Written Exam for the B.Sc. or M.Sc. in Economics summer 2014

Industrial Organization

Final Exam

May 30, 2014

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam paper consists of three pages in total, including this one

Question 1: Quantity setting, product differentiation, and strategic delegation

Consider a market with two (ex ante identical) firms, firm 1 and firm 2. The utility of a representative consumer, who consumes the quantities q_1 and q_2 of firm 1's and firm 2's goods, is given by

$$U(q_0, q_1, q_2) = a(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + q_2^2\right) - dq_1q_2 + q_0,$$

where q_0 is the quantity of a "Hicksian composite good" and $a > 0$ and $d \in \left(-\sqrt{2}, \sqrt{2}\right)$ are exogenous parameters. The consumer faces the following budget constraint: $p_0q_0 + p_1q_1 + p_2q_2 \leq m$, where p_i is the price of good i and m is the consumer's income. If we normalize the price of the Hicksian composite good to one ($p_0 = 1$), then the consumer's utility maximization problem leads to the following indirect demand functions:

$$p_1 = a - q_1 - dq_2, \qquad p_2 = a - q_2 - dq_1.$$

All consumers are assumed to be identical, so the above functions indeed represent market (not only individual) demand. Each firm has a constant marginal cost that is denoted by $c \in (0, a)$.

Firm i ($i = 1, 2$) is owned by individual O_i and managed by some other individual M_i. Each owner O_i can give an instruction to his or her own manager M_i about which objective function to maximize. In particular, M_i's objective function is (for $i = 1, 2$ and $i \neq j$):

$$V_i = \pi_i + x_iq_i = (a - c + x_i - q_i - dq_j)\, q_i,$$

where the variable x_i is chosen by the owner O_i. This means that if, for example, O_i chooses $x_i = c$, then this is an instruction to M_i to maximize the firm's revenues. However, O_i is allowed to choose any $x_i \in \Re$, so many other instructions are also possible.

The sequence of events is as follows:

1. O_1 and O_2 simultaneously choose x_1 and x_2, respectively.
2. M_1 and M_2 observe both x_1 and x_2. They then simultaneously choose q_1 and q_2, respectively.

The objective of owner O_i at stage 1 is to maximize the own firm's profits: $\pi_i = (a - c - q_i - dq_j)\, q_i$. The objective of manager M_i at stage 2 is to maximize the function V_i defined above.

(a) Solve for the subgame-perfect equilibrium values of x_1 and x_2. You may assume that the second-order conditions are satisfied.

(b) Suppose $d < 0$ and consider the four statements below. Which one of the statements is correct? Answer only with a number: (i), (ii), (iii) or (iv).

- (i) The goods that the firms produce are, in the eyes of the consumers, **substitutes**. At stage 2, the managers' choice variables q_1 and q_2 are **strategic substitutes**.
- (ii) The goods that the firms produce are, in the eyes of the consumers, **substitutes**. At stage 2, the managers' choice variables q_1 and q_2 are **strategic complements**.
- (iii) The goods that the firms produce are, in the eyes of the consumers, **complements**. At stage 2, the managers' choice variables q_1 and q_2 are **strategic substitutes**.
- (iv) The goods that the firms produce are, in the eyes of the consumers, **complements**. At stage 2, the managers' choice variables q_1 and q_2 are **strategic complements**.

(c) [*You are encouraged to answer this question even if having failed to solve question (a).*] What is the sign of the equilibrium values of x_1 and x_2 (positive, negative or zero) if $d > 0$, if $d < 0$ and if $d = 0$, respectively? Explain the intuition.[1]

(d) In the model described above, the demand functions are derived with the help of the so-called representative consumer approach. What is the other approach for deriving a demand function that we discussed in the course? Explain briefly the idea behind this other approach.

Question 2: Collusion in a Cournot oligopoly with a fixed production cost

Consider a market with two profit-maximizing, quantity-setting firms, indexed by $i = 1, 2$. Firm

[1] It is not obvious that there is one single correct answer to the question about the intuition. The students will be given credit for their ability to understand how the model works and their ability to reason about possible effects that may explain the results.

i's cost function is given by

$$C(q_i) = \begin{cases} 0 & \text{if } q_i = 0 \\ q_i + 8 & \text{if } q_i > 0, \end{cases}$$

where $q_i \geq 0$ is firm i's output. The inverse demand function is given by $p = 13 - q_1 - q_2$, where p is the market price. Also assume that there are infinitely many discrete time periods and that in each period the firms simultaneously choose their quantities. The firms' (common) discount factor is denoted by $\delta \in (0, 1)$. After each period the firms perfectly observe the rival's chosen quantity.

Consider the following grim trigger strategy for player 1:

$$q_{1,t} = \begin{cases} 6 & \text{...if } t \text{ is } \textbf{odd} \text{ and there have been} \\ & \text{no deviations in any period } \tau < t; \\ 0 & \text{...if } t \text{ is } \textbf{even} \text{ and there have been} \\ & \text{no deviations in any period } \tau < t; \\ 4 & \text{...if there has been at least one} \\ & \text{deviation in any period } \tau < t, \end{cases}$$

where $q_{1,t}$ is firm 1's quantity in period t. And consider the following grim trigger strategy for player 2:

$$q_{2,t} = \begin{cases} 6 & \text{...if } t \text{ is } \textbf{even} \text{ and there have been} \\ & \text{no deviations in any period } \tau < t; \\ 0 & \text{...if } t \text{ is } \textbf{odd} \text{ and there have been} \\ & \text{no deviations in any period } \tau < t; \\ 4 & \text{...if there has been at least one} \\ & \text{deviation in any period } \tau < t, \end{cases}$$

where $q_{2,t}$ is firm 2's quantity in period t.

(a) Investigate under what conditions the strategies above constitute a subgame-perfect Nash equilibrium of the infinitely repeated game.[2]

(b) In the context of infinitely repeated games, what is meant by the so-called folk theorem? Also, discuss briefly some criteria used by IO economists to select among multiple equilibria in an infinitely repeated oligopoly game.

End of Exam

[2] You may want to use the formula for an infinite geometric series: $\sum_{t=0}^{\infty} \delta^t = 1/(1-\delta)$ for $\delta \in (0, 1)$.