Written Exam for the B.Sc. or M.Sc. in Economics summer 2014

Industrial Organization

Final Exam

August 14, 2014

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam paper consists of three pages in total, including this one

Question 1: Collusion with fluctuating demand

Consider the following version of the Rotemberg-Saloner model.[1] In a market there are n ex ante identical firms, indexed by i. They produce a homogeneous good and each firm has a constant marginal cost $c \geq 0$. There are infinitely many, discrete time periods t (so $t = 1, 2, 3, \ldots$), and at each t the firms simultaneously choose their respective price, p_i^t. The firms' common discount factor is denoted by $\delta \in (0,1)$. As the good is homogeneous, demand is a function of the lowest price, $p^t = \min\{p_1^t, p_2^t, \ldots, p_n^t\}$. Demand is stochastic: with probability $\lambda \in (0,1)$, demand in period t is high, $q^t = D_H(p^t)$; and with probability $1-\lambda$, demand in period t is low, $q^t = D_L(p^t)$, with $D_H(p^t) > D_L(p^t)$ for all p^t. Demand realizations are independent across time. If two or more firms charge the same price, then these firms share the demand equally between themselves.

The firms can observe all rival firms' choice of price once it has been made. Moreover, the firms can observe the current period's demand realization, before choosing their price. However, the demand realizations in future periods are not known to the firms.

Let p_s^m be the state $s \in \{L, H\}$ monopoly price, i.e., the price that maximizes $(p-c)\,D_s(p)$. Exactly as in the course, consider a grim trigger strategy in which each firm starts out charging the price $p_s^t = p_s^m$ if the period t state is s. However, if there has been any deviation from that behavior by anyone of the firms in any previous period, then each firm plays $p_s^t = c$.

(a) Derive a (necessary and sufficient) condition for when the above grim trigger strategy is part of a subgame-perfect Nash equilibrium.[2] In particular, state the condition as $\delta \geq \delta_0$, where δ_0 is a function of n, λ, and the maximized industry profits in state s [i.e., of $\pi_s^m \equiv (p_s^m - c)\,D_s(p_s^m)$], but not a function of δ.

(b) *[You are encouraged to answer this question even if having failed to solve question (a).]* When is full collusion most difficult to sustain—in a high or in a low state? Explain the intuition. Answer verbally only.

(c) In the above model it was assumed that demand realizations are independent across time. Suppose that was not the case. In particular, suppose that there was a (strong) positive correlation between the state in period t and the state in period $t+1$. In this environment, when would you expect full collusion to be most difficult to sustain—in a high or in a low state? Explain how you reason. Answer verbally only.[3]

[1] Relative to the model in the course, this version is extended in two ways: the number of firms is arbitrary and the probability of a high demand state is not necessarily one-half. Otherwise the setup is identical to the one in the lecture slides and in the book.

[2] You may want to use the formula for an infinite geometric series: $\sum_{t=0}^{\infty} \delta^t = 1/(1-\delta)$ for $\delta \in (0,1)$.

Question 2: Discrimination against minorities and strategic incentives

The following is a model of discrimination, where this term is understood as a firm's refusal to serve members of a minority. It builds on Hotelling's linear city model, which we studied in the course.

There are two restaurants that are exogenously located at each end of Hotelling's linear city (as illustrated below).

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Restaurant 1		Restaurant 2

There are two groups of customers: *minority customers* (who have green hair) and *non-minority customers* (who have pink hair). Within each group, customers differ from each other with respect to their location on the Hotelling line, and for both groups the distribution of locations is uniform. The mass of all customers is normalized to one, and the fraction of minority customers equals $\gamma \in \left(0, \frac{1}{10}\right)$. All customers have so-called unit demand, meaning that they want to visit at most one restaurant. In particular, their preferences are exactly as in our textbook version of the model. Assume that the parameters of the model are such that the market is covered (i.e., all customers who are allowed to visit at least one of the restaurants find it worthwhile to do so). As we showed in the course, for any given prices p_1 and p_2, the location of the customer who is indifferent between the two restaurants equals

$$\overline{\theta} = \frac{p_2 - p_1 + 1}{2},$$

[3] It is not obvious that there is one single correct answer to this question. The students will be given credit for their ability to understand how the model works and their ability to reason about possible effects.

where the parameter τ in the customer's transportation cost function has been set equal to one. Each restaurant has a constant marginal cost of production, which is normalized to zero.

The timing of events is as follows.

1. The two restaurants simultaneously decide whether or not to serve the minority costumers. Denote this strategy by $x_i \in \{n, d\}$, where $x_i = d$ means that Restaurant i does not serve the minority costumers.

2. The restaurants observe x_1 and x_2 and then simultaneously choose p_1 and p_2. Price discrimination is not allowed: minority costumers, if they are served, must be charged the same price as non-minority costumers.

3. The customers observe the decisions at stages 1 and 2 and then decide which restaurant to visit. A minority customer cannot visit a restaurant that does not serve those customers. Instead such a customer must visit the other restaurant (if there is such a non-discriminating restaurant) or not visit any restaurant at all (if both restaurants refuse to serve members of the minority). The demands facing the two restaurants are therefore as in Table 1. Restaurant i's profit at stage 2, given some (x_1, x_2), can accordingly be written as

$$\pi_i = p_i D_i (p_1, p_2).$$

(a) Solve for all subgame-perfect Nash equilibria of the game described above (however, do not bother about the mixed-strategy equilibrium at stage 1).

Hint 1: The result should be that, at stage 1, the only (pure strategy) equilibria are $(x_1, x_2) = (d, n)$ and $(x_1, x_2) = (n, d)$; that is, one of the restaurants discriminates whereas the other one does not.

Hint 2: When solving for the equilibrium prices in the two symmetric subgames at stage 2, you are allowed to assume $p_1 = p_2$.

	$D_1(p_1, p_2)$	$D_2(p_1, p_2)$
$(x_1, x_2) = (n, n)$	$\overline{\theta}$	$1 - \overline{\theta}$
$(x_1, x_2) = (d, d)$	$(1-\gamma)\overline{\theta}$	$(1-\gamma)(1-\overline{\theta})$
$(x_1, x_2) = (d, n)$	$(1-\gamma)\overline{\theta}$	$1-(1-\gamma)\overline{\theta}$
$(x_1, x_2) = (n, d)$	$1-(1-\gamma)(1-\overline{\theta})$	$(1-\gamma)(1-\overline{\theta})$

Table 1: Demand functions

(b) Interpret your results: what is the economic logic that explains why the restaurants at stage 1 make the choices they make in the equilibria that you derived? When explaining that logic, make sure you answer the following two questions: (i) At stage 2, are the restaurants' choice variables strategic substitutes or strategic complements, and what is the significance of this? (ii) What is the significance of the assumption that each firm can observe the other firm's decision whether to discriminate before choosing the price at stage 2?

You are encouraged to attempt part (b) even if you have not been able to answer part (a). You can base your answer to part (b) on the suggestion in the first hint in part (a).

End of Exam