Written Exam for the M.Sc. in Economics summer 2013

Financial Markets

Final Exam

April 6, 2013 at 10.00 until April 8, 2013 at 10.00

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The paper must be uploaded as <u>one PDF document</u> (including the standard cover and the appendices). The PDF document must be named with exam number only (e.g. '1234.pdf') and uploaded to Absalon.

Focus on Exam Cheating

In case of presumed exam cheating, which is observed by either the examination registration of the respective study programmes, the invigilation or the course lecturer, the Head of Studies will make a preliminary inquiry into the matter, requesting a statement from the course lecturer and possibly the invigilation, too. Furthermore, the Head of Studies will interview the student. If the Head of Studies finds that there are reasonable grounds to suspect exam cheating, the issue will be reported to the Rector. In the course of the study and during examinations, the student is expected to conform to the rules and regulations governing academic integrity. Academic dishonesty includes falsification, plagiarism, failure to disclose information, and any other kind of misrepresentation of the student's own performance and results or assisting another student herewith. For example failure to indicate sources in written assignments is regarded as failure to disclose information. Attempts to cheat at examinations are dealt with in the same manner as exam cheating which has been carried through. In case of exam cheating, the following sanctions may be imposed by the Rector:

● 1. A warning

● 2. Expulsion from the examination

● 3. Suspension from the University for at limited period or permanent expulsion.

Please answer all 3 problems and all sub-questions below.

Problem 1:

(a) On pages 86–87, the textbook discusses the effects of imperfect competition among inventory-holding dealers. Taking for granted that $(M-1)/[(M-2)M]$ is a decreasing function of M for $M > 2$, argue that the market is more liquid, the greater the number of dealers. Intuitively explain this effect.

(b) On pages 166–167, the textbook discusses the difference among two definitions of the transitory component of returns. Intuitively explain why this difference arises, and discuss the possible interpretations of the transitory component in the two models.

(c) On pages 3–5 of Foucault, Hombert and Roşu (2012), they mention one particular feature of their model with the speed advantage. Higher precision of the public information will increase liquidity and generate a greater participation rate of the informed investor. Intuitively explain and discuss these two effects.

Problem 2:

This problem reconsiders the limit order book model of Section 8.1 in the textbook under a different set of technical assumptions about the information structure.

The final value of the risky asset is v, where $\Pr(v = 1) = \Pr(v = 0) = 1/2$.

Informed traders privately observe the signal $s \in [0, 1]$ which has conditional densities $f(s|v = 1) = 2s$ and $f(s|v = 0) = 2(1 - s)$. Note for later that the cumulative distribution functions are $F(s|v = 1) = s^2$ and $F(s|v = 0) = 2s - s^2$.

Informed traders are risk-averse with simple mean-variance preferences. The parameter $\rho > 0$ captures the degree of risk aversion, and the competitive demand function is to purchase x units of the risky asset where

$$x = \frac{\mathrm{E}[v|s] - p}{\rho \mathrm{Var}[v|s]},$$

when the price is given as p. The informed traders are endowed with zero units of the risky

asset.

Part of the asset demand comes from uninformed noise traders. There is chance α that the next arriving trader is informed, and chance $1 - \alpha$ that the trader is uninformed. An uninformed trader will place a market order for the risky asset, with probability $1/2$ to buy $I > 0$ units, with probability $1/2$ to sell I units. This order size $I > 0$ is a fixed parameter in our model. The arrivals process and the uninformed order direction are stochastically independent of the asset value v.

As in Section 8.1, competitive risk-neutral market makers post limit orders. The limit order book is represented by $p(q)$ which is the average price paid for a trade of size q. The total price is $P(q) = p(q) q$. By construction of a limit order book, $P'(q)$ is strictly increasing, as it represents the marginal price quoted for market orders which reach size q.

(a) Given a limit order book, argue that an informed trader with signal s will submit a market order of size $x(s)$ which solves

$$x = \frac{\mathrm{E}\left[v|s\right] - P'\left(x\right)}{\rho \mathrm{Var}\left[v|s\right]}.$$

(b) When an informed trader observes signal s, the posterior probability that $v = 1$ can be calculated by Bayes' rule as

$$\Pr\left(v = 1|s\right) = \frac{\Pr\left(v = 1\right) f\left(s|v = 1\right)}{\Pr\left(v = 1\right) f\left(s|v = 1\right) + \Pr\left(v = 0\right) f\left(s|v = 0\right)}.$$

Verify that this rule gives $\Pr\left(v = 1|s\right) = s$ in this model. Check also that $\mathrm{E}\left[v|s\right] = s$ and $\mathrm{Var}\left[v|s\right] = s\left(1 - s\right)$.

(c) For a given trade size $q > 0$, show that $x\left(s\right) \geq q$ if and only if

$$s - P'\left(q\right) \geq \rho q s\left(1 - s\right). \tag{1}$$

[Hint: combine (a) and (b).] Next, show that (1) is quadratic in s, false when $s = P'\left(q\right)$, and true when $s = 1 > P'\left(q\right)$. Conclude that there exists a unique threshold $\hat{s}\left(q\right)$ such that $x\left(s\right) \geq q$ if and only if $s \in \left[\hat{s}\left(q\right), 1\right]$. Show that the threshold solves

$$P'\left(q\right) = \hat{s}\left(q\right) - \rho q \hat{s}\left(q\right)\left(1 - \hat{s}\left(q\right)\right). \tag{2}$$

(d) On the ask side of the book, market makers adjust limit orders until $P'(q)$ is the expected asset value given that the incoming market order Q walks through quantity q. Thus, $P'(q) = \mathrm{E}(v = 1|Q \geq q) = \Pr(v = 1|Q \geq q)$. Verify for any $q \in [0, I]$ that

$$\Pr(Q \geq q|v = 1) = \alpha\left(1 - \hat{s}^2(q)\right) + (1 - \alpha)/2,$$

$$\Pr(Q \geq q|v = 0) = \alpha\left(1 - \hat{s}(q)\right)^2 + (1 - \alpha)/2,$$

and use Bayes' rule

$$\Pr(v = 1|Q \geq q) = \frac{\Pr(v = 1)\Pr(Q \geq q|v = 1)}{\Pr(v = 1)\Pr(Q \geq q|v = 1) + \Pr(v = 0)\Pr(Q \geq q|v = 0)}$$

to conclude that

$$P'(q) = \frac{1}{2} + \frac{\alpha\hat{s}(q)(1 - \hat{s}(q))}{2\alpha(1 - \hat{s}(q)) + (1 - \alpha)} \tag{3}$$

on quantities q where the ask side of the limit order book is active.

(e) For every active quantity $q \in [0, I]$, it is necessary that the limiting price $P'(q) \in (1/2, 1)$ and the trader threshold $\hat{s}(q) \in (0, 1)$ satisfy the pair of equations (2) and (3). It is further necessary that the resulting $P'(q)$ is a strictly increasing function. Note that both (2) and (3) have been written on the form that $P'(q)$ equals some function of $\hat{s}(q)$. Sketch graphs for the two functions, and argue that the equation pair has a solution where $P'(q) \in (1/2, 1)$ and $\hat{s}(q) \in (0, 1)$.

(f) Note that q affects the solution to the equation pair through its direct effect on (2). Show that $P'(q)$ is increasing when α and ρ are close to zero, but not so if α or ρ are too large. You are welcome to make this argument with reference to your graphs from (e). Interpret and discuss this last result, that there may not exist an increasing solution to the necessary equations.

Problem 3:

Below is an excerpt of an article from the Economist magazine on August 11, 2012. Please write a short essay discussing to which extent the course readings can relate to the issue of this text. In particular, consider the risk/return trade-off from acting as a trading

intermediary, and consider which damage to the market could lead the SEC to call this event unacceptable. If you wish to elaborate your answer beyond the syllabus, you are welcome to seek more information about Knight Capital or the related events on the internet.

"AT THE close of trading on August 6th Bloomberg's Broker Rankings reported that Knight Capital held the top position for the volume of shares traded on American exchanges for the day. Since it had also held that spot a few days earlier, that might seem unsurprising. Given that it had started the day facing bankruptcy, it was something of a miracle.

Founded in 1995, Knight has spent the past decade turning itself into a trusted trading intermediary for many of America's largest mutual-fund companies and retail brokers. All those years of work short-circuited in just a few minutes on August 1st, when a software glitch caused a flurry of unintended trades. It appears that the opening prices of more than 100 securities might have been directly affected, with a particularly large impact on half a dozen shares. What exactly went wrong is not yet known, although a new program is thought to be culpable. Nor is it clear which investors won and lost as a result of these erroneous trades, with the obvious exception of Knight itself, which was left with a hole in its accounts of $440m and promptly saw most of its customers stampede in the other direction.

Over the next four days, the firm's management faced horrendous challenges, including righting technical problems, retaining skittish employees, pacifying irate regulators and luring back customers while securing a financing package compelling enough to restore confidence. There would have been little surprise if Knight had gone bust. But its management was broadly respected; and its customers, and even some of its competitors, wanted it to survive. In some ways the clock was on its side, too. Both Knight's management and potential investors were aware that there was no time for prolonged haggling.

Even so, it was a close-run thing. After a weekend of fraught negotiations, a deal was announced just before the market reopened on August 6th with critical details missing that had to be filled in via a second announcement. In short, Knight received a capital injection of $400m in equity from a consortium of financial firms, including Jefferies Group, an investment bank, which took the lead; Blackstone, a private-equity firm;.GETCO, a Chicago-based competitor; and two brokers, Stifel Financial and TD Ameritrade.

The cost looks steep—in excess of 70% of the equity of the firm was surrendered to the new investors—until you consider how slim the chances of survival are for a securities firm that has lost the confidence of the market. Knight did not qualify for too-big-to-fail perks and the Securities and Exchange Commission (SEC) reportedly rejected the firm's pleas for the mistaken trades to be voided. If it could not arrange its own bail-out, it was dead.

Big firms such as E*Trade, Scottrade and Vanguard have said they are using Knight once again. The recovery has come far more quickly than many expected, although has a long way to go. Employees with long-term equity incentives have seen their stakes wiped out,

and may be hard to retain. Customers that have returned realise that they face their own reputational risks if the firm fails again.

Despite Knight's escape, it is clear that just a few moments of electronic idiocy can cost a firm everything. Knight's travails are the latest in a string of technology snafus, such as glitches on Facebook's first day of trading on the NASDAQ stock exchange and a botched debut for BATS Global Market on its own electronic exchange. The SEC has issued the expected statement of outrage about Knight's trading problems. "Unacceptable," says Mary Schapiro, its chairman. No one really disagrees, least of all Knight itself."