Written Exam at the Department of Economics winter 2019-20

Contract Theory

Final Exam

January 17, 2020

(3-hour closed book exam)

Answers only in English.

This exam question consists of six pages in total (including this one)

Falling ill during the exam

If you fall ill during an examination at Peter Bangs Vej, you must:

- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

Be careful not to cheat at exams!

You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

1 Question 1: Insurance and adverse selection

The following is a model of a monopoly insurance market with adverse selection. It is identical to one that we studied in a problem set in the course (except that the verbal (d) part of the question is added).

The principal (P) is a monopoly insurance company and the agent (A) is a car owner who may want to purchase a car insurance. Depending on how skillful A is as a driver, she may or may not have an accident. The probability of having an accident depends on A's type. A skillful (and therefore a *low*-demand) driver has an accident with probability $\underline{\theta}$, and a less skillful (and therefore a *high*-demand) driver has an accident with probability $\overline{\theta}$. Assume that $0 < \underline{\theta} < \overline{\theta} < 1$.

A's disutility of having an accident, measured in monetary terms as a deduction from her income, is denoted by $d > 0$, and A's monetary income is denoted by $w > d$. Moreover, A's payment *to* P in case there is no accident is denoted by p; and the *net* compensation A *receives* from P in case there indeed is an accident (i.e., the net indemnity) is denoted by a. A is risk averse and her utility function is denoted by u (where $u' > 0$ and $u'' < 0$). Therefore, A's utility if buying the insurance is

$$\begin{cases} u\left(w - d + a\right) & \text{if having an accident} \\ u\left(w - p\right) & \text{if not having an accident.} \end{cases}$$

P is risk neutral and wants to maximize the company's expected profits. P does not know the type of A, but assigns the probability $v \in (0,1)$ to the event that $\theta = \underline{\theta}$.

P offers a menu of two distinct contracts to A. As in the course, the contract variables are indicated either with "upper-bars" or "lower-bars," depending on which type the contract is aimed at. The contract variables are p and a. However, to solve the problem it is more convenient to think of P as choosing the utility levels directly, instead of the contract variables. Thus introduce the following notation:

$$\overline{u}_N \stackrel{\text{def}}{=} u\left(w - \overline{p}\right), \quad \overline{u}_A \stackrel{\text{def}}{=} u\left(w - d + \overline{a}\right), \quad \underline{u}_N \stackrel{\text{def}}{=} u\left(w - \underline{p}\right), \quad \underline{u}_A \stackrel{\text{def}}{=} u\left(w - d + \underline{a}\right).$$

Also, let h be the inverse of u (hence $h' > 0$ and $h'' > 0$). We can now rewrite the problem as follows. Given that P is risk neutral and wants to maximize its expected profit, P's objective function can be written as

$$\begin{aligned} V &= v\left[(1-\underline{\theta})\,\underline{p}-\underline{\theta}\underline{a}\right]+(1-v)\left[(1-\overline{\theta})\,\overline{p}-\overline{\theta}\overline{a}\right] \\ &= v\left[w-\underline{\theta}d-(1-\underline{\theta})\,h\left(\underline{u}_N\right)-\underline{\theta}h\left(\underline{u}_A\right)\right]+(1-v)\left[w-\overline{\theta}d-\left(1-\overline{\theta}\right)h\left(\overline{u}_N\right)-\overline{\theta}h\left(\overline{u}_A\right)\right]. \end{aligned}$$

P wants to maximize V w.r.t. $(\underline{u}_N, \underline{u}_A, \overline{u}_N, \overline{u}_A)$, subject to the following four constraints:

$$\left(1-\overline{\theta}\right)\overline{u}_N+\overline{\theta}\overline{u}_A \geq \overline{U}^*, \tag{IR-high}$$

$$\left(1-\underline{\theta}\right)\underline{u}_N+\underline{\theta}\underline{u}_A \geq \underline{U}^*, \tag{IR-low}$$

$$\left(1-\overline{\theta}\right)\overline{u}_N+\overline{\theta}\overline{u}_A \geq \left(1-\overline{\theta}\right)\underline{u}_N+\overline{\theta}\underline{u}_A, \tag{IC-high}$$

$$\left(1-\underline{\theta}\right)\underline{u}_N+\underline{\theta}\underline{u}_A \geq \left(1-\underline{\theta}\right)\overline{u}_N+\underline{\theta}\overline{u}_A, \tag{IC-low}$$

where

$$\overline{U}^* \stackrel{\text{def}}{=} \left(1-\overline{\theta}\right)u\left(w\right)+\overline{\theta}u\left(w-d\right)$$

and

$$\underline{U}^* \stackrel{\text{def}}{=} \left(1-\underline{\theta}\right)u\left(w\right)+\underline{\theta}u\left(w-d\right)$$

are the two types' outside options.

(a) At the first-best optimum (i.e., the optimum when A's type is observable), both types are offered a contract with full insurance (so that $\overline{u}_N = \overline{u}_A$ and $\underline{u}_N = \underline{u}_A$). Explain, in words, the economic logic behind this result.

(b) Show that the constraints IC-high and IC-low jointly imply that $\underline{u}_N - \underline{u}_A \geq \overline{u}_N - \overline{u}_A$.

(c) Assume that the constraints IR-high and IC-low are lax at the second-best optimum[1] (so that they can be disregarded). Show that, at the second-best optimum, the high type is fully insured ($\overline{u}_N = \overline{u}_A$) whereas the low-type is underinsured ($\underline{u}_N > \underline{u}_A$).

(d) In some other adverse selection models that we studied, the outside option for the "good" type was (sufficiently much) more attractive than the "bad" type's outside option. This gave rise to a phenomenon called "countervailing incentives." Answer, in words, the following questions: (i) What is by meant by "countervailing incentives"? (ii) What are the possible consequences of this phenomenon in terms of efficiency and rent extraction at the second-best optimum? (iii) What is the intuition for the results under (ii)?

[1]The second-best optimum is, as usual, defined as the optimum when A's type is *not* observable—i.e., the solution to the problem described in the text above.

Question 2: A patent race among n firms, with moral hazard

The following is a model of moral hazard with two outcome levels and a continuum of effort levels. It builds on a similar model that we studied in the course.

In a market there are $n \geq 2$ firms that compete with each other in a patent race. Exactly one of the firms wins the race. The value of winning is $v > 0$, whereas the value of not winning is zero. Each firm has an owner (and each owner owns only one firm). The owner does not run the firm's daily business herself but instead hires a manager to do that. In particular, it is the manager who decides how much effort to exert in the patent race. The interaction between each pair of owner and manager is characterized by moral hazard: the owner cannot observe her manager's effort but only whether the own firm won the race or not. This means that the wage that the owner pays the own manager can be made contingent only on whether the race was won or not.

The owner of firm i has the following payoff function:

$$V_i = p_i(\mathbf{x})\left(v - \bar{t}_i\right) - \left[1 - p_i(\mathbf{x})\right]\underline{t}_i.$$

Here, $\bar{t}$ (respectively, $\underline{t}$) is manager i's wage if firm i won (respectively, did not win) the race, $\mathbf{x} = (x_1, x_2, \cdots, x_n)$, $x_i \geq 0$ is firm i's effort, and $p_i(\mathbf{x})$ is the probability that firm i wins the race. The manager of firm i has the following payoff function

$$U_i = p_i(\mathbf{x})\,\bar{t}_i + \left[1 - p_i(\mathbf{x})\right]\underline{t}_i - x_i,$$

and his outside option payoff equals zero. Each manager i is protected by limited liability; in particular, the owner must choose a wage that is non-negative: $\underline{t} \geq 0$ and $\bar{t} \geq 0$.

The timing of events is as follows:

1. All n firm owners simultaneously and independently choose the own contract variables. Then each manager accepts or rejects the contract offer.[2] The contract choice and the acceptance/rejection decision in firm i cannot be observed by the owner or the manager of any other firm.

2. The manager of each firm i chooses the effort x_i. These choices are made simultaneously.

3. The owners and managers of all firms receive their payoffs.

At stage 1 and 2, a firm's owner and manager take the behavior of the owners and managers of all other firms as given. Thus, the optimization problem of owner i when choosing the optimal contract variables can be written as follows:

$$\max_{\underline{t},\bar{t},x_i} \left\{ p_i(\mathbf{x})\left(v - \bar{t}_i\right) - \left[1 - p_i(\mathbf{x})\right]\underline{t}_i \right\}, \qquad \text{subject to}$$

[2] That is, the owners do not compete for managers—each manager has the option of working for his own (potential) employer or not all. Moreover, it is assumed that if a manager rejects his offer, then that firm cannot participate in the patent race and the owner's payoff equals zero.

$$p_i(\mathbf{x})\,\bar{t}_i + [1 - p_i(\mathbf{x})]\,\underline{t}_i - x_i \geq 0, \tag{IR}$$

$$x_i \in \arg\max_{\tilde{x}_i} \left\{ p_i(\mathbf{x})\,\bar{t}_i + [1 - p_i(\mathbf{x})]\,\underline{t}_i - \tilde{x}_i \right\}, \tag{IC}$$

$$\underline{t}_i \geq 0, \qquad \bar{t}_i \geq 0. \tag{LL}$$

Assume that the probability-of-winning function is given by

$$p_i(\mathbf{x}) = \begin{cases} \frac{x_i}{\sum_{j=1}^n x_j} & \text{if } \sum_{j=1}^n x_j > 0 \\ \frac{1}{n} & \text{if } \sum_{j=1}^n x_j = 0. \end{cases} \tag{1}$$

(a) Solve for a symmetric Nash equilibrium of the model. That is, solve the optimization problem of owner i stated above (taking the behavior of all other firms as given); after having taken the first-order condition(s), you can impose symmetry across the choice variables of different firms. You should state the (symmetric) equilibrium values of the three variables $\underline{t}$, $\bar{t}$, and x_i. For full credit, you should verify that all the second-order conditions are satisfied.

- Hint #1: Use the first-order approach.
- Hint #2: Guess that the IR constraint is satisfied at the optimum and check this after having found a candidate solution.

Consider another version of the model, where the firm owners do *not* delegate the effort decision to a manager and thus there is no moral hazard problem. Instead, each owner exerts the effort herself (and she herself therefore incurs the associated effort cost). In particular, in this model, owner i's payoff function is given by $\pi_i = p_i(\mathbf{x})\,v_i - x_i$, where $p_i(\mathbf{x})$ is again given by (1). This model has an equilibrium in which each owner chooses the effort $x^* = \frac{(n-1)v}{n^2}$, and her equilibrium payoff equals $\pi^* = \frac{v}{n^2}$.

You are encouraged to answer the (b) part also if you have failed to solve the (a) part.

(b) Compare the equilibrium outcomes of the model without moral hazard, described immediately above, and the moral hazard model that you solved in part (a). In particular, at the symmetric equilibrium,

(i) which model yields the highest payoff for the owners, and

(ii) which model yields the highest total surplus (defined as the sum of the owner's and the manager's payoffs)?

In order to answer these questions, you do not need to do any math (and if you nevertheless do that, you will not get any credit for this). Instead, you should explain verbally how we should expect the logic of the two models to work and why we should expect your particular answers to the two questions to be correct. If you think there are different effects that work in opposite directions and that the answer to one or both questions therefore is ambiguous (without looking at the specific math results), then you should answer that—but you should also explain your reasoning and what these different effects are.

- Hint: Note that, in the model without moral hazard, if the owners could enter a binding agreement with each other that required them all to choose a zero effort, then each owner's payoff would equal $\pi^{coop} = \frac{v}{n}$, which is strictly larger than the non-cooperative payoff $\pi^* = \frac{v}{n^2}$. In this sense, the fact that there is competition between the owners hurts them.

End of Exam