

Written Exam for the M.Sc. in Economics winter 2013-14

Contract Theory

Final Exam / Master's Course

January 8, 2014

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam paper consists of four pages in total, including this page.

Attempt both questions

Question 1 (adverse selection)¹

In a country there are a continuum of citizens (the agent of the model), who differ from each other only with respect to their ability of working and producing output. In particular, each citizen is either “able” or “not able”, with the proportion of able citizens being equal to $\nu \in (0, 1)$. Each citizen can choose how many hours to work. If working q hours, a citizen incurs a cost $C(q, \theta) = \theta q$, where $\theta = \underline{\theta}$ if the citizen is able and $\theta = \bar{\theta}$ if he is not able, with $0 < \underline{\theta} < \bar{\theta}$. The surplus that is generated if the citizen works q hours is given by the function $S(q)$, which satisfies

$$S'(q) > 0, \quad S''(q) < 0, \quad S(0) = 0.$$

This surplus enters the budget of the country’s government (the principal of the model). However, the government can choose to transfer an amount t of money to the citizen. The citizen’s payoff if working for $q \geq 0$ hours and receiving a transfer t from the government is given by

$$U = t - \theta q.$$

The government is benevolent and cares about the payoffs of both groups of citizens. In particular, using the standard notation from the course with “lower bars” and “upper bars”, the government’s payoff can be written as (this is where the model differs from the one in the course)

$$\begin{aligned} V &= \nu \underline{U} + (1 - \nu) \bar{U} - \varphi(|\underline{U} - \bar{U}|) \\ &= \nu (\underline{t} - \underline{\theta} \underline{q}) + (1 - \nu) (\bar{t} - \bar{\theta} \bar{q}) - \varphi[|(\underline{t} - \underline{\theta} \underline{q}) - (\bar{t} - \bar{\theta} \bar{q})|], \end{aligned}$$

where the function φ satisfies $\varphi' > 0$, $\varphi'' > 0$ and $\varphi(0) = 0$. This formulation implies that the government, everything else equal, dislikes inequality. The government’s budget constraint is given by

$$\nu S(\underline{q}) + (1 - \nu) S(\bar{q}) \geq \nu \underline{t} + (1 - \nu) \bar{t}.$$

The government can enter binding agreements with its citizens. However, a citizen’s type is private information to that citizen and the government can only observe the number of hours worked, q . Therefore, the contract can only specify the quantity q and the transfer t . The government has all the bargaining power and makes a take-it-or-leave-it offer to the citizen. It is assumed that the citizen has no outside option and must accept one of the contracts in the menu of contracts offered by the government. Suppose that the government wants to offer different contracts to the two types of citizens.

¹This is a slightly tweaked version of the optimal taxation model that Laffont and Martimort discuss in their book, and it is identical to a problem that you discussed in an exercise class (PS6, problem 2).

- a) Consider the second-best problem, where the government cannot observe whether any given citizen is able or not. You may suppose that the incentive compatibility constraint of the agent-type that is “not able” does not bind. You may also assume that the budget constraint is binding. Show that there is “efficiency at the top” (i.e., that $\underline{q}^{SB} = \underline{q}^{FB}$). Also show that the second-best quantity of the agent-type that is “not able” is lower than his first-best quantity (i.e., that $\bar{q}^{SB} < \bar{q}^{FB}$).
- b) Explain the nature of the trade-off that the government faces. Also explain the intuition for the results that $\underline{q}^{SB} = \underline{q}^{FB}$ and $\bar{q}^{SB} < \bar{q}^{FB}$.
- c) Consider the more general version of the adverse selection model discussed in Laffont and Martimort’s book (and in the lecture slides). There it was assumed that the agent’s cost function satisfies $C_{qq\theta}(q, \theta) \geq 0$. (i) Where in the analysis did we make use of that assumption? (ii) Explain the economic implication of this assumption and why, in economic terms, it helps us with the issue asked about in part (i).

Question 2 (moral hazard)²

In a land far, far away lies the Kingdom of Contractland, a small nation ruled by the dictator King Principal (who indeed is the principal of our model). The king’s subjects, who jointly are the agent of the model, are all farmers. We model the farmers as a continuum with mass one. Each farmer decides how hard to work: $e \in [0, 1]$. The effort level e then determines the probability with which the farmer earns a high ($w \in (0, 1)$) or a low (equal to zero) income; in particular, $\Pr[\text{income} = w] = e$.³ By exerting the effort e the farmer incurs the cost $\psi(e) = \frac{1}{2}e^2$. The farmer is risk neutral and his utility equals

$$u = \begin{cases} w - \tau - \psi(e) \equiv \bar{u} & \text{if earning a high income} \\ b - \psi(e) \equiv \underline{u} & \text{if earning a low income.} \end{cases}$$

Here, τ is a tax that the farmer must pay to the king when earning a high income, and b is an income support that the farmer receives from the king when the farmer’s income is low. The variables τ and b are chosen by the king, in a way that respects the king’s budget constraint. By the law of large numbers, the budget constraint can be written as

$$e\tau - (1 - e)b \geq 0, \tag{budget}$$

where the first term on the left-hand side represents the total tax revenues and the second term is the total amount of income support paid out.

²This moral hazard model is a variation of the setup with two outcomes and a continuum of effort levels that we studied in the course.

³The farmers’ income draws are independent.

The king's objective function is given by

$$V = e\tau - (1 - e)b - \alpha |\bar{u} - \underline{u}| + \beta E[u], \quad (1)$$

where $\alpha \in (0, w)$ and $\beta \in (0, 1)$ are parameters and $E[u] \equiv e(w - \tau) + (1 - e)b - \psi(e)$ is the expected utility of a farmer. That is, the king cares about (i) the rents that he can keep for himself (the first two terms in (1)), (ii) the level of inequality in the country (the third term) and (iii) the farmers' expected income (the last term). The timing of events is as follows:

1. The king commits to some values of τ and b .
2. Each farmer chooses e .
3. Given e , the farmers' income levels are realized and all parties receive their payoffs.

The king does not observe the effort choice e , which is why he cannot directly dictate to a farmer what effort he should exert. The king therefore chooses only τ and b . Also, the king cannot choose a tax that is larger than the farmer's high income and he cannot make the income support negative (our two limited liability constraints):

$$\tau \leq w, \quad (\text{LL-high})$$

$$b \geq 0. \quad (\text{LL-low})$$

- a) [*The first best solution*] Suppose, in contrast to what was said above, that the king can indeed observe e and therefore dictate to each farmer what effort to exert. Everything else is as stated in the model description above. What values of τ , b and e will the king choose, and why is that his optimal choice? Do not solve the model mathematically but provide verbal answers to those questions (if you nevertheless make some calculations, do not include them in your answers).
- *Hint:* Since $\beta < 1$, maximizing the own rents is more important to the king than making the farmers' expected utility large.
- b) [*The second best solution*] Suppose, as in the original model description, that the king cannot observe e . Show formally (using mathematics) what the king's optimal choice of τ , b and e is.
- c) In the article by Canice Prendergast (JEL, 1999) there is a discussion about three kinds of drawbacks with using so-called subjective performance evaluations. Explain what is meant by subjective performance evaluations and provide a brief summary of Prendergast's discussion of the three kinds of drawbacks.

END OF EXAM