

Written re-exam for the M. Sc. in Economics. Summer 2016

Economic Growth

Master's Course

August 29, 2016

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam question consists of 4 pages in total including this page.

The weighting of the problems is: Problem 1: 40%, Problem 2: 45%, and Problem 3: 15%.¹

¹The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

Problem 1 Let aggregate output be produced through a differentiable three-factor production function \tilde{F} :

$$Y = \tilde{F}(K, L_1, L_2, t),$$

where K is capital input, L_1 is input of unskilled labor (below also called blue-collar labor), and L_2 is input of skilled labor. The dating of the variables is suppressed unless needed for clarity. Suppose technological change is such that the production function can be rewritten

$$\tilde{F}(K, L_1, L_2, t) = F(K, H(L_1, L_2, t)),$$

where the “nested” function $H(L_1, L_2, t)$ represents input of a “human capital” aggregate. Let F be CRS-neoclassical w.r.t. K and H and let H be CRS-neoclassical w.r.t. (L_1, L_2) . Let $\partial H/\partial t > 0$ so that “technical change” amounts to “technical progress”. Assume markets are competitive. Finally, let the real wages of unskilled and skilled labor be denoted w_1 and w_2 , respectively.

- a) Find an expression for the relative wage (also called the *skill premium*), showing that the relative wage depends only on t and the factor ratio L_2/L_1 . *Hint:* useful tools here are the chain rule and Euler’s theorem: if the differentiable function $f(x, y)$ is homogeneous of degree one, then the partial derivatives of f are homogeneous of degree zero.
- b) In terms of the function H define the Hicksian concepts of *skill-biased* technical change, *skill-neutral* technical change, and *blue collar-biased* technical change. *Hint:* the general Hicksian definition is: technical change is *neutral* vis-a-vis two production factors, if the production function changes over time in such a way that the relative factor incomes under perfect competition remain unchanged along any hypothetical path where the factor ratio is constant.

In the US the skill premium (measured by the wage ratio for college graduates vis-a-vis high school graduates) has had an upward trend since 1950. In the same period also the supply of skilled labor relative to unskilled labor has been rising, in fact even faster than the skill premium.

- c) In terms of the mentioned Hicksian concepts, how will you characterize an economic evolution with these two properties? Comment.
- d) Could there be a causal link between the two observed phenomena? Briefly explain.

We will now consider another concept you should know: *capital-skill complementarity*. Let the aggregate production function be

$$Y = \tilde{F}(K, L_1, L_2, t) = F(K, A_{1t}L_1, A_{2t}L_2) = (K + A_{1t}L_1)^\alpha (A_{2t}L_2)^{1-\alpha}, \quad 0 < \alpha < 1,$$

where K , L_1 , and L_2 have the same meaning as above, whereas A_{1t} and A_{2t} are technical coefficients that may be rising over time. Markets are still assumed competitive.

- e) In terms of the function \tilde{F} , define the property *capital-skill complementarity*. Is the property present, given the specification of \tilde{F} ? Why or why not?
- f) Find an expression for the skill premium, showing that it depends only on α and two generally variable “factors”, namely the ratios $(K + A_{1t}L_1)/(A_{2t}L_2)$ and A_{2t}/A_{1t} .

Consider a path, which we name P , along which the real interest rate is a positive constant, r . Let also the capital depreciation rate be a constant, δ .

- g) Assuming the representative firm maximizes profit, show that along the path P , the ratio $(K + A_{1t}L_1)/(A_{2t}L_2)$ will be constant. *Hint*: find an expression for $(K + A_{1t}L_1)/(A_{2t}L_2)$ in terms of the capital cost, $r + \delta$.

Suppose the skill premium is rising along the path P .

- h) Given the model, what must be the explanation of this?

Problem 2 Consider a closed economy with profit maximizing firms, operating under perfect competition. The size of the labor force (= employment = population) is L_t . Aggregate output (GDP) at time t is Y_t per time unit. Output is used for consumption and investment in physical capital, K_t , so that $\dot{K}_t = Y_t - C_t - \delta K_t$, where C_t is consumption and δ is the rate of physical decay of capital, $\delta \geq 0$. The initial value $K_0 > 0$ is given. There is a perfect market for loans with interest rate r_t . Time is continuous and there is no uncertainty.

The production function of firm i is

$$Y_{it} = K_{it}^\alpha (A_t L_{it})^{1-\alpha}, \quad 0 < \alpha < 1, \quad i = 1, 2, \dots, N,$$

where A_t is the economy-wide technology level, $\sum_i K_{it} = K_t$, $\sum_i L_{it} = L_t$, and N is “large”. Suppose each firm is small relative to the economy as a whole and perceives it has no influence on aggregate variables, including A_t .

- a) In general equilibrium, determine r at time t . *Hint*: look at first-order conditions of a single firm, then observe a certain similarity across firms concerning the chosen capital-labor ratio, then use clearing in the factor markets.
- b) In general equilibrium, determine the aggregate production function at time t .
- c) For a given A_t , find the TFP level (total factor productivity) at time t .

For any variable $x > 0$, let g_x denote its growth rate, \dot{x}/x .

- d) Following the basic idea in growth accounting, express g_Y analytically in terms of the “contributions” from growth in K , L , and a residual, respectively.

- e) Find expressions for the TFP growth rate, the gross income share of capital (aggregate gross income to capital owners divided by GDP = GNP), and the labor income share, respectively.

From now, suppose A_t evolves according to

$$A_t = e^{\varepsilon t} K_t^\lambda, \quad \varepsilon > 0, \quad 0 < \lambda < 1, \quad (*)$$

where ε and λ are given constants.

- f) Briefly interpret (*). Given (*), express g_Y analytically in terms of the “contributions” from growth in K , L , and a residual, respectively.
- g) As a thought experiment, suppose we have empirical data for this economy. Will applying standard growth accounting on the basis of these data lead to over- or underestimation of the “contribution” to output growth from growth in capital? Why?

Let $L_t = L_0 e^{nt}$, where n is a positive constant.

- h) Determine the growth rate of $y \equiv Y/L$ under balanced growth, assuming saving is positive. *Hint:* use a certain general balanced growth property.
- i) Briefly explain what constitutes the ultimate source of per capita growth according to the model. Compare with what the growth accounting in d) suggested.

Problem 3 Two basic models of innovation-based endogenous growth are the Romer-Jones horizontal innovations model and the Schumpeterian model of vertical innovations.

- a) From both a theoretical and an empirical point of view, the Schumpeterian model has in at least one respect a certain strength compared with the horizontal innovations model. What is this strength, why is it a strength, and from what difference in the two model structures does it derive? In connection with the last question, a few formulas are likely to be useful.
- b) The bulk of empirical evidence suggests that market economies do too little R&D investment compared to the optimal level as defined from the perspective of a social planner respecting the preferences of an assumed representative infinitely-lived household. Are both models consistent with this evidence? Are there in both models theoretically possible combinations of parameter values that may give rise to the opposite conclusion? Explain. Make sure you include all relevant parameters.
- c) In spite of their differences, the models have several elements in common. One common element is a static “distortion” related to market power. What is this “distortion” and what kind of policy remedy could be suggested? Explain verbally and possibly with a little math.