Written exam at the Department of Economics
Summer 2021
Economic growth
Final exam (re-exam), August 12

3-hour closed book exam. Answers only in English.

This exam question consists of 6 pages in total, including this front page.

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1 Short essay questions

Question 1.1

Based on the readings of this course, briefly discuss whether further movement toward free trade is desirable.

Question 1.2

What is an O-ring production function? Explain why such a production function may entail a correlation between the human capital of a firms' workers and the complexity of its products.

Question 1.3

Suppose new technologies gradually make more and more jobs redundant because of automation. Explain why workers might not be adversely affected by this process in the long run. Be specific about the mechanisms at work.

Question 1.4

Postulate: "If the aggregate production function exhibits constant returns to scale in the reproducible factor of production (asymptotically, at least) then positive growth in income per capita is ensured". Do you agree or disagree? Explain why/why not.

Question 1.5

There is currently an active debate on the future of economic growth. One argument, originally associated with the economist Robert Gordon, is that productivity growth is likely to decline because we already have harvested the "low hanging fruits", technologically speaking. Explain how one can turn this somewhat vague notion into a testable implication, and whether the data supports Robert Gordons' hypothesis.

Question 1.6

Suppose a group of countries establish a "common market", which ensures a fully integrated labor market and more competition over-all. Imagine these are the only effects of the common market. Will the common market initiative necessarily increase growth in the member state countries according to an Aghion-Howitt model? Explain why/why not.

2 Skill biased technical change in the task-based model

Consider the task-based model in Autor and Acemoglu (2011). Aggregate production is a Cobb-Douglas aggregate of a continuum of distinct tasks:

$$Y = \exp\left[\int_0^1 \ln y(i)\, di\right]$$

Each task is produced according to

$$y(i) = A_L \alpha_L(i)\, l(i) + A_M \alpha_M(i)\, m(i) + A_H \alpha_H(i)\, h(i)$$

where $l(i)$, $m(i)$ and $h(i)$ denote low-skilled labor, medium-skilled labor, and high skilled labor, respectively, employed in producing task i. A_L, A_M, A_H are general skill-specific productivity levels, and α_L, α_M and α_H are task *and* skill-specific productivity levels. Let $p(i)$ denote the price of task i. Assume that $\alpha_L(i)/\alpha_M(i)$ and $\alpha_M(i)/\alpha_H(i)$ are continuously differentiable and strictly decreasing such that low-skilled workers produce tasks $i < I_L$, high-skilled workers produce tasks $i > I_H$, and medium-skilled workers produce tasks $I_L \leq i \leq I_H$. Lastly, labor markets clear such that:

$$\int_0^1 l(i)\, di = L, \quad \int_0^1 m(i)\, di = M, \quad \textit{and} \quad \int_0^1 h(i)\, di = H,$$

Question 2.1

Show that the equilibrium labor allocation is as follows:

$$\begin{aligned} l(i) &= \frac{L}{I_L} \text{ for any } i < I_L \\ m(i) &= \frac{M}{I_H - I_L} \text{ for any } L_L < i < I_H \\ h(i) &= \frac{H}{1 - I_H} \text{ for any } i > I_H \end{aligned}$$

Hint: You will need to derive the wage rates for each type of worker, and use the property of the symmetric Cobb-Douglas production function that expenditures on each task are identical.

Question 2.2

Derive the two no-arbitage conditions for labor at I_L and I_H:

$$\frac{A_M \alpha_M (I_H) M}{I_H - I_L} = \frac{A_H \alpha_H (I_H) H}{1 - I_H} \quad (1)$$

$$\frac{A_L \alpha_L (I_L) L}{I_L} = \frac{A_M \alpha_M (I_L) M}{I_H - I_L} \quad (2)$$

Question 2.3

Suppose that medium-skilled workers are replaced by machines in some of the tasks they produce. Explain intuitively what happens to the two cut-off values I_H and I_L. Use your answer and the no-arbitage conditions to illustrate what happens in a (I_H, I_L)-diagram.

Question 2.4:

Based on your answer to the previous question, explain (in words) what happens to the relative wage rates $\frac{w_H}{wM}$ and $\frac{w_M}{w_L}$. Is the model's predictions for $I_L, I_H, \frac{w_H}{wM}$, and $\frac{w_M}{w_L}$ consistent with what we observe in the data? Discuss.

3 Accidental bequests and inequality

Consider an overlapping generations economy where economic activity extends into the infinite future. The economy is closed and all markets are competitive. All individuals survive at most for two periods. During the first period people work and decide on how much to save for retirement. During this period they also have an off-spring (costlessly), which will ensure the population always is populated. There is no population growth, so each young person "spawns" one off-spring. The size of each generation is normalized to one to keep notation simple.

Surviving to retirement is not guaranteed however. Only with probability π will the individual manage to experience retirement after which they die for sure, as in a standard Diamond model. With probability $(1 - \pi)$ they die prematurely. This will leave unclaimed savings, since this fraction of people do not get to consume the savings they build up during their working years. We assume that this income flow is simply passed on to their off-spring as "accidental bequest". Notice that this creates inequality since some households receive bequest while others do not. We will study the dynamics of inequality below.

Individual i maximizes expected utility

$$\ln c_{1t}^i + \pi \ln c_{2t+1}^i,$$

The first period budget constraint is $c_{1t}^i + s_t^i = w_t^i + b_t^i \equiv I_t$ and $c_{2t+1}^i = (1 + r_{t+1}) s_t^i$. The notation is c for consumption (subscript 1 for consumption during youth and 2 for consumption during old age), s is savings of the young, b is an accidental bequest, and w is lifetime wage income. It can be shown that expected or average (since the is no aggregate uncertainty) savings of the young is:

$$E\left(s_t^i\right) = s_t = \frac{\pi}{1+\pi} E\left(w_t^i + b_t^i\right) = \frac{\pi}{1+\pi}\left(w_t + b_t\right). \tag{3}$$

Expected - or average - bequest (since the is no aggregate uncertainty) is given by:

$$E(b_t) = b_t = (1-\pi)(1+r_t)\, E\left(s_{t-1}\right) = (1-\pi)(1+r_t)\, s_{t-1}, \tag{4}$$

where s_{t-1} is average savings of the "parent" generation. So average bequest is the probability of premature death multiplied by the capitalized value of the savings of their "parents".

Denote by $\sigma_{b,t}^2$ as the variance of bequests at time t Assume that the expected (average) wage is $E(w_t^i) = w_t$ and that the variance of wage income at time t is σ_w^2 and constant over time. Moreover, we assume that there is *no* correlation between individual level bequests and individual level wage income.

Question 3.1

Assume the real rate of return is constant over time, $r_t = r$, and that wages grow at the rate of technological change, $A_t = (1+g)\, A_{t-1}$. Show that average bequests in efficiency units are

$$\tilde{b}_t = (1-\pi)\frac{(1+r_t)}{1+g}\frac{\pi}{1+\pi}\left(\tilde{w}_{t-1} + \tilde{b}_{t-1}\right)$$

where the generic variable $\tilde{x_t} = x_t/A_t$.

Question 3.2

Using the above equation and the information given in the assignment, show that bequest inequality (in efficiency units) measured by the variance of $\tilde{b}_t$ is:

$$\sigma_{\tilde{b},t}^2 = \left[\frac{(1-\pi)\,\pi}{1+\pi}\frac{1+r}{1+g}\right]^2 \sigma_{\tilde{w}}^2 + \left[\frac{(1-\pi)\,\pi}{1+\pi}\frac{1+r}{1+g}\right]^2 \sigma_{\tilde{b},t-1}^2 \tag{5}$$

(Hint: suppose X, Z, Y are stochastic variables, where $X = aY + bZ$ and a, b are constants. Then the following holds: $VAR(X) = a^2 VAR(Y) + b^2 VAR(Z) + 2COV(aY, bZ)$, where the last term is the covariance between the two right hand side variables.)

Question 3.3

Assume $r > g$. Draw the transition diagram for the model, using the dynamic equation you just derived (Equation 5). Under what parameter restriction does a unique stable steady state exist? Show that the steady state variance of bequest is

$$\sigma_{\tilde{b}}^2 = \frac{\left[\frac{(1-\pi)\pi}{1+\pi}\frac{1+r}{1+g}\right]^2}{1-\left[\frac{(1-\pi)\pi}{1+\pi}\frac{1+r}{1+g}\right]^2}\sigma_{\tilde{w}}^2$$

Question 3.4

In the last question we assumed $r > g$. Is this assumption an empirically reasonable, as a general regularity? Explain why you say yes/no.

Question 3.5

Thomas Piketty has argued that $r > g$ leads to greater inequality. What is the impact from an increase in $r - g$ on bequest inequality, according to the present model? Provide an economic interpretation of the result.