Written Exam at the Department of Economics winter 2019-20

Contract Theory

Final (Resit) Exam

February 12, 2020

(3-hour closed book exam)

Answers only in English.

This exam question consists of five pages in total (including this one)

Falling ill during the exam
If you fall ill during an examination at Peter Bangs Vej, you must:
- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

Be careful not to cheat at exams!
You cheat at an exam, if during the exam, you:
- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Life insurance with adverse selection

The following is a model of adverse selection in a life insurance market. It builds on the standard adverse selection model that we studied in the course. The principal (P) is a monopoly insurance company and the agent (A) is an individual who may want to purchase a life insurance.

There are three time periods: 1, 2, and 3. The individual A lives for sure in period 1 but will die either (with probability θ) between periods 1 and 2 or otherwise (with probability $1-\theta$) between periods 2 and 3. If A chooses to purchase a life insurance, he must pay a premium p_1 in period 1. Moreover:

- In case A dies between periods 1 and 2, then in period 2 the insurance company pays out a compensation a_2 to B, who is a close relative of A and whom A cares about.
- In case A dies between periods 2 and 3, then there is no compensation paid out by the company in period 2. Instead, A must pay a new premium p_2 in period 2, and in period 3 (when A is dead) the insurance company pays out a compensation a_3 to B.

When making decisions at the beginning of period 1, A maximizes the following expected utility:

$$Eu = u(w_A - p_1) + \theta u(w_B + a_2) + (1-\theta)\left[u(w_A - p_2) + u(w_B + a_3)\right], \tag{1}$$

where w_A is A's income, w_B is B's income, and u is A's and B's common utility function. It is assumed that $w_A > w_B > 0$ and that $u' > 0$ and $u'' < 0$.[1] We can interpret the specification in (1) as follows. A assigns the same weight to his period 1 payoff as to any future payoff (so there is no time discounting). He also assigns the same weight to his own utility as to that of B's; however, A cares about B's utility only in the period immediately after A's death.

A has private information about his type $\theta \in \{\underline{\theta}, \overline{\theta}\}$, which we can interpret as the individual's health status. A healthy (and therefore a *low*-demand) type dies between periods 1 and 2 with probability $\underline{\theta}$, and a less healthy (and therefore a *high*-demand) type dies between periods 1 and 2 with probability $\overline{\theta}$. We assume that $0 < \underline{\theta} < \overline{\theta} < 1$. The insurance company, P, assigns the probability $v \in (0,1)$ to the event that $\theta = \underline{\theta}$.

[1] We also assume that the utility function satisfies a standard Inada condition, $u'(0) = \infty$, which guarantees that a corner solution is never optimal.

The company P is risk neutral and wants to maximize its expected profit. Moreover, when making its decisions in period 1, it does not discount its profits in future periods. At the beginning of period 1, P offers a menu of two distinct contracts to A, who accepts exactly one of the contracts or rejects both of them. As in the course, the contract variables are indicated either with "upper-bars" or "lower-bars," depending on which type the contract is aimed at. The contract variables are the premium levels and the compensation levels for each type. However, to solve the problem it is more convenient to think of P as choosing the utility levels directly, instead of the contract variables. Thus introduce the following notation:

$$\underline{u}_1 \stackrel{\text{def}}{=} u(w_A - \underline{p}_1), \quad \underline{u}_2^A \stackrel{\text{def}}{=} u(w_A - \underline{p}_2), \quad \underline{u}_2^B \stackrel{\text{def}}{=} u(w_B + \underline{a}_2), \quad \underline{u}_3 \stackrel{\text{def}}{=} u(w_B + \underline{a}_3),$$

$$\overline{u}_1 \stackrel{\text{def}}{=} u(w_A - \overline{p}_1), \quad \overline{u}_2^A \stackrel{\text{def}}{=} u(w_A - \overline{p}_2), \quad \overline{u}_2^B \stackrel{\text{def}}{=} u(w_B + \overline{a}_2), \quad \overline{u}_3 \stackrel{\text{def}}{=} u(w_B + \overline{a}_3).$$

Letting h be the inverse of the utility function (so $h \stackrel{\text{def}}{=} u^{-1}$), we can now write P's expected profit as follows:

$$\begin{aligned} V = K - \nu \left[h\left(\underline{u}_1\right) + \underline{\theta} h\left(\underline{u}_2^B\right) + (1-\underline{\theta})\left(h\left(\underline{u}_2^A\right) + h\left(\underline{u}_3\right)\right)\right] \\ - (1-\nu)\left[h\left(\overline{u}_1\right) + \overline{\theta} h\left(\overline{u}_2^B\right) + (1-\overline{\theta})\left(h\left(\overline{u}_2^A\right) + h\left(\overline{u}_3\right)\right)\right], \end{aligned} \tag{2}$$

where K is a constant. P wants to maximize V w.r.t. $\left(\underline{u}_1, \underline{u}_2^A, \underline{u}_2^B, \underline{u}_3, \overline{u}_1, \overline{u}_2^A, \overline{u}_2^B, \overline{u}_3\right)$, subject to the following four constraints:

$$\underline{u}_1 + \underline{\theta}\underline{u}_2^B + (1-\underline{\theta})\left(\underline{u}_2^A + \underline{u}_3\right) \geq \underline{U}^*, \tag{IR-low}$$

$$\overline{u}_1 + \overline{\theta}\overline{u}_2^B + (1-\overline{\theta})\left(\overline{u}_2^A + \overline{u}_3\right) \geq \overline{U}^*, \tag{IR-high}$$

$$\underline{u}_1 + \underline{\theta}\underline{u}_2^B + (1-\underline{\theta})\left(\underline{u}_2^A + \underline{u}_3\right) \geq \overline{u}_1 + \underline{\theta}\overline{u}_2^B + (1-\underline{\theta})\left(\overline{u}_2^A + \overline{u}_3\right), \tag{IC-low}$$

$$\overline{u}_1 + \overline{\theta}\overline{u}_2^B + (1-\overline{\theta})\left(\overline{u}_2^A + \overline{u}_3\right) \geq \underline{u}_1 + \overline{\theta}\underline{u}_2^B + (1-\overline{\theta})\left(\underline{u}_2^A + \underline{u}_3\right), \tag{IC-high}$$

where $\overline{U}^*$ and $\underline{U}^*$ are the two types' outside option payoffs.

Assume that the parameters of the model are such that, at the optimum, the constraints IR-high and IC-low are lax. Then the Lagrangian associated with the insurance company's maximization problem can be written as

$$\begin{aligned} \mathcal{L} = K - \nu \left[h\left(\underline{u}_1\right) + \underline{\theta} h\left(\underline{u}_2^B\right) + (1-\underline{\theta})\left(h\left(\underline{u}_2^A\right) + h\left(\underline{u}_3\right)\right)\right] \\ - (1-\nu)\left[h\left(\overline{u}_1\right) + \overline{\theta} h\left(\overline{u}_2^B\right) + (1-\overline{\theta})\left(h\left(\overline{u}_2^A\right) + h\left(\overline{u}_3\right)\right)\right] \\ + \mu\left[\underline{u}_1 + \underline{\theta}\underline{u}_2^B + (1-\underline{\theta})\left(\underline{u}_2^A + \underline{u}_3\right) - \underline{U}^*\right] \\ + \lambda\left[\overline{u}_1 + \overline{\theta}\overline{u}_2^B + (1-\overline{\theta})\left(\overline{u}_2^A + \overline{u}_3\right) - \underline{u}_1 - \overline{\theta}\underline{u}_2^B - (1-\overline{\theta})\left(\underline{u}_2^A + \underline{u}_3\right)\right], \end{aligned}$$

where μ and λ are the shadow prices associated with IR-low and IC-high, respectively.

(a) Show, by solving as much as you need of the maximization problem, how the four utility levels for the **high** ($\overline{\theta}$) type relate to each other at the optimum. That is, are the utility levels all equal (so that the high type is fully insured)? If not, which utility levels are higher and which ones are lower? Answer by stating a ranking like, for example, $a = b = c = d$ or $a < b = c \leq d$, where the letters a-d represent the four utility levels for the high type. Also make sure that you have explained how you obtained this ranking.

(b) Show, by solving as much as you need of the maximization problem, how the four utility levels for the **low** ($\underline{\theta}$) type relate to each other at the optimum. That is, are the utility levels all equal (so that the low type is fully insured)? If not, which utility levels are higher and which ones are lower? Answer by stating a ranking like, for example, $a = b = c = d$ or $a < b = c \leq d$, where the letters a-d represent the four utility levels for the low type. Also make sure that you have explained how you obtained this ranking.

You are encouraged to answer the (c) part also if you have failed to solve the (a) and (b) parts.

(c) Explain in words the logic for why we should expect the low type and high type, respectively, to be fully insured or not fully insured.

Question 2: Moral hazard with mean-variance preferences

Consider the following moral hazard model with mean-variance preferences that we studied in the course. There is one (single) agent, A, and one principal, P. A chooses an effort level $e \in \Re_+$, thereby incurring the cost $c(e) = \frac{1}{2}e^2$. Given a choice of e, the output (i.e., A's performance) equals $q = e + z$, where z is an exogenous random term drawn from a normal distribution with mean zero and variance v. It is assumed that P can observe q but not e. Moreover, neither P nor A can observe z. A's wage (i.e., the transfer from P to A) can only be contingent on the output q. It is restricted to be linear in q:

$$t = \alpha + \beta q = \alpha + \beta (e + z).$$

A is risk averse and has a CARA utility function: $U = -\exp[-r(t - c(e))]$, where r (> 0) is the coefficient of absolute risk aversion. Therefore A's expected utility is

$$EU = -\int_{-\infty}^{\infty} \exp[-r(t - c(e))] f(z)\, dz,$$

where $f(z)$ is the density of the normal distribution. P's objective function is

$$V = q - t = q - \alpha - \beta q = (1 - \beta)(e + z) - \alpha,$$

which in expected terms becomes $EV = (1-\beta)\,e - \alpha$. It is also assumed that A's outside option utility is $\widehat{U} = -\exp\left[-r\widehat{t}\right]$, where $\widehat{t} > 0$. The timing of events is as follows.

1. P chooses the contract parameters, α and β.
2. A accepts or rejects the contract and, if accepting, chooses an effort level.
3. The noise term z is realized and A and P get their payoffs.

Answer the following questions:

(a) Solve for the β parameter in the second-best optimal contract, denoted by β^{SB} (you do not need to solve for α^{SB}, and you will not get any credit if you nevertheless do that). You should make use of the following (well-known) result:

$$EU = -\exp\left[-r\left(\alpha + \beta e - \frac{1}{2}e^2 - \frac{1}{2}\nu r \beta^2\right)\right].$$

You are encouraged to attempt parts (b)–(d) also if you have not been able to answer part (a).

(b) Does the agent get any rents at the second-best optimum? Do not only answer yes or no, but also explain how you can tell.

(c) The first-best values of the effort level and the β parameter equal $e^{FB} = 1$ and $\beta^{FB} = 0$, respectively. How do these values relate to the corresponding second-best values? In particular, is there under- or overprovision of effort at the second-best optimum?

(d) Consider the limit case where $r \to 0$. Explain what happens to the relationship between the second-best and the first-best effort levels. Also explain the intuition for this result.

End of Exam