Written Exam at the Department of Economics winter 2018-19

Contract Theory

Final (Resit) Exam

February 15, 2019

(3-hour closed book exam)

Answers only in English.

This exam consists of four pages in total, including this one

NB: If you fall ill during an examination at Peter Bangs Vej, you must contact an invigilator who will show you how to register and submit a blank exam paper. Then you leave the examination. When you arrive home, you must contact your GP and submit a medical report to the Faculty of Social Sciences no later than seven (7) days from the date of the exam.

Be careful not to cheat at exams!

- You cheat at an exam, if during the exam, you:
- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Optimal taxation

Consider the following version of the optimal taxation model that we studied in the course. In a country there are a continuum of citizens (the agent of the model), who differ from each other only with respect to their ability of working and producing output. In particular, each citizen is either "able" or "not able", with the proportion of able citizens being equal to $\nu \in (0, 1)$. Each citizen can choose how many hours to work. If working q hours, a citizen incurs a cost $C(q, \theta) = \theta q$, where $\theta = \underline{\theta}$ if the citizen is able and $\theta = \overline{\theta}$ if he is not able, with $0 < \underline{\theta} < \overline{\theta}$. The surplus that is generated if the citizen works q hours is given by the function $S(q)$, which satisfies

$$S'(q) > 0, \qquad S''(q) < 0, \qquad S(0) = 0.$$

This surplus enters the budget of the country's government (the principal of the model). However, the government can choose to transfer an amount t of money to the citizen. The citizen's payoff if working for $q \geq 0$ hours and receiving a transfer t from the government is given by

$$U = t - \theta q.$$

The government is benevolent and cares about the payoffs of both groups of citizens. In particular, using the standard notation from the course with "lower bars" and "upper bars", the government's payoff can be as written as

$$V = \nu G(\underline{U}) + (1 - \nu) G(\overline{U}) = \nu G\left(\underline{t} - \underline{\theta}\underline{q}\right) + (1 - \nu) G\left(\overline{t} - \overline{\theta}\overline{q}\right),$$

where the function G satisfies $G' > 0$, $G'' < 0$. The government's budget constraint is given by

$$\nu S\left(\underline{q}\right) + (1 - \nu) S(\overline{q}) \geq \nu \underline{t} + (1 - \nu) \overline{t}.$$

The government can enter binding agreements with its citizens. However, a citizen's type is private information to that citizen and the government can only observe the number of hours worked, q. Therefore, the contract can only specify the quantity q and the transfer t. The government has all the bargaining power and makes a take-it-or-leave-it offer to the citizen. It is assumed that the citizen has no outside option and must accept one of the contracts in the menu of contracts offered by the government. Suppose that the government wants to offer different contracts to the two types of citizens.

(a) Derive the first-best solution; that is, characterize the optimal menu of contracts under the assumption that the government actually can observe whether any given citizen is able or not. Explain the economic intuition behind your result.

Now consider the second-best problem, where the government cannot observe whether any given citizen is able or not. You may suppose that the incentive compatibility constraint of the agent-type that is "not able" does not bind.

(b) Show that there is "efficiency at the top" (i.e., that $\underline{q}^{SB} = \underline{q}^{FB}$). Also show that the second-best quantity of the agent-type that is "not able" is lower than his first-best quantity (i.e., that $\overline{q}^{SB} < \overline{q}^{FB}$). Explain the nature of the trade-off that the government faces.

Question 2: Moral hazard with three effort and output levels

Prometheus Sørensen (the principal, P for short) owns a factory producing pencils and wants to hire Absalon Nielsen (the agent, A for short) to work there. If hired, A's task will be to operate a pencil machine. How smoothly the machine runs depends, in a stochastic fashion, on the level of effort that A chooses to exert. In particular, A can choose one of three effort levels, $e \in \{0, 1, 2\}$, where $e = 2$ means "making a big effort", $e = 1$ means "making a small effort", and $e = 0$ means "not making an effort". A's effort cost equals

$$\psi(e) = \begin{cases} 2\psi & \text{if } e = 2 \\ \psi & \text{if } e = 1 \\ 0 & \text{if } e = 0, \end{cases}$$

with $\psi > 0$. The number of pencils that come out of the machine, q, is either large ($q = q_L$), medium ($q = q_M$), or small ($q = q_S$), with $q_L > q_M > q_S > 0$. The probability of each of the three output levels depends on A's effort, as specified by the following table:

$\Pr[q \mid e]$	$e = 0$	$e = 1$	$e = 2$
$q = q_S$	π_S	$\pi_S - a$	$\pi_S - a - b$
$q = q_M$	π_M	$\pi_M + a$	$\pi_M + a$
$q = q_L$	π_L	π_L	$\pi_L + b,$

with $\pi_S + \pi_M + \pi_B = 1$, $a > 0$, $b > 0$, and all probabilities in the table being strictly between zero and one. In addition, the following condition is assumed to hold:

$$b\pi_M < a\pi_L. \tag{1}$$

P (and the court) can observe which quantity q that is realized but not the effort level chosen by A. It is assumed that P has all the bargaining power and makes a take-it-or-leave-it offer to A. A contract can specify three numbers, t_S, t_M, and t_L, where t_i is the payment to A if $q = q_i$ (for $i \in \{S, M, L\}$). P is risk neutral and her payoff, given a quantity q and a payment t, equals

$$V = q - t.$$

A is also risk neutral and his payoff, given a payment t and an effort level e, equals

$$U = t - \psi(e).$$

A is protected by limited liability, meaning that $t_S \geq 0$, $t_M \geq 0$, and $t_L \geq 0$. A's outside option would yield the payoff zero.

If P wanted to induce A to choose $e = 0$, then the optimal contract would be to set all three payments equal to zero ($t_S = t_M = t_L = 0$). Thus, P's maximized second-best payoff if inducing $e = 0$ is given by

$$V_0^{SB} = \pi_S q_S + \pi_M q_M + \pi_L q_L.$$

(a) Consider the case where P induces A to choose the effort $e = 1$. Derive, for this case, the optimal payment levels (any method—graphical or non-graphical—is fine, as long as the results are shown). Moreover, compute P's maximized second-best payoff (i.e, the expected output minus the expected payments) if inducing $e = 1$, and denote this by V_1^{SB}. What is the condition required for having $V_1^{SB} \geq V_0^{SB}$? Interpret this condition.

(b) Consider the case where P induces A to choose the effort $e = 2$. Derive, for this case, the optimal payment levels (any method—graphical or non-graphical—is fine, as long as the results are shown). Moreover, compute P's maximized second-best payoff (i.e, the expected output minus the expected payments) if inducing $e = 2$, and denote this by V_2^{SB}. What is the condition required for having $V_2^{SB} \geq V_0^{SB}$ and $V_2^{SB} \geq V_1^{SB}$? Interpret these conditions.

End of Exam