Written Exam for the B.Sc. in Economics summer 2012

Industrial Organization

Final Exam

August 16, 2012

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students’ self-service system.

Attempt both questions. Make sure that you explain all the steps of your analysis and that you define any new notation that you use.

Question 1

Consider a market in which consumers have unit demand,[1] with reservation prices r. The r's are distributed on the unit interval $[0,1]$. Let $f(r)$ denote the mass of consumers with reservation price r. We assume that

$$f(r) = \begin{cases} mx(1-r)^{x-1} & \text{if } r \in [0,1] \\ 0 & \text{if } r \notin [0,1], \end{cases}$$

where $m > 0$ and $x > 0$ are exogenous parameters. This means that the total mass of customers equals m:

$$\int_0^1 f(r)\,dr = \int_0^1 mx(1-r)^{x-1}\,dr = m.$$

A consumer buys if and only if the reservation price weakly exceeds the price:

$$r \geq p.$$

a) Show that the inverse demand function for this market is given by

$$p = 1 - bQ^y,$$

where Q is total industry output, $b \equiv m^{-\frac{1}{x}}$, and $y \equiv \frac{1}{x}$.

b) Assume Cournot competition among n firms, all with constant marginal cost $c \in [0,1)$. Solve for the equilibrium firm quantities, given the inverse demand function that is stated in the a) question. Assume that a typical firm's second-order condition is satisfied (so that the first-order condition indeed characterizes the firm's optimal quantity).

[You are encouraged to attempt part b) even if you have not been able to answer part a).]

c) Prove that the second-order condition in the b) question is indeed satisfied (so that we can infer that the first-order condition characterizes the firm's optimal quantity). If you believe that we need to make more specific assumptions about the value of the parameter y in order to prove this, then specify those assumptions. Otherwise, prove that the second-order condition is satisfied for all $y > 0$.

Question 2

Consider a market in which there are two firms, firm 1 and firm 2. The firms produce identical products and they face the following inverse demand function: $p = 45 - 9(q_1 + q_2)$, where p is market price, q_1 is firm 1's output, and q_2 is

[1] This means that the consumers demand either exactly one unit of the good or no unit at all.

firm 2's output. Both firms' marginal cost of producing the product is constant and equal to 9, and there are no fixed costs. Moreover, the firms compete in quantities. Firm i ($i = 1, 2$) is owned by individual O_i and managed by some other individual M_i. Each owner O_i can give an instruction to his or her own manager M_i whether to try to maximize the firm's profits or its revenues. The sequence of events of the game is as follows:

1. O_1 and O_2 simultaneously choose whether to instruct its manager to maximize profits (P) or revenues (R).

2. M_1 and M_2 observe their own instruction and the other manager's instruction. Then they simultaneously choose the own firm's output, trying to maximize either the profits or the revenues (depending on the instruction they received).

The objective of each owner is to maximize the own firm's profits.

a) Solve for all subgame-perfect Nash equilibria of the game described above.

b) Interpret your results: what is the economic logic that explains why the owners make the choices they make in the equilibrium (or the equilibria) that you derived? Are the managers' choice variables strategic substitutes or strategic complements, and what is the significance of this? What is the significance of the assumption that each manager can observe also the other manager's instruction before making the output decision?

[You are encouraged to attempt part b) even if you have not been able to answer part a).]

END OF EXAM