Answer Sheet to the Written Exam

Financial Markets

April 2013

In order to achieve the maximal grade 12 for the course, the student must excel in all three problems.

Problem 1:

This problem focuses on testing part 1 of the course's learning objectives, that the students show "The ability to readily explain and discuss key theoretical concepts and results from academic articles, as well as their interpretation." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

(a) Intuitively, each dealer has some market power, but the greater the number of dealers, the closer the market to a competitive situation (Cournot style). Dealers earn their non-competitive profit through excessively raising the price charged to buyers of the asset, and excessively depressing the price paid to sellers of the asset. In short, less competition makes the market less deep. Conversely, when a greater number of dealers bring the market closer to competition, the market is more liquid in the sense of being deeper.

(b) In the simplest structural model on page 166, with $p_t = p_t^* + s_t$, it is natural to label $s_t = u_t$ as the transitory component of the observed price. The permanent component is η_t, and the return is then given by equation (9.36). In the moving average model used for comparison, the return is given by equation (9.40). From (9.40), one particular shock ϵ_t raises r_t one-for-one, but reduces r_{t+1} by a factor a. The permanent component of a shock on prices is therefore $(1-a)\epsilon_t$, and the temporary component is $a\epsilon_t$ (which is reversed in the next period). For comparison, note the similar role played by the shock u_t in equation (9.36), temporarily raising r_t to the exact same extent to which u_t will reduce r_{t+1}.

Given these definitions, on page 167 the book points out that the estimated size of the temporary component will be different in the two models. Intuitively, the difference can arise because the permanent component η_t is stochastically independent of the transitory component u_t in the structural model, while the permanent component $(1-a)\epsilon_t$ is correlated with the temporary component $a\epsilon_t$ in the moving average model. The two models are not identical. Both definitions appear to make sense, but the example shows the point that the exact model used for defining a term, here the transitory component, has implications for the phrasing of conclusions drawn from the data.

(c) It is helpful to note that the claims made in the introduction of the article are discussed at greater length in Section 4.1 of the article. As explained earlier on the article's page 3, the informed investor trades partly on the market makers' forecast error, partly on the advantage of reacting faster to news. Intuitively, when the public information is more precise, the market makers' forecast error is smaller, and the investor therefore shifts weight on to news trading. Greater weight on news trading explains the investor's greater participation rate. As discussed on page 22, liquidity is higher since greater public information precision reduces the market makers' asymmetric information problem, as in the Kyle model. The issue is subtle, as also acknowledged on page 22, because the greater precision of the news flow, to which the informed investor can react with a speed advantage, actually gives the informed investor a greater short-run information advantage.

Problem 2:

This problem focuses on testing part 2 of the course's learning objectives, that the students show "The ability to carefully derive and analyze results within an advanced, mathematically specified theoretical model." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

(a) A market order of size x will give the informed trader final wealth $w = xv - P(x) + m$ where m denotes the endowment of cash. To maximize the mean-variance utility $\mathrm{E}[w|s] - \rho\mathrm{Var}[w|s]/2$, note first that $\mathrm{E}[w|s] = x\mathrm{E}[v|s] - P(x) + m$ and $\mathrm{Var}[w|s] = x^2\mathrm{Var}[v|s]$. The objective function is quadratic in x, and the first order condition for the maximum is

$$0 = \mathrm{E}[v|s] - P'(x) - x\rho\mathrm{Var}[v|s].$$

This is easily rewritten as the expression for x given in the question. With these mean-variance preferences, the amount of safe cash m does not affect the demand for the risky asset. Likewise, the demand for the risky asset is determined by the price for the marginal unit, $P'(x)$, without reference to the total amount spent on the asset, which is $P(x)$ in the limit order book, but would be $P'(x)x$ in a perfectly competitive setting. Thus, the demand is the same as in the competitive model.

(b) Inserting the given $\Pr(v = 1) = \Pr(v = 0) = 1/2$ with $f(s|v = 1) = 2s$ and $f(s|v = 0) = 2(1 - s)$ into the Bayesian formula, it is simple to reduce the expression:

$$\frac{(1/2)2s}{(1/2)2s + (1/2)2(1-s)} = \frac{s}{s+1-s} = s.$$

With the two possible asset values 0 and 1, we soon find that $\mathrm{E}[v|s] = \Pr(v = 1|s)\,1 + \Pr(v = 0|s)\,0 = s$ and that $\mathrm{Var}[v|s]$ is

$$\Pr(v = 1|s)\,(1-s)^2 + \Pr(v = 0|s)\,(0-s)^2 = s(1-s)^2 + (1-s)s^2 = s\,(1-s)\,(s+1-s) = s\,(1-s)\,.$$

(c) The equation in (a) characterizes $x(s)$. Observe that in this equation, the left hand side is strictly increasing, while the right hand side is decreasing (because $P'(x)$ is increasing). It follows that a certain q is below the equation's solution $x(s)$ if and only if the left hand side is below the right hand side when evaluated at $x = q$. Inserting from (b) that $\mathrm{E}[v|s] = s$ and $\mathrm{Var}[v|s] = s(1-s)$, the inequality (1) follows.

The right hand side of (1) is a quadratic function of s, which is zero where $s = 0$ and $s = 1$, and maximal where $s = 1/2$. The left hand side is linear, so that the whole inequality is quadratic. With equality, there can be at most two solutions to the quadratic equation (2). Evaluated at $s = 0$, the left hand side is negative, while at $s = 1$ it is positive. Hence, there exists exactly one solution $\hat{s}\,(q)$ to the equation in this interval. Finally, $s \geq \hat{s}(q)$ is the region where the left hand side of (1) exceeds the right hand side, which by our first argument happens if and only if $x\,(s) \geq q$.

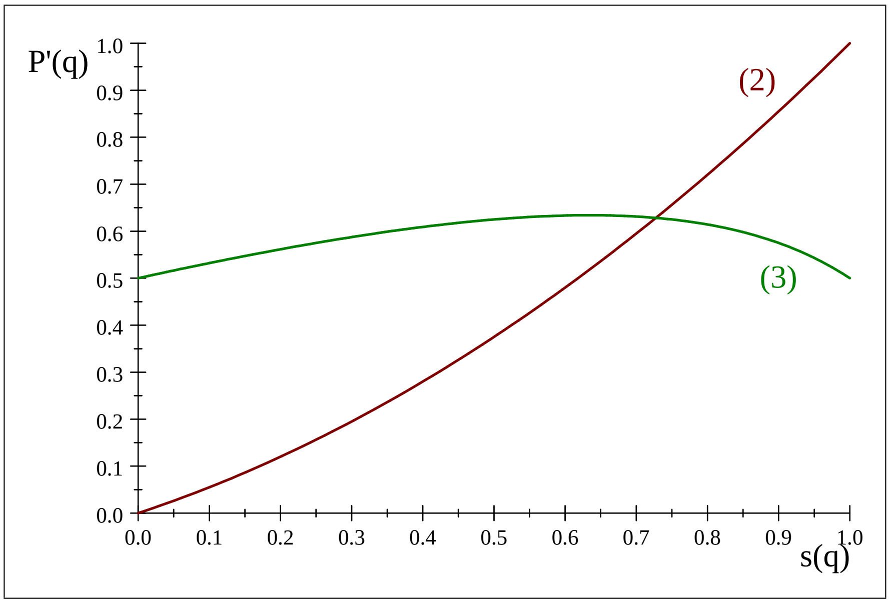
(d) When $q \in [0, I]$, the incoming market order Q walks through quantity q in two cases: either the next arriving trader is uninformed and buys I, or the arriving trader is informed and has received a signal $s \geq \hat{s}(q)$ (according to part (c)). The conditional probability that $s \geq \hat{s}(q)$ is given by $1 - F\,(\hat{s}(q)|v = 1) = 1 - (\hat{s}(q))^2$ when $v = 1$ and by $1 - F\,(\hat{s}(q)|v = 0) = 1 - 2\hat{s}(q) + (\hat{s}(q))^2 = (1 - \hat{s}(q))^2$ when $v = 0$. This explains the expressions for $\Pr(Q \geq q|v = 1)$ and $\Pr(Q \geq q|v = 0)$.

In the following calculation, it's useful to simplify notation by writing $\hat{s}$ in place of $\hat{s}(q)$. Recalling that $\Pr(v = 1) = \Pr(v = 1) = 1/2$, Bayes' rule then gives

$$\begin{aligned}
\Pr(v = 1|Q \geq q) &= \frac{\alpha(1-\hat{s}^2) + (1-\alpha)/2}{\alpha(1-\hat{s}^2) + (1-\alpha)/2 + \alpha(1-\hat{s})^2 + (1-\alpha)/2} \\
&= \frac{1}{2} + \frac{2\alpha(1-\hat{s}^2) + (1-\alpha) - \alpha(1-\hat{s}^2 + (1-\hat{s})^2) - (1-\alpha)}{2\alpha(1-\hat{s}^2 + (1-\hat{s})^2) + 2(1-\alpha)}.
\end{aligned}$$

Since $1 - \hat{s}^2 = (1-\hat{s})(1+\hat{s})$, this reduces to the expression in (3). In the numerator, $1-\hat{s}^2-(1-\hat{s})^2 = (1-\hat{s})(1+\hat{s}-1+\hat{s}) = 2(1-\hat{s})\hat{s}$, and in the denominator $1-\hat{s}^2+(1-\hat{s}))^2 = (1-\hat{s})(1+\hat{s}+1-\hat{s}) = 2(1-\hat{s})$.

(e) The right-hand side in equation (2) is 1 at $\hat{s}\,(q) = 1$, and when $\hat{s}\,(q) \leq 1/2$ it is less than $1/2$ since $\rho q \hat{s}\,(q)\,(1-\hat{s}\,(q)) \geq 0$. The right hand side of (3) is equal to $1/2$ when evaluated at $\hat{s}\,(q) = 0$ and $\hat{s}\,(q) = 1$, and since the large fraction is positive, it's above $1/2$ on the interval where $0 < \hat{s}\,(q) < 1$. Both functions are continuous, and since (2) is below (3) at $1/2$ and (3) is below (2) at 1, there exists a crossing of the two functions where $\hat{s}(q) \in (1/2, 1)$. Again, (3) is always above 1/2, while (2) must be always less than one, so $P'(q) \in (1/2, 1)$. You can draw some graphs by plugging in parameter values for α, ρ and q. It is not hard to see that the quadratic function in (2) is convex. At some work, differentiating twice, it can be checked that the function (3) is concave, reaching its maximum at the point $\bar{s} > 1/2$ where $2\alpha(1-\bar{s}) = (1-\alpha)(2\bar{s}-1)$. Here is a graph where $\alpha = \rho = 1/2$ and $q = 1$.



(f) I apologize for a mistake in this question. The question says "Show that $P'(q)$ is increasing when α and ρ are close to zero, but not so if α or ρ are too large," but it should say "Show that $P'(q)$ is increasing when $1-\alpha$ and ρ are close to zero, but not so if $1-\alpha$ or ρ are too large." The mistake was reported by a student, and I announced the correction on the course homepage and via an email to all students on Sunday afternoon between 15:00 and 16:00, towards the end of the exam. Grading the answers, it will be taken into account that attempts to answer this question may have taken extra time.

The intersection of (2) and (3) as drawn in the graph from (e), is always such curve (2) starts below curve (3) at 1/2 and crosses up to end above curve (3) at 1. The direct effect on our two equations from an exogenous increase in q is only to move curve (2) downwards, as only the term $\rho q \hat{s}(q)(1-\hat{s}(q))$ is affected. It is not hard to verify that the increase in q thus implies that $\hat{s}(q)$ rises. What is less certain is whether the corresponding $P'(q)$ at the crossing will go up or down. Since the crossing follows curve (3) which is never moved by the change in q, the value for $P'(q)$ is marginally rising (as desired) if and only if curve (3) is increasing at the crossing point. Recall that (3) is concave with a maximum at $\bar{s}$ where $2\alpha(1-\bar{s}) = (1-\alpha)(2\bar{s}-1)$. This point $\bar{s}$ rises from 1/2 to 1 as α rises from 0 to 1, so the curve is more likely to be increasing at the crossing if α is large, i.e., $1-\alpha$ is close to zero.

Also, it is easier for curve (2) to hit curve (3) on its increasing part if ρ is small, since curve (2) then lies higher. When $1-\alpha$ and ρ are sufficiently close to zero, the solution for $P'(q)$ will be increasing all the way as q rises from 0 to I. Conversely, when $1-\alpha$ or ρ are too large, the solution for $P'(q)$ is not increasing.

Intuitively, in (d) we determined $P'(q)$ as the expected value of the risky asset given that the arriving trader places a buy order of size at least q. From (a) and (c), the larger is q, the larger needs the realized signal s to be in order for an informed trader to purchase at least q. Hence, as q rises, it becomes less likely that informed investors will desire to pass that order size — however, those who do, have stronger reasons (higher signals) to trade that much. The first effect pulls in the direction that the incoming large trade is more likely to be uninformed, and hence market makers should not so much adjust their estimate of the asset value. The second effect, that the informed has stronger information, suggests that market makers should further revise their estimate of the asset value. Our analysis shows that the first effect is not very important when α is high because market makers can feel quite certain that the arriving trader is informed, and that the second effect is important when ρ is low because informed traders are willing to trade large quantities when they are not very risk averse.

Problem 3:

This problem focuses on testing part 3 of the course's learning objectives, that the students show "The ability to apply the most relevant theoretical apparatus to analyze a given, new case-based problem." The maximal grade is given for an excellent presentation that demonstrates a high level of command of all aspects of the relevant material and containing no or only few minor weaknesses.

Below are some suggested applications of the course literature to this case. It is important to note that these applications have shortcomings which should be discussed.

- Being an intermediary trading fairly large volumes, Knight Capital can be reasonably expected to be an important player when it comes to setting prices in markets. If Knight Capital were small, and could take prices as given in very liquid markets, even a set of false trading orders would not create any serious losses. Illiquidity is thus an important part of the story.

- The intermediary faces a serious risk of direct losses arising from conducting trades in illiquid markets, at prices which can be far from fair asset values. The risk here seems different in nature from the asset value risk which plays a role in the textbook's models, but we might expect to see the intermediary trying to earn an average return to compensate for the risk, as in Chapter 5.

- The story from the Economist doesn't mention the counterparties to the trades, who presumably gained as much as was directly lost by Knight Capital in connection with these trades. Part of the profits that can be earned by other intermediaries could apparently come from being counterpart to such unintended trades.
- The story focuses on the reputational damage to the business model of Knight Capital. Trading on behalf of clients, it's important to signal that clients can expect trades to be executed at competitive prices. Beyond the direct loss from the trades carried out, Knight Capital apparently risked additional losses.
- The SEC may be concerned that well-trusted intermediaries are an important element of well-functioning financial markets, where everyone who has an economic reason to trade can show up and expect to engage in trades on fair terms.