Written exam for the M. Sc. in Economics. Summer 2015

**Economic Growth**

Master's Course

June 4, 2015

(3-hours closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam question consists of 5 pages in total including this page.

The weighting of the problems is: Problem 1: 35%, Problem 2: 55%, and Problem 3: 10%.[1]

[1] The percentage weights should only be regarded as indicative. The final grade will ultimately be based on an assessment of the quality of the answers to the exam questions in their totality.

**Problem 1** Consider an economy with aggregate production function

$$Y_t = \tilde{F}(K_t, L_t, t),$$

where $\tilde{F}$ is a neoclassical production function w.r.t. $K$ and $L$, $Y$ is GNP, $K$ is capital input, and $L$ is labor input. Let time be continuous. For any variable $z$ which is a differentiable function of time, $t$, we apply the notation $g_z \equiv \dot{z}/z$, where $\dot{z} \equiv dz/dt$.

a) By the standard growth accounting method, find a formula for the TFP growth rate, $x$. Interpret the concept TFP growth rate.

Assume from now on that $\tilde{F}$ has CRS w.r.t. $K$ and $L$. Let $y_t \equiv Y_t/L_t$ and $k_t \equiv K_t/L_t$.

b) On this basis express $g_y$ in terms of $x$, $g_k$, and the output elasticity w.r.t. $K$.

From now on we also assume that $\tilde{F}$ can be written

$$\tilde{F}(K_t, L_t, t) = F(K_t, A_t L_t), \tag{*}$$

where the technology level $A_t$ grows at a given constant rate $g > 0$ and employment grows at a given constant rate $n > 0$. Moreover, the increase in capital per time unit is given by

$$\dot{K}_t = S_t - \delta K_t \equiv Y_t - C_t - \delta K_t, \qquad \delta \geq 0, \tag{**}$$

where $C$ is aggregate consumption and not all of $Y$ is consumed.

c) Determine $g_Y$ and $g_K$ along a balanced growth path (BGP). *Hint:* in view of the given information we know something about the relationship between $g_Y$ and $g_K$ along the BGP.

d) Determine $g_y$ along the BGP. Is there a sense in which technical progress, along the BGP, explains more than what the growth accounting under a) and b) suggests? Explain.

e) Let markets be competitive and suppose firms maximize profits. Let the labor income share be denoted $SL$ ("share of labor"). Find an expression for $SL$ along the BGP such that the expression contains only the effective capital-labor ratio. Will $SL$ necessarily be constant along the BGP? Why or why not?

The French economist Thomas Piketty is sceptical towards Kaldor's stylized facts and predicts that in the next many decades ahead the effective capital-labor ratio will be rising. How this affects $SL$ depends, within neoclassical thinking, on the elasticity of substitution between $K$ and $L$, which in turn is determined by the production function in intensive form and the effective capital-labor ratio $\tilde{k} \equiv K/(AL)$. Let the elasticity of substitution between $K$ and $L$ be denoted $\sigma(\tilde{k})$, where $\sigma(\tilde{k})$ *may* be a constant.

f) In the context of a cost-minimizing firm, $\sigma(\tilde{k})$ coincides with $\mathrm{E}\ell_{\tilde{w}/\hat{r}}\tilde{k}$, that is, the elasticity of $\tilde{k}$ w.r.t. the relative factor price $\tilde{w}/\hat{r}$, where $\tilde{w} \equiv w/A$ is the real price per unit of effective labor, $w$ being the real wage, and $\hat{r}$ is the rental rate of capital. Piketty interprets the empirical evidence such that $\sigma(\tilde{k})$ tends to be above 1. In this case, what is the implied forecast regarding the direction of movement of the labor income share? Why? *Hint:* $SL = \frac{wL}{\hat{r}K+wL} \equiv \frac{\tilde{w}/\hat{r}}{\tilde{k}}/(1+\frac{\tilde{w}/\hat{r}}{\tilde{k}})$.

g) An alternative interpretation of the bulk of empirical evidence is that $\sigma(\tilde{k})$ tends to be below 1. In this case, what is the implied forecast regarding the direction of movement of the labor income share? Why?

**Problem 2** We consider a Barro-style model of a closed economy. There is a constant population of size $L$, and $L$ also indicates the number of households, each of which supplies inelastically one unit of labor per time unit. There are $M$ profit maximizing firms, operating under perfect competition ($L$ and $M$ are constant and "large"). There is also a government that, free of charge, supplies a service $G_t$ per time unit. Time, $t$, is continuous. Aggregate output is $Y_t$ per time unit and output is used for private consumption, $C_t \equiv c_t L$ , the public service, $G_t$, and investment, $I_t$, in (physical) capital, i.e., $Y_t = C_t + G_t + I_t$. The aggregate stock of capital, $K_t$, changes according to $\dot{K}_t = I_t - \delta K_t$, where $\delta \geq 0$ is the capital depreciation rate. The initial value $K_0 > 0$ is given. The capital stock in the economy is owned, directly or indirectly (through bonds and shares), by the households. Markets are competitive. The equilibrium real wage is denoted $w_t$. There is a perfect market for loans with a real interest rate, $r_t$, and there is no uncertainty.

There is a given tax rate, $\tau$, on private financial wealth. Aggregate private financial wealth is denoted $V_t$ and equals the aggregate capital stock since there is no government debt and natural resources are ignored. The service $G_t$ is the only public expenditure and the government budget is balanced at every $t$:

$$G_t = \tau V_t = \tau K_t, \qquad \tau > 0. \tag{GBC}$$

The production function for firm $i$ is

$$Y_{it} = AK_{it}^{\alpha}(G_t L_{it})^{1-\alpha}, \quad A > 0, 0 < \alpha < 1, \qquad i = 1, 2, ..., M. \tag{*}$$

a) Comment on the nature of $G_t$.

b) Show that in equilibrium the interest rate is

$$r_t = \alpha A(L\tau)^{1-\alpha} - \delta \equiv r,$$

a constant, and that aggregate output is

$$Y_t = \sum_i Y_{it} = \cdots = AK^{\alpha}(GL)^{1-\alpha} = \cdots = A(\tau L)^{1-\alpha}K_t \equiv \bar{A}(\tau)K_t,$$

where you should fill in the lacking steps indicated by dots.

Suppose the households, all alike, have infinite horizon, a constant rate of time preference $\rho > 0$, and an instantaneous utility function with (absolute) elasticity of marginal utility of consumption equal to a constant $\theta > 0$. For any variable $z$ which is a differentiable function of time, we apply the notation $g_z \equiv \dot{z}/z$, where $\dot{z} \equiv dz/dt$.

c) What is the before-tax and the after-tax rate of return, respectively, on a household's financial wealth?

d) Set up the optimization problem of an individual household with initial financial wealth $a_0$ $(= V_0/L)$. Derive the first-order conditions and the transversality condition. Finally, derive the Keynes-Ramsey rule for this problem. Comment.

e) Let $k \equiv K/L$ and $y \equiv Y/L$. Find $g_k$ and $g_y$ in general equilibrium of this economy (an informal argument, based on your general knowledge about reduced-form AK models, is enough). In case you need restrictions on some parameters to ensure existence of equilibrium with growth, state them.

f) Sign $\partial g_y/\partial L$ and $\partial g_y/\partial \tau$, respectively (it can not be ruled out that at least one of the signs depends on a certain parameter condition). Provide the intuition behind your results.

g) There are at least two mutually related distinctive features of this Barro-style model that may be disputed (in a similar way as the learning-by-investing model by Paul Romer contain two mutually related problematic features). What are these?

We now change the model. Let $L$ grow at the constant rate $n \geq 0$, replace the technology assumption (*) by the assumption

$$Y_{it} = AK_{it}^{\alpha}(G_t^{\lambda}L_{it})^{1-\alpha}, \qquad A > 0, 0 < \alpha < 1, 0 < \lambda \leq 1, \qquad i = 1, 2, ..., M. \qquad (**)$$

h) Show that in equilibrium the aggregate production can be written $Y = AK^{\alpha}(G^{\lambda}L)^{1-\alpha}$.

i) Let $0 < \lambda < 1$. With (GBC) still in force, find $g_Y$, $g_y$, and $\partial g_y/\partial \tau$, respectively, along a balanced growth path with positive saving. *Hint:* (i) if a production function $Y = F(K, XL)$ is homogeneous of degree one, then

$$\frac{Y}{K} = F(1, \frac{XL}{K});$$

(ii) combine this with the balanced growth equivalence theorem; (iii) use (GBC).

j) Compare with the result concerning $\partial g_y/\partial \tau$ at f) above. Comment.

k) An "all-knowing and all-powerful" social planner with the same criterion function as the representative household will, for any given $K_t$ and $L_t$, choose $G_t$ such that $G_t$ is proportional to $Y_t$ as given at h) with factor of proportionality equal to $\lambda(1-\alpha)$. Show this and provide the intuition. *Hint:* consider $\partial(Y - G)/\partial G$.

**Problem 3** *Short questions*

a) It is sometimes argued that results like $g_y = \lambda n/(1-\lambda)$ (from Arrow's version of the learning-by-investing model, standard notation) are from an empirical point of view falsified by the fact that cross-country growth regressions do not tend to indicate a positive correlation between per capita economic growth and population growth. Briefly evaluate this argument.

b) In our syllabus there are two expanding-input-variety models with knowledge spillovers. The aggregate invention "production functions" in these two models are quite similar, but there is nevertheless a difference that has far-reaching implications. Give a brief account.

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