Written Exam for the M.Sc. in Economics 2014 (Fall Term)

Financial Econometrics A: Volatility Modelling

Final Exam: Masters course

January 19 2015 | 3-hour closed book exam.

Notes on Exam: Please note that there are a total of 9 questions which should all be answered.

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

Question 1:

Q1.1:

Solution: In ARCH_{abs} σ_t^2 is proportional to $|r_{t-1}|$ while in the classic ARCH(1) it is proportional to r_{t-1}^2. In both the response is symmetric to large/small values of returns, but with different powers. That is, they are both of the form $|r_{t-1}|^\delta$ and one would probably prefer to also estimate δ in an application as seen in the exercises, and which would allow for possible asymmetry in the response. Relevance (as also reflected by the proposed δ-extension) is determined by empirical fit of the model.

Q1.2:

Solution: $E(\delta(r_t)|r_{t-1} = r) = 1 + a + b|r| = \frac{(1+a+b|r|)}{1+r^2}\delta(r)$. As $r^2 \to \infty$ faster than $|r|$, clearly $\frac{(1+a+b|r|)}{1+r^2} < \beta < 1$ for $|r| > M$ for some M. Hence in particular for $(1, 10)$ this is the case.

Q1.3:

Solution: Use BFGS or some other optimizer in ox to produce $\hat{a}$ by optimizing the log-likeihood function. In case of t_v innovations this would be QMLE rather than MLE. A discussion of this would be good.

Q1.4:

Solution: At the true value (a_0, b_0) (here one may use $(1, 10)$ or just leave it as general), $(r_t/\sigma_t)^2 - 1 = z_t^2 - 1$ which is iid mean 0. Hence by the weakly mixing property, the score is asymptotically Gaussian if

$$\frac{1}{T}\sum_{t=1}^{T}\frac{1}{\sigma_t^4} \to^p E\left(\sigma_t^{-4}\right) = \phi < \infty.$$

This holds as, $\frac{1}{\sigma_t^4} \leq \frac{1}{a_0}$.

Q1.5:

Solution: Standard application to compute sde and for testing (χ^2 inference). The "sandwich" form can be useful if z_t are not iid Gaussian but for example iid t_v distributed and hence the model misspecified.

Question 2:

Q2.1:

Solution: Clearly misspecified - showing up as IGARCH (usual story expected here) and in residuals being non-Gaussian. Also the fact that there is no ARCH effects left is a stylized finding when applying GARCH(1,1) to remove ARCH(∞) effects.

Q2.2:

Solution: Straightforward.

Q2.3:

Solution: This is the EM-Gaussian-likelihood - explain details for EM algorithm here.

Q2.4:

Solution: If $\mathbf{P}$ satisfies the standard conditions for stationarity all is fine (listing these would be very good) - conflict as $\hat{p}_{11}$ is close to one - would be interpreted as almost absorbing "low" volatility regime. Note missing std deviations - means in fact zero information.