# Written Exam for the B.Sc. or M.Sc. in Economics summer 2013 <br> Industrial Organization 

Final Exam

June 11, 2013
(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

This exam paper consists of four pages in total including this page.

> Attempt both questions. Make sure that you explain all the steps of your analysis and that you define any new notation that you use.

There is no need to restate model assumptions and/or questions from the exam paper in the your answers (and it will be easier for the examiners to read the answers if you do not do this.)

## Question 1 (informative advertising)

Consider the following model of informative advertising (which is almost identical to the one we studied in the course). There are two firms that are exogenously located at each end of Hotelling's linear city (as illustrated below).

$$
\underset{\text { Firm } 1}{0}-------------------\underset{\text { Firm } 2}{1}
$$

The consumers are uniformly distributed on the line. They have so-called unit demand, meaning that they want to consume at most one unit of the good that the firms sell. A consumer with location $x$ gets the net utility $U_{1}(x)=$ $r-\tau x^{2}-p_{1}$ if buying from firm 1 and the net utility $U_{2}(x)=r-\tau(1-x)^{2}-p_{2}$ if buying from firm 2, where the notation is the same as in the course. If not buying from either firm, a consumer gets zero net utility. A consumer has the option to buy from firm $i$ if and only if she has been reached by an ad from that firm. The fraction of consumers who receive an ad from firm $i$ is denoted by $\lambda_{i}$. Therefore, the fraction $\lambda_{1}\left(1-\lambda_{2}\right)$ can buy from firm 1 only; the fraction $\lambda_{2}\left(1-\lambda_{1}\right)$ can buy from firm 2 only; the fraction $\lambda_{1} \lambda_{2}$ can buy from either firm (as in the standard Hotelling model). The remaining consumers have not seen any ad and cannot buy the good at all. The firms have a constant marginal cost of production, denoted by $c$; their cost of advertising equals $\frac{a}{2} \lambda_{i}^{2}$; and their objective is to maximize their profits. It is assumed that $2 a>\tau$.

The timing of events is as follows.

1. Each firm $i$ chooses its price, $p_{i}$, and how much to advertise, $\lambda_{i}$. The choices of $p_{1}, p_{2}, \lambda_{1}, \lambda_{2}$ are made simultaneously.
2. The consumers observe the stage 1 choices and then make their consumption decisions.

Assume that the parameters are such that the market is covered (i.e., all consumers who have seen at least one ad find it worthwhile to buy from one of the firms). Then one can verify (as we did in the course) that the full information demand that firm 1 faces equals

$$
\widehat{x}=\frac{p_{2}-p_{1}+\tau}{2 \tau}
$$

while firm 2's full information demand equals $1-\widehat{x}$.
a) Solve for the equilibrium prices and advertising levels.
b) The equilibrium profit levels are

$$
\pi_{1}^{*}=\pi_{2}^{*}=\pi^{*}=\frac{2 a}{\left[1+\sqrt{\frac{2 a}{\tau}}\right]^{2}} .
$$

We have $\frac{\partial \pi^{*}}{\partial a}>0$; that is, a firm's equilibrium profits go up if advertising becomes more costly. Explain the logic behind this comparative statics result.
c) Explain (i) what kind of model Kreps and Scheinkman (Bell Journal of Economics, 1983) studied and (ii) what result they could show. Also, (iii) discuss the limitations and implications of their analysis and result.

## Question 2 (network externalities and price discrimination)

Consider a market in which a monopoly firm is producing and selling a good. The firm is a price-setter and it is, for simplicity, assumed that it does not have any production costs. Consumers have unit demand; that is, they want to consume one unit of the good or otherwise nothing. There are two groups of (potential) consumers: "young consumers" and "old consumers". The mass of all consumers equals one. The fraction of old consumers is $\gamma \in(0,1)$, and the fraction of young consumers is $1-\gamma$. Within each group there is heterogeneity with respect to a taste parameter $\theta \in[0,1]$. The distribution of $\theta$ 's is the same for young and old consumers: it is uniform on $[0,1]$. The firm is able to price discriminate across the two groups (but not among consumers within each group). Thus, the price that any member of the group of young consumers must pay is denoted by $p_{y}$, and the price that any member of the group of old consumers must pay is denoted by $p_{o}$.

The net utility of an old consumer with taste parameter $\theta \in[0,1]$ is given by $\theta-p_{o}$ if buying the good, and zero otherwise. The net utility of a young consumer with taste parameter $\theta \in[0,1]$ is given by $\theta+\nu n_{o}^{e}-p_{y}$ if buying the good, and zero otherwise. Here, $\nu \in(0,4)$ is an exogenous parameter and $n_{o}^{e}$ is the number of old consumers that the young consumer expects will buy the good. Denote the number of young (old, respectively) consumers that actually buy the good by $n_{y}$ ( $n_{o}$, respectively).

One can show that the demand originating from the group of old consumers is given by

$$
n_{o}=\left\{\begin{array}{cc}
\gamma & \text { if } p_{o} \leq 0  \tag{1}\\
\gamma\left(1-p_{o}\right) & \text { if } p_{o} \in[0,1] \\
0 & \text { if } p_{o} \geq 1
\end{array}\right.
$$

One can also show that, if $p_{o} \in[0,1]$, then the fulfilled-expectations demand originating from the group of young consumers ${ }^{1}$ is given by

$$
n_{y}=\left\{\begin{array}{cc}
1-\gamma & \text { if } p_{y} \leq \nu \gamma\left(1-p_{o}\right)  \tag{2}\\
(1-\gamma)\left(1+\nu \gamma-p_{y}-\nu \gamma p_{o}\right) & \text { if } \nu \gamma\left(1-p_{o}\right) \leq p_{y} \leq 1+\nu \gamma\left(1-p_{o}\right) \\
0 & \text { if } p_{y} \geq 1+\nu \gamma\left(1-p_{o}\right)
\end{array}\right.
$$

[^0]The firm wants to choose $p_{o}$ and $p_{y}$ so as to maximize its profits, $\pi=p_{o} n_{o}+p_{y} n_{y}$. Denote the optimal choices of $p_{o}$ and $p_{y}$ by $p_{o}^{*}$ and $p_{y}^{*}$, respectively.

Assumption 1. The parameters of the model are such that $p_{o}^{*} \in[0,1]$ and $p_{y}^{*} \in\left[\nu \gamma\left(1-p_{o}^{*}\right), 1+\nu \gamma\left(1-p_{o}^{*}\right)\right]$.
a) Solve for the firm's optimal choices of $p_{o}$ and $p_{y}$, under Assumption 1. (Hint: The optimal choices will satisfy $p_{o}^{*}<\frac{1}{2}<p_{y}^{*}$ ).
b) Suppose a law was introduced that forced the firm to charge the same price, which we denote by $\bar{p}$, to all consumers. Assuming that $n_{o}$ is given by the middle line of (1) and that $n_{y}$ is given by the middle line of (2), one can show that the firm's optimal choice of this uniform price equals $\bar{p}^{* *}=\frac{1}{2}$. The consumer surplus of the group of old consumers and young consumers, respectively, given the optimal uniform price $\bar{p}^{* *}$, is defined by
$S_{o}^{* *} \equiv \gamma \int_{\bar{p}^{* *}}^{1}\left(\theta-\bar{p}^{* *}\right) d \theta \quad$ and $\quad S_{y}^{* *} \equiv(1-\gamma) \int_{\bar{p}^{* *}-\nu n_{o}^{* *}}^{1}\left(\theta+\nu n_{o}^{* *}-\bar{p}^{* *}\right) d \theta$,
where $n_{o}^{* *}=\gamma / 2$ is the number of old consumers who purchase the good. In contrast, the consumer surplus of the group of old consumers and young consumers, respectively, given the prices $p_{o}^{*}$ and $p_{y}^{*}$ that are optimal when price discrimination is allowed, is defined by
$S_{o}^{*} \equiv \gamma \int_{p_{o}^{*}}^{1}\left(\theta-p_{o}^{*}\right) d \theta \quad$ and $\quad S_{y}^{*} \equiv(1-\gamma) \int_{p_{y}^{*}-\nu n_{o}^{*}}^{1}\left(\theta+\nu n_{o}^{*}-p_{y}^{*}\right) d \theta$,
where $n_{o}^{*}=\gamma\left(1-p_{o}^{*}\right)$ is the number of old consumers who purchase the good. Use verbal reasoning to answer the following two questions. Which surplus should we expect to be the largest, $S_{o}^{* *}$ or $S_{o}^{*}$ ? Similarly, which surplus should we expect to be the largest, $S_{y}^{* *}$ or $S_{y}^{*}$ ? Make sure that you identify any effects that suggest that the ban on price discrimination is good, respectively bad, for the group's surplus. It may be that there are effects pointing in opposite directions and that it is hard to say anything about the net effect.
[You should answer the b) question only in terms of a verbal reasoning. Formal calculations will not be given any credit and must not be part of the answer.]
c) Derive the demand function stated in (2) (which is valid for $p_{o} \in[0,1]$ ).

## END OF EXAM


[^0]:    ${ }^{1}$ This fulfilled-expectations demand function specifies the number of young consumers who want to buy, given some prices $p_{y}$ and $p_{o}$ and given some beliefs $n_{o}^{e}$ about the number of old consumers who buy. Moreover, those beliefs are correct, $n_{o}^{e}=n_{o}$.

