Written Exam for the B.Sc. or M.Sc. in Economics summer 2015

Industrial Organization

Final Exam

May 27, 2015

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

This exam paper consists of four pages in total, including this page

Attempt both questions.
Explain all the steps of your analysis and define any new notation that you use.
Show all the calculations that your analysis relies on.

Question 1: Informational advertising and strategic incentives

Consider the following model of informative advertising. There are two firms that are exogenously located at each end of Hotelling's linear city. The consumers, who have so-called unit demand,[1] are uniformly distributed on the line and their total mass equals one. A consumer with location x receives the net utility $U_1(x) = r - \tau x^2 - p_1$ if buying from firm 1 and the net utility $U_2(x) = r - \tau(1-x)^2 - p_2$ if buying from firm 2, where the notation is the same as in the course.[2] If not buying from either firm, a consumer gets zero net utility.

A consumer has the option to buy from firm i if and only if she has been reached by an ad from that firm. The fraction of consumers who receive an ad from firm i is denoted by λ_i. Therefore, the fraction $\lambda_1(1-\lambda_2)$ can buy from firm 1 only; the fraction $\lambda_2(1-\lambda_1)$ can buy from firm 2 only; the fraction $\lambda_1\lambda_2$ can buy from either firm (as in the standard Hotelling model). The remaining consumers have not seen any ad and cannot buy the good at all. The firms have a constant marginal cost of production, denoted by c; their cost of advertising equals $a_i\Phi(\lambda_i)$; and their objective is to maximize their profits. It is assumed that $a_1 \geq a_2 > 0$ and that, for all $\lambda_i \in [0,1]$,

$$\Phi'(\lambda_i) > 0, \qquad \Phi''(\lambda_i) > 0, \qquad \Phi(0) = 0.$$

The timing of events is as follows.

1. Each firm i chooses how much to advertise, $\lambda_i \in [0,1]$. The choices of λ_1 and λ_2 are made simultaneously.
2. The firms observe the stage 1 decisions. Then each firm i chooses its price, $p_i \geq 0$. The choices of p_1 and p_2 are made simultaneously.
3. The consumers observe the stage 1 and 2 choices and then make their consumption decisions.

Assume that the parameters are such that the market is covered (i.e., all consumers who have seen at least one ad find it worthwhile to buy from one of the firms). Then one can verify (as we did in the course) that the *full information*[3] demand that firm 1 faces equals

$$\widehat{x} = \frac{p_2 - p_1 + \tau}{2\tau},$$

while firm 2's full information demand equals $1 - \widehat{x}$. Firm 1's profit can be written as

$$\pi_1 = \lambda_1(p_1 - c)\left[1 - \lambda_2 + \lambda_2\left(\frac{p_2 - p_1 + \tau}{2\tau}\right)\right] - a_1\Phi(\lambda_1),$$

and firm 2's profit is defined analogously.

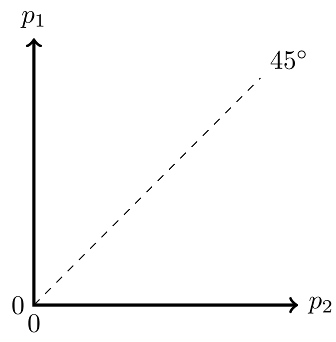
(a) Given some stage 1 choices $\lambda_1 > 0$ and $\lambda_2 > 0$, let p_1^* and p_2^* denote the equilibrium prices at stage 2. Solve for these equilibrium prices (using mathematics). Also illustrate this stage 2 equilibrium (as the intersection of the firms' best reply functions) in a diagram like the one below. Are the firms' stage 2 choice variables (i) strategic substitutes, (ii) strategic complements or (iii) neither? With the help of the

[1] This means that the consumers demand either exactly one unit of the good or no unit at all.

[2] So p_i is firm i's price, $r > 0$ is the utility the consumer would receive from consuming the good for free and without having to travel, and $\tau > 0$ is a parameter.

[3] A firm's *full information demand* is the demand that the firm would face if all consumers were able to buy from either firm (so if $\lambda_1 = \lambda_2 = 1$).

diagram you have drawn, explain what the effect of an increase in λ_1 is on p_1^* and p_2^*. Do these prices go up or down?



At stage 1, firm 1 maximizes the following profit expression with respect to λ_1:

$$\pi_1 = \lambda_1 \left(p_1^* - c\right) \left[1 - \lambda_2 + \lambda_2 \left(\frac{p_2^* - p_1^* + \tau}{2\tau}\right)\right] - a_1 \Phi\left(\lambda_1\right). \quad (1)$$

Suppose the second-order condition to this problem is satisfied and that the optimal choice of λ_1 is interior (so $\lambda_1 \in (0,1)$). Differentiating (1) w.r.t. λ_1 yields

$$\frac{\partial \pi_1}{\partial \lambda_1} = \left(p_1^* - c\right) \left[1 - \lambda_2 + \lambda_2 \left(\frac{p_2^* - p_1^* + \tau}{2\tau}\right)\right] + \frac{\lambda_1 \lambda_2 \left(p_1^* - c\right)}{2\tau} \frac{\partial p_2^*}{\partial \lambda_1} - a_1 \Phi'\left(\lambda_1\right). \quad (2)$$

(b) Copy equation (2) and indicate in your copy which terms in (2) that represent a *direct* effect on profit of increasing λ_1 and which terms represent an *indirect* (or *strategic*) effect on profit of increasing λ_1. Explain these two effects in economic terms.

(c) Next, set $c = 0$ and $a_1 = a_2 = a$. Consider the possibility of an equilibrium of the overall game that is symmetric — that is, an equilibrium in which $p_1 = p_2 = p^*$ and $\lambda_1 = \lambda_2 = \lambda^*$.

- *Claim: An advertising level λ^* is part of such a symmetric equilibrium if and only if the following condition holds:*

$$f(\lambda^*) = a\Phi'\left(\lambda^*\right). \quad (3)$$

Specify a function $f(\lambda^*)$ that makes the claim true.

Question 2: Cournot competition with asymmetric firms

Consider a market in which $n \geq 2$ firms produce and sell a homogeneous good and compete à la Cournot. The firms interact just once and they make their output decisions simultaneously. As the products are identical, inverse demand is a function of the firms' total output:

$$p = P\left(\sum_{i=1}^{n} q_i\right),$$

where q_i is firm i's output. This demand function is downward-sloping, $P'\left(\sum_{i=1}^{n} q_i\right) < 0$. Firm i's cost function is denoted by $C_i\left(q_i\right)$. This cost function is strictly increasing and weakly convex, $C_i'\left(q_i\right) > 0$ and $C_i''\left(q_i\right) \geq 0$. Firm i's profit function is

$$\pi_i\left(q_1, \ldots, q_n\right) = q_i P\left(\sum_{j=1}^{n} q_j\right) - C_i\left(q_i\right).$$

Assume that the second-order condition associated with firm i's problem of maximizing π_i w.r.t. q_i is satisfied. Denote the equilibrium quantities by $(q_1^*, \ldots, q_n^*)$.

Let the Lerner index for firm i, given equilibrium play, be defined by

$$L_i \stackrel{\text{def}}{=} \frac{P\left(\sum_{j=1}^{n} q_j^*\right) - C_i'\left(q_i^*\right)}{P\left(\sum_{j=1}^{n} q_j^*\right)}.$$

Also, let firm i's market share be denoted by $\alpha_i \stackrel{\text{def}}{=} q_i^* / \left(\sum_{j=1}^{n} q_j^*\right)$. Finally, let the *inverse* of the price elasticity of demand be denoted by

$$\frac{1}{\eta} \stackrel{\text{def}}{=} -\frac{\left[\sum_{j=1}^{n} q_j^*\right] P'\left(\sum_{j=1}^{n} q_j^*\right)}{P\left(\sum_{j=1}^{n} q_j^*\right)}.$$

(a) Show that $L_i = \alpha_i/\eta$. Also, explain in words why firm i's Lerner index is increasing in α_i and decreasing in η (i.e., explain the economic intuition behind these two comparative statics results).

Define the "average Lerner index" as follows:

$$\widehat{L} \stackrel{\text{def}}{=} \sum_{j=1}^{n} \alpha_j L_j = \frac{P\left(\sum_{j=1}^{n} q_j^*\right) - \sum_{j=1}^{n} \alpha_j C_j'(q_j)}{P\left(\sum_{j=1}^{n} q_j^*\right)}.$$

(b) Show that

$$\widehat{L} = \frac{I_H}{\eta}, \tag{4}$$

where $I_H \stackrel{\text{def}}{=} \sum_{i=1}^{n} \alpha_i^2$ is the Herfindahl index. Also, consider the following claim: *The result in (4) supports the idea that market concentration is associated with market power.* Explain the reasoning behind this claim (make sure that your explanation clarifies what the Herfindahl index is meant to measure).

Now consider the following special case of the above model. Indirect demand is given by

$$p = a - b \sum_{j=1}^{n} q_j,$$

where $a > 0$ and $b > 0$ are parameters. Firm i has a (constant) marginal cost c_i (with $a > c_i \geq 0$), and no fixed costs. Therefore firm i's profit is

$$\pi_i = \left(a - c_i - b \sum_{j=1}^{n} q_j \right) q_i.$$

(c) Assume that the parameters of this model are such that, at the equilibrium, all firms are active (i.e., $q_i^* > 0$ for all i). Solve for firm i's equilibrium quantity. Also, use the result that you obtain, and any further calculations that you may require, to argue formally that a firm in this market gains (in terms of its profit) by being relatively efficient (i.e., by having a cost parameter that is low relative to the rivals' cost parameters).

End of Exam