Written Exam for the M.Sc. in Economics autumn 2011-2012

Contract Theory

Final (Resit) Exam / Master's Course

February 24, 2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Attempt both questions

Question 1 (adverse selection)

This adverse selection model with two types is identical to one that we studied in the course.

A firm (the agent, A) interacts with a government procurement agency (the principal, P). A produces office material that P wants to purchase. A's cost of producing q units of office material is given by the function $C(q,\theta)$, where θ is an efficiency parameter. This function satisfies

$$C(0,\theta)=0, \qquad C_q>0, \qquad C_{qq}\geq 0, \qquad C_\theta>0, \qquad C_{q\theta}>0, \qquad C_{qq\theta}\geq 0.$$

The value for P of receiving q units of office material is given by the function $S(q)$, which satisfies

$$S'(q)>0, \qquad S''(q)<0, \qquad S(0)=0.$$

The efficiency parameter θ can take two values: $\theta\in\left\{\underline{\theta},\overline{\theta}\right\}$, with $0<\underline{\theta}<\overline{\theta}$. A knows the value of θ perfectly. However, P only knows that

$$\Pr\left[\theta=\underline{\theta}\right]=\nu \qquad \text{and} \qquad \Pr\left[\theta=\overline{\theta}\right]=1-\nu,$$

with $0<\nu<1$. The procurement agency has all the bargaining power and makes a take-it-or-leave-it offer to the firm. A contract can specify the quantity q that A must produce and deliver and the payment t that A will receive. Suppose that P wants to offer different contracts to the two types of firms. P is risk neutral and its payoff, given a quantity q and a payment t, equals

$$V=S(q)-t.$$

A is also risk neutral and its payoff, given a quantity q and a payment t, equals

$$U=t-C(q,\theta).$$

A's outside option (the same for both types) would yield the payoff zero.

P offers a menu of two distinct contracts to A. As in the course, the contract variables are indicated either with "upper-bars" or with "lower-bars", depending on which type the contract is aimed at. P's problem is to choose $\left(\underline{t},\underline{q},\overline{t},\overline{q}\right)$ so as to maximize

$$\nu\left[S\left(\underline{q}\right)-\underline{t}\right]+(1-\nu)\left[S\left(\overline{q}\right)-\overline{t}\right]$$

subject to the following four constraints:

$$\overline{t}-C\left(\overline{q},\overline{\theta}\right)\geq 0, \tag{IR-bad}$$

$$\underline{t}-C\left(\underline{q},\underline{\theta}\right)\geq 0, \tag{IR-good}$$

$$\overline{t}-C\left(\overline{q},\overline{\theta}\right)\geq \underline{t}-C\left(\underline{q},\overline{\theta}\right), \tag{IC-bad}$$

$$\underline{t}-C\left(\underline{q},\underline{\theta}\right)\geq \overline{t}-C\left(\overline{q},\underline{\theta}\right). \tag{IC-good}$$

a) Explain in words what each one of the four constraints says and why it must be satisfied at the optimum.

b) Prove that incentive compatibility and Spence-Mirrlees ($C_{q\theta} > 0$) imply monotonicity; that is, show that if the inequalities defining incentive compatibility hold and if the Spence-Mirrlees condition is satisfied, then the quantity offered to the $\underline{\theta}$-type agent is at least as large as the one offered to the $\overline{\theta}$-type agent.

c) The first best optimal quantities are defined by $S'\left(\underline{q}^{FB}\right) = C_q\left(\underline{q}^{FB}, \underline{\theta}\right)$ and $S'\left(\overline{q}^{FB}\right) = C_q\left(\overline{q}^{FB}, \overline{\theta}\right)$, respectively. Assume that the constraints (IR-good) and (IC-bad) are lax at the second-best optimum (so that they can be disregarded). Show that, at the second-best optimum, the good type's quantity is *not* distorted relative to the first best ($\underline{q}^{SB} = \underline{q}^{FB}$) and that the bad type's quantity is distorted downwards ($\overline{q}^{SB} < \overline{q}^{FB}$).

d) Explain the intuition for the results you were asked to show under c). Also explain the nature of the trade-off that the principal faces.

Question 2 (moral hazard)

This is a model of so-called sharecropping that lets the farmer's effort choice be continuous. It builds on similar model from the course, which assumed a binary effort choice.

A landlord (the principal, P) owns a piece of land and wants to lease the land to a poor farmer (the agent, A). If entering such an agreement, A will, when farming the land, choose what effort to make, $e \in [0, 1]$. The associated effort cost equals $\psi(e)$, where this function satisfies

$$\psi' > 0, \quad \psi'' > 0, \quad \psi(0) = \psi'(0) = 0, \quad \lim_{e \to 1} \psi'(e) = \infty.$$

Depending on A's effort and on the weather, the output that is produced may be high ($q = \overline{q}$) or low ($q = \underline{q}$, with $0 \leq \underline{q} < \overline{q}$). The probability that output is high equals the effort level: $\Pr(q = \overline{q} \mid e) = e$. The market price of the output equals unity. Therefore, q is also the market value of the output.

P (and the court) can observe which quantity that is realized ($\overline{q}$ or $\underline{q}$) but not whether A has worked hard or not. Therefore, in principle, the contract between P and A could consist of two numbers, indicating how much A should pay P in each state (a high-output state or a low-output state). However, the contract that is actually used is a so-called sharecropping contract, which is characterized by a single number, $\alpha \in [0, 1]$. The number α is the share of output that A is allowed to *keep*, whereas the remaining share $1 - \alpha$ is paid to P. Therefore, P's expected profit equals

$$V = (1 - \alpha)\left[e\overline{q} + (1 - e)\underline{q}\right].$$

Moreover, A's expected utility equals $U = \alpha\left[e\overline{q} + (1 - e)\underline{q}\right] - \psi(e)$. A's outside option would yield the utility zero. A is protected by limited liability, meaning that a contract cannot stipulate that A must pay, in net terms, some amount of money to P. It is assumed that P has all the bargaining power and makes a take-it-or-leave-it offer to A.

a) Characterize the second best optimal values of α and e, using the first-order approach. Assume that the functional forms are such that the second-order conditions are satisfied.

b) In a richer model with both a continuum of effort levels and output levels, what are the two conditions needed to ensure that the first-order approach is valid? You do not have to state the conditions formally — it suffices if you do it in words.

c) Instead of the model discussed above, consider the following moral hazard model with a risk neutral principal and a risk neutral agent who is protected by limited liability. There are two effort levels (0 and 1) and four output levels (y_1, y_2, y_3 and y_4). The probabilities with which the different output levels realize, given the two different effort levels, are indicated in the following table:

	Effort $= 0$	Effort $= 1$
y_1	$\pi_{10} = 0.3$	$\pi_{11} = 0.1$
y_2	$\pi_{20} = 0.4$	$\pi_{21} = 0.4 + 2x$
y_3	$\pi_{30} = 0.2$	$\pi_{31} = 0.3 - x$
y_4	$\pi_{40} = 0.1$	$\pi_{41} = 0.2 - x,$

where $x \in [0, 0.1]$. What is the condition that we need to impose on the model to ensure that the principal's optimal contract is such that the agent's payment is strictly increasing in the level of output that is realized? For what values of x is this condition satisfied? Explain the intuition for why this condition matters.

END OF EXAM