Written Exam at the Department of Economics
Summer 2021
Economic Growth
Final Exam, June 3, 9am-noon

3-hour closed book exam. Answers only in English.

This exam question consists of 5 pages in total, including this front page.

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- submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

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You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
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- Or if you otherwise violate the rules that apply to the exam

1 Short essay questions

Question 1.1

Jones and Klenow (2016) provide an approach to comparing “welfare” across countries. They show that (consumption) inequality negatively influences country-level welfare . Why is that so?

Question 1.2

(a) The core principle which underlies the assumption of constant returns to capital and labor in the aggregate is the so-called replication argument. Explain what the replication argument is. (b) The conventional wisdom is that in order for GDP per capita to grow in the long run technology will have to improve. What is the core principle which leads to the insight that per capita growth requires technological change? Explain why this principle does not violate the replication argument.

Question 1.3

Bloom et al (2020) show that the number of transistors on an integrated circuits has doubled roughly every other year since 1970. (a) Why do Bloom et al nevertheless think research productivity has fallen? (b) What model(s) of economic growth is consistent with their evidence? (c) Can Moore’s law be a steady state? Explain.

Question 1.4

Explain why it is challenging to estimate the causal effect of the adoption of industrial robots on labor market outcomes. Explain further the approach Acemoglu and Restrepo (2019) take to solve this problem.

Question 1.5

Explain why the arrival of entirely new products can bias conventional estimates of economic growth.

Question 1.6

What is wage polarization and job polarization, and how does the task based model help us to understand the two phenomena?

2 Intermediate goods and comparative development

Consider the following model formulated in discrete time:

$$Q_t = \bar{A}\left(K_t^{\alpha}L^{1-\alpha}\right)^{1-\sigma}X_t^{\sigma} \tag{1}$$

$$X_t = \bar{x}Q_t \tag{2}$$

$$Y_t = \left(1-\bar{x}\right)Q_t \tag{3}$$

$$K_{t+1} = sY_t + \left(1-\delta\right)K_t \tag{4}$$

where Q_t is gross output, X_t is intermediate good input, Y_t is GDP, $\bar{A}$ is a fixed productivity level, and K_t is capital. The parameters $\alpha, \sigma, \bar{x}$, and s are all assumed to be between 0 and 1.

Question 2.1

The assumptions of the model ensures the existence of a steady state. Show that GDP per worker in steady state can be written as:

$$y = \left\{\bar{A}\left(1-\bar{x}\right)^{1-\sigma}\bar{x}^{\sigma}\right\}^{\frac{1}{(1-\alpha)(1-\sigma)}}\left(\frac{K}{Y}\right)^{\frac{\alpha}{(1-\alpha)}} \tag{5}$$

Question 2.2

Why and how does the steady state differ from the steady state in a standard Solow model? Explain the intuition (no math is required in your answer to this question).

Question 2.3

Suppose now that $\bar{x}$, s, δ, α, and σ are identical across countries, but $\bar{A}$ differs due to difference in technology or human capital. Derive an expression for the relative GDP per capita of two countries, denoted Country 1 and Country 2, as a function of their relative productivity levels, $A_1$and A_2, and the parameters of the model.

Question 2.4

(a) What is the optimal value of $\bar{x}$ in the model outlined above? Explain how your result can be used to calibrate the value of σ, and what an empirically reasonable value of σ might be. (b) What is a reasonable guess for the size of α, and why is it a reasonable guess?

Question 2.5

Use the proposed values for α and σ from your previous answer to quantitatively compare Country 1 and Country 2 assuming that $\frac{A_1}{A_2} = \frac{1}{2}$. Compare to a situation with $\sigma = 0$, and explain why intermediate goods and weak links are useful for explaining comparative development.

3 Premature death in an OLG model

Consider an overlapping generations economy where economic activity extends into the infinite future. The economy is closed and all markets are competitive. All individuals survive at most for two periods. During the first period people work and decide on how much to save for retirement. During this period they also have an off-spring (costlessly), which will ensure the population always is populated. There is no population growth, so each young person "spawns" just one off-spring. The size of each generation is normalized to one for notational simplicity.

Getting to experience retirement is not guaranteed however. Only with probability π will the individual manage to experience retirement after which they die for sure, as in a standard Diamond model. With probability $(1-\pi)$ they die prematurely. This will leave unclaimed savings, since this fraction of people do not get to consume the savings they build up during their working years. We assume that this income flow is simply passed on to their off-spring as "accidental bequest".

Specifically, individuals maximize expected utility

$$\ln c_{1t} + \pi \ln c_{2t+1}, \tag{6}$$

The first period budget constraint is $c_{1t} + s_t = w_t + b_t \equiv I_t$ and $c_{2t+1} = (1 + r_{t+1})\, s_t$. The notation is c_i for consumption ($i = 1$ for consumption during youth and $i = 2$ for consumption during old age), s is savings of the young, and b_t is accidental bequest. For future reference note that

$$b_t = (1-\pi)(1+r_t)\, s_{t-1}. \tag{7}$$

That is, expected (and since there is no aggregate uncertainty: average) bequest is the probability of premature death multiplied by the capitalized value of the savings of their "parents".

Question 3.1

Show that savings of the young is given by:

$$s_t = \frac{\pi}{1+\pi}\left(w_t + b_t\right). \tag{8}$$

Question 3.2

The amount of capital available for the economy in the next generation is determined by the savings of the currently young generation, just as in a standard Diamond model $K_{t+1} = s_t$. Show that the law of motion for capital can be written as:

$$K_{t+1} = s^w w_t + s^r (1 + r_t) K_t, \tag{9}$$

where $s^w = \frac{\pi}{1+\pi}$ and $s^r = \frac{\pi(1-\pi)}{1+\pi}$. (Hint: To figure out how to elimate bequest you need to use equation (7) and that current period savings of the young determines the next period capital stock).

Question 3.3

We assume firms operate a standard Cobb-Douglas production technology:

$$Y_t = K_t^{\alpha} (A_t)^{1-\alpha}, \tag{10}$$

where $A_{t+1} = (1 + g) A_t$ is exogenous technological change and the size of the labor force, recall, is normalized to one. Firm maximize profits:

$$Y_t - w_t - (1 + r_t) K_t, \tag{11}$$

where we assume capital depreciates fully during a period. Solve the profit maximization problem of the firm and proceed to show that the law of motion of capital in efficiency units can be written as:

$$k_{t+1} = s_t = \frac{\theta}{1 + g} k_t^{\alpha}, \tag{12}$$

where $\theta = \frac{\pi}{1+\pi} (1 - \alpha) + \frac{\pi}{1+\pi} (1 - \pi) \alpha$ and $k_t = K_t / A_t$.

Question 3.4

Illustrate the transition diagram for the model. Does a steady state exist? Is it unique? Stable?

Question 3.5

What is the real rate of return in the steady state? In recent decades the real rate of return has exhibited a secular decline. Discuss what the model suggests could be driving forces behind this decline? Compare with what the (empirically based) conventional wisdom is.