FB}$? How does $\underline{q}^{SB}$ relate to $\underline{q}^{FB}$? Solve as much as you need of the problem in order to answer those questions. You do not need to show that the second-order condition is satisfied (and you will not get any credit if you nevertheless do that), but otherwise you should prove all your claims.

[You are encouraged to attempt part (b) also if you have not been able to answer part (a).]

(b) Explain the intuition/the economic logic behind the results that you find. If you think it it sheds light on the logic for the problem here, you are encouraged to relate to other results/arguments that we have studied and discussed in the course.

Question 2: Moral hazard with mean-variance preferences

Consider the following moral hazard model with mean-variance preferences that we studied in the course. There is one (single) agent, A, and one principal, P. A chooses an effort level $e \in \Re_+$, thereby incurring the cost $c(e) = \frac{1}{2}e^2$. Given a choice of e, the output (i.e., A's performance) equals $q = e + z$, where z is an exogenous random term drawn from a normal distribution with mean zero and variance ν. It is assumed that P can observe q but not e. Moreover, neither P nor A can observe z. A's wage (i.e., the transfer from P to A) can only be contingent on the output q. It is restricted to be linear in q:

$$t = \alpha + \beta q = \alpha + \beta (e + z).$$

A is risk averse and has a CARA utility function: $U = -\exp[-r(t - c(e))]$, where r (> 0) is the coefficient of absolute risk aversion. Therefore A's expected utility is

$$EU = -\int_{-\infty}^{\infty} \exp[-r(t - c(e))] f(z)\, dz,$$

where $f(z)$ is the density of the normal distribution. P's objective function is

$$V = q - t = q - \alpha - \beta q = (1 - \beta)(e + z) - \alpha,$$

which in expected terms becomes $EV = (1 - \beta)e - \alpha$. It is also assumed that A's outside option utility is $\widehat{U} = -\exp\left[-r\widehat{t}\right]$, where $\widehat{t} > 0$. The timing of events is as follows.

1. P chooses the contract parameters, α and β.
2. A accepts or rejects the contract and, if accepting, chooses an effort level.
3. The noise term z is realized and A and P get their payoffs.

Answer the following questions:

(a) Solve for the β parameter in the second-best optimal contract, denoted by β^{SB} (you do not need to solve for α^{SB}, and you will not get any credit if you nevertheless do that). You should make use of the following (well-known) result:

$$EU = -\exp\left[-r\left(\alpha + \beta e - \frac{1}{2}e^2 - \frac{1}{2}\nu r \beta^2\right)\right].$$

[You are encouraged to attempt parts (b)–(d) also if you have not been able to answer part (a).]

(b) Does the agent get any rents at the second-best optimum? Do not only answer yes or no, but also explain how you can tell.

(c) The first-best values of the effort level and the β parameter equal $e^{FB} = 1$ and $\beta^{FB} = 0$, respectively. How do these values relate to the corresponding second-best values? In particular, is there under- or overprovision of effort at the second-best optimum?

(d) Consider the limit case where $r \to 0$. Explain what happens to the relationship between the second-best and the first-best effort levels. Also explain the intuition for this result.

End of Exam