Written Exam at the Department of Economics winter 2019-20

Financial theory and models

Exam

January 23, 2020

(3-hour open book exam)

Answers only in English.

This exam question consists of 1 page in total

Falling ill during the exam

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- contact an invigilator who will show you how to register and submit a blank exam paper.
- leave the examination.
- contact your GP and submit a medical report to the Faculty of Social Sciences no later than five (5) days from the date of the exam.

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You cheat at an exam, if during the exam, you:

- Make use of exam aids that are not allowed
- Communicate with or otherwise receive help from other people
- Copy other people's texts without making use of quotation marks and source referencing, so that it may appear to be your own text
- Use the ideas or thoughts of others without making use of source referencing, so it may appear to be your own idea or your thoughts
- Or if you otherwise violate the rules that apply to the exam

Problem 1. Consider a stock in a binary one-period model with value 100 USD at time $t = 0$. We assume that the value increases by 7 percent in the up-state and decreases with 3 percent in the down state at $t = 1$. The risk free interest rate r is 5 percent.

(i) Calculate the risk neutral probability distribution.

(ii) Calculate the arbitrage free price c of a European call option written on this stock with strike price $K = 67$ USD.

Problem 2. Patients arrive independently and at random to a medical facility with a mean of 5 patients per hour.

(i) What is the distribution of the number of patients arriving during a period of two hours?

(ii) What is the probability that exactly k patients arrive during a period of two hours for each k?

(iii) Let Y_1 denote the stochastic variable that measures the time from the opening of the clinic until the first (or more) patients arrive. What is the distribution of Y_1.

Problem 3. Let B_t denote the Brownian motion. Show that the expectation

$$E\big[\exp(B_t)\big] = e^{t/2} \qquad t \geq 0.$$

Problem 4. Let B_t denote the Brownian motion and take $c > 0$. Show that the process

$$\hat{B}_t = \frac{1}{c} B_{c^2 t} \qquad t \geq 0$$

also is a Brownian motion.

Problem 5. We consider the Heath-Jarrow-Morton (HJM) framework for interest rate models. The forward rates $F(t, T)$ are modeled by specifying the dynamics

$$dF(t,T) = \alpha(t,T)\,dt + \sigma(t,T)\,dB_t\,.$$

Assume that the forward rate volatility $\sigma(t, T)$ is given by

$$\sigma(t,T) = \sigma e^{-(T-t)} \qquad 0 \leq t \leq T,$$

where $\sigma > 0$ is a constant.

(i) Calculate the forward rate drift $\alpha(t, T)$ for $0 \leq t \leq T$.

(ii) Show that $\alpha(t, T) \to 0$ for $t \to T$.