Written Exam at the Department of Economics Summer 2021

Financial Frictions, Liquidity and the Business Cycle
Final Exam

From June 9 10 AM to June 11 10 AM, 2021

Answers only in English.

A take-home exam paper cannot exceed 10 pages – and one page is defined as 2400 keystrokes

This exam question consists of 5 pages in total

The paper must be uploaded as one PDF document. The PDF document must be named with exam number only (e.g. '127.pdf') and uploaded to Digital Exam.

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Exam cheating is for example if you:

- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Reuse parts of a written paper that you have previously submitted and for which you have received a pass grade without making use of quotation marks or source references (self-plagiarism)
- Receive help from others in contrary to the rules laid down in part 4.12 of the Faculty of Social Science's common part of the curriculum on cooperation/sparring

You can read more about the rules on exam cheating on your Study Site and in part 4.12 of the Faculty of Social Science's common part of the curriculum.
Exam cheating is always sanctioned by a written warning and expulsion from the exam in question. In most cases, the student will also be expelled from the University for one semester.

1 (20 points) Answer true, false, or uncertain. Justify your answer.

A shock that reduces entrepreneurs' wealth in the Bernanke and Gertler (1990) model improves the average quality of financed projects as the worst entrepreneurs are now unable to borrow.

2 (20 points) Answer true, false, or uncertain. Justify your answer.

Optimal liquidation in a model with moral hazard and reinvestment needs can be implemented via credit lines only when the economy has sufficient outside liquidity.

3 (20 points) Answer true, false, or uncertain. Justify your answer.

A more complete network always reduces the risk of systemic risk in the Allen and Gale (2000) model.

4 (120 points) Consider an economy that lasts for three periods, $t = 0, 1, 2$, and has one consumption good. To transfer wealth from $t = 0$ to later periods there are two technologies. One is a storage technology with unitary gross return, such that a unit of the good invested in t gives a unit of the good in $t + 1$. The other technology is a long term investment project that gives $R > 1$ units of the good in date 2 per unit of the good invested in date 0. If liquidated at date 1, the return is $L < 1$.

There are a continuum of banks and a continuum of consumers, both of mass one. Consumers are endowed with a unit of the good at date 0. Of these, a fraction π will be "impatient", and prefer to consume in $t = 1$, while the rest will be "patient" and prefer to consume in $t = 2$. Their type is private information and only revealed at date $t = 1$. Their preferences are given by

$$\begin{array}{ll} u(c_1) & \text{for impatient consumers,} \\ \rho u(c_2) & \text{for patient consumers.} \end{array}$$

Where c_i represents consumption in date i, and $u' > 0$, $u'' < 0$. Assume all consumers use banks to save their endowment for future consumption needs.

At time $t = 0$ banks do not know the share of patient and impatient consumers that constitute their clients. Banks invest share I of the resources deposited in them at $t = 0$ in the long run technology, and share $1 - I$ is invested in the short run technology.

At $t = 1$, consumers' type is realized and banks discover their liquidity needs. We assume only two possible cases: with probability P_H, they have a share π_H of impatient clients, and with probability $P_L = 1 - P_H$ the share of impatient depositors is $\pi_L < \pi^H$.

Note that $P_H \pi_H + P_L \pi_L = \pi$, the economy-wide probability of being an impatient consumer.

i. Characterize optimal investment and consumption plans (i.e. the social planner's allocation).

For the remainder of the problem assume parameters are such that $c_1^* \leq c_2^*$.

ii. Characterize investment and consumption plans when banks are in autarky. By this we mean that each bank has to withstand their liquidity shock by offering a deposit contract that is contingent on the realization of the share of impatient consumers they serve. *HINT:* Note that knowing they are in autarky, banks will not necessarily invest the same amount in the long-run technology as in optimal allocation.

iii. Now assume there is an interbank market that opens at $t = 1$ that allows banks to trade liquidity, and that banks' liquidity shock is observable. Show that this interbank market allows the decentralization of the first best. Estimate the amounts of liquidity traded at $t = 1$ and find the equilibrium interest rate of interbank loans. *HINT:* Note this is a different interbank market as the one used in Allen and Gale (2000).

iv. Now assume there is an interbank market but that banks' liquidity shock is unobservable (i.e. banks could "lie" about their type of liquidity shock and the market would not be aware of this). What condition must be satisfied by the interbank interest rate to ensure that they reveal their type truthfully? Explain.

For the remainder of the problem assume that $\pi_L = \pi_H = \pi$, such that banks are homogeneous.

v. Is it possible to have bank runs? If so, for what parameter values? Explain why runs are inefficient.

vi. Assume that a bank run equilibrium is possible. Assume households have an endowment e_1 at date 1 and the government can raise a tax from all households at date 1, the proceeds of which are used to make whole all depositors who have not been repaid by their bank. Derive the optimal banking contract under this policy and find the optimal tax. Characterize the minimum e_1 that would prevent bank runs.

5 (100 points) Consider the following version of Geanakoplos (2010). There is one consumption good, two dates, and a continuum of risk neutral traders that have resources at date 0, and consume at date 1. There are two assets in the economy, a risky one (or market) with unit supply and state contingent payoffs, and a riskless one. There are two states of the world, "up" and "down". The riskless rate is $R_f \geq 1$. The market return, $\tilde{r}$, is u in the up state and d in the down state, where $u > R_f > d$.

Traders' beliefs about the probability of state "up" are uniformly distributed over $[0, 1]$. Trader with type $\pi \in [0, 1]$ believes probability of u is π.

Each type starts with exogenous net worth W_0. Assume that every financial contract that is written is fully collateralized (e.g. if a trader borrows to purchase a unit of the risky asset the contractual repayment cannot be higher than d).

i. Assume that there is a borrowing constraint such that traders cannot borrow more than ϕ (if $\phi = 0$, then no borrowing is allowed). Characterize the identity of the marginal trader and equilibrium prices in terms of R_f, u, d, and ϕ. Show that both are increasing functions of ϕ. Interpret.

ii. Now assume that there are no borrowing constraints and that traders are also allowed to short sell the market (again using fully collateralized contracts). Characterize the identities of the marginal traders (on the long and the short) and equilibrium prices. What is the effect on the market price of allowing for short sales? Explain.

For the rest of the problem assume traders have log utility. They invest x in the market and $W_0 - x$ in the riskless asset. Assume there are no borrowing, nor short-selling, constraints.

iii. Show that the optimal choice of x is

$$x = W_0 R_f \frac{\pi(u - R_f) + (1 - \pi)(d - R_f)}{(u - R_f)(R_f - d)}.$$

iv. You don't know what the investor thinks π is. But suppose that in equilibrium the investor's wealth is fully invested in the market. Find the expected excess return on the market (i.e. the expected return over the risk free return) in terms of R_f, u, and d. *HINT:* Your answer should not involve π.

v. Write down an expression for the risk-neutral probability of an up-move (the risk neutral probability is the probality under which risky asset prices can be written as $p = E^*[\tilde{r}]/R_f$; here it is $\pi_u^* = (R_f - d)/(u - d)$), and show that the risk-neutral expectation of the return on the market is R_f. Find the risk-neutral variance of the return on the market in terms of u, d, and R_f, simplifying your answer as much as possible.