Written Exam for the M.Sc. in Economics Autumn 2014 (Fall Term)

Financial Econometrics A: Volatility Modelling

Final Exam: Masters course

Exam date: **FEBRUARY 20 2015**

3-hour closed book exam.

Please note there are a total of 9 questions which should **all be replied to**.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Question 1:

Q1.1:

Solution: Fat tails – and $E|z_t|^3$ not finite as $v = 3$ (so $3 - \delta$ moments are finite).

Q1.2

Solution: $E(\delta(x_t)|x_{t-1} = x) = 1 + \alpha x^2$ - hence $\alpha < 1$.

Q1.3

Solution: Standard application of the CLT for weakly mixing processes as y_t has mean zero, and hence $E(\cdot) = 0$ (mgd). Also the fact that x_t is weakly mixing with $E\left(x_{t-1}^2/1 + 0.5x_{t-1}^2\right) \leq 1$ should be mentioned. Indentical arguments as for ARCH(1) Gaussian case (given the fact $Ey_t = 0$ and $Ey_t^4 < \infty$ which should not be derived).

0.1 Q1.4

Solution: Information (second order derivative at θ_0) converge in probability; Third order derivative (uniformly) bounded. Different levels of details and explanation allowed.

Q1.5

Solution: As in the standard conditional Gaussian case - except the quantile has to be backed out from scaled t_3 distribution. So non-standard. Would be good if formula(e) are provided.

Question 2:

Q2.1

Solution: Use $p_{21} = 1 - p_{22}$ such that $\hat{p}_{22} = 0.93$ - hence both regimes close to boundary - misspecified model(?) reflecting common stylized features similar to IGARCH. High/low regime for log-returns Δy_t.

Q2.2

Solution: Gaussian density - and smoothed prob. Different density (t) and hence different smoothed probabilities (may refer to recursions which were in the notes for general density but not needed to state these at all).

Q2.3

Solution: Fig 2.2 shows the smoothed probabilities $p_t^*(1)$ estimated- seems likely to spend much time in one (the high vol) regime.

Q2.4:

Solution: $\hat{p}_{21}$ and

$$\begin{aligned}
P\left(s_{T+2}=1|s_T=2\right) &= P\left(s_{T+2}=1|s_{T+1}=1\right)P\left(s_{T+1}=1|s_T=2\right) \\
&\quad + P\left(s_{T+2}=1|s_{T+1}=2\right)P\left(s_{T+1}=2|s_T=2\right) \\
&= p_{11}p_{21}+p_{21}p_{22} \\
\hat{p}_{11}\hat{p}_{21}+\hat{p}_{21}\hat{p}_{22} &= 0.98\cdot 0.07+0.07\cdot 0.93=0.13
\end{aligned}$$