

Written Exam for the B.Sc. or M.Sc. in Economics summer 2015

**Industrial Organization**

Final (Resit) Exam

August 13, 2015

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title which was followed by “eksamen på dansk” in brackets, you must write your exam paper in Danish.

**This exam paper consists of four pages in total, including this page**

Attempt both questions.

Explain all the steps of your analysis and define any new notation that you use.

Show all the calculations that your analysis relies on.

## Question 1: Can price signal quality?

Consider a market that is served by a monopoly firm. This firm sells a single good, it wants to maximize its profit, and its choice variable is the price of the good,  $p$ . The firm's marginal production cost is constant and denoted by  $c \in [0, 1)$ . The potential consumers in the market have so-called unit demand<sup>1</sup> and they differ from each other with respect to a taste parameter  $\theta$ . The distribution of  $\theta$ 's is uniform on the interval  $[0, 1]$  and the mass of all potential consumers equals one. The net utility of a person with taste parameter  $\theta$  is given by

$$\begin{cases} \theta + s - p & \text{if buying} \\ 0 & \text{if not buying,} \end{cases}$$

where  $s \in (0, 1)$  is an exogenous parameter that represents the quality of the good — the larger is  $s$ , the higher is the quality.

*Claim:* The demand for the firm's good is given by the following expression:

$$Q(p) = \begin{cases} 1 & \text{if } p < s \\ 1 + s - p & \text{if } p \in [s, 1 + s] \\ 0 & \text{if } p > 1 + s. \end{cases}$$

(a) Prove the above claim (i.e., derive the demand for the firm's good). Moreover, show that the firm's optimally chosen price and its maximized profit, respectively, equal

$$p^* = \frac{1 + s + c}{2} \quad \text{and} \quad \pi^* = \frac{(1 + s - c)^2}{4}.$$

Now consider the following variation of the game described above. First set  $c = 0$ . Moreover, suppose that the quality is either good or bad ( $s = s_g$

<sup>1</sup>That is, they either buy one unit of the good or nothing at all.

or  $s = s_b$ , with  $0 < s_b < s_g < 1$ ). The firm itself knows the value of  $s$  from the outset of the game. A fraction  $\lambda \in (0, 1)$  of the consumers also know  $s$  from the start of the game. However, the remaining consumers initially know only the prior distribution of  $s$ , which is given by  $\Pr[s = s_g] = \mu \in (0, 1)$ . The distribution of the informed and the uninformed consumers' taste parameters  $\theta$  is the same (namely, uniform on the interval  $[0, 1]$ ).

The sequence of events is as follows: (1) The firm, knowing  $s$ , chooses its price  $p$ . (2) All consumers observe the chosen price, and the uninformed consumers use this observation to update their beliefs about  $s$ . (3) All consumers decide, simultaneously, whether or not to buy the good.

Within this framework we want to study the following question: Can the high-quality type of firm, by charging a particularly high price, signal to the uninformed consumers that its good indeed is of a high quality? To answer this question, consider the possibility of a separating equilibrium in which the low-quality firm (the bad type) charges the price  $p_b$  and the high-quality firm (the good type) charges the price  $p_g$ , with  $p_b \neq p_g$ .<sup>2</sup> One can show that, in any separating equilibrium, the bad type chooses the price that maximizes its profits, given that the consumers believe that the quality is bad ( $s = s_b$ ). That is, the firm's chosen price and its maximized profit, respectively, equal

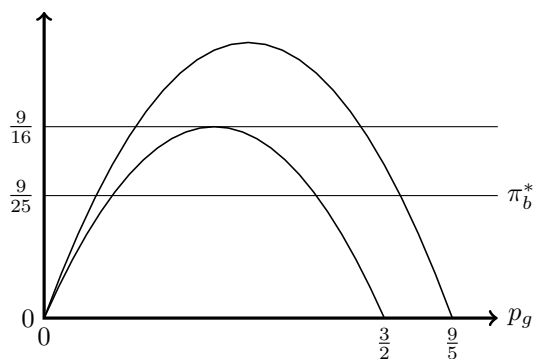
$$p_b^* = \frac{1 + s_b}{2} \quad \text{and} \quad \pi_b^* = \frac{(1 + s_b)^2}{4}.$$

Assume the following out-of-equilibrium beliefs: Consumers believe that the quality is bad if they observe any price that differs from the equilibrium prices  $p_b$  and  $p_g$ .

<sup>2</sup>The equilibrium concept that we employ is, as in our analysis of the Milgrom-Roberts model in the course, perfect Bayesian equilibrium.

(b) Formulate two necessary conditions that a separating equilibrium must satisfy and which, given the out-of-equilibrium beliefs assumed above, also are sufficient for such an equilibrium to exist. State the two conditions in terms of mathematics. Also explain in words what each one of the conditions says.

(c) Assume next that  $s_g = \frac{4}{5}$ ,  $s_b = \frac{1}{5}$ , and  $\lambda = \frac{1}{2}$ . Then the two conditions can be illustrated with the help of the figure below. Copy this figure exactly and indicate in your version of the figure where the high-quality type's price in a separating equilibrium ( $p_g^*$ ) can be located.



(d) In the course we studied another signaling game — the limit pricing model due to Milgrom and Roberts (or, rather, Tirole's simplified version of their analysis). In relation to that model, explain the following things (in words only):

- (i) What is meant by limit pricing and predatory pricing?
- (ii) Scholars at the University of Chicago have argued that predatory pricing cannot be rational (and, hence, it does not occur). Explain the reasoning behind this argument.
- (iii) How did Milgrom and Roberts (in Tirole's simplified version) model limit pricing? Focus on the key model assumptions and explain how the logic of the model works.

## Question 2: A market with vertically related firms

Consider a market in which consumers have unit demand, with valuations  $r$ . The  $r$ 's are uniformly distributed on the unit interval  $[0, 1]$  and the total mass of consumers equals one. The good in this market is sold by a monopoly retailer, Firm D (the downstream firm). At the outset of the game the consumers are not aware of the existence of this firm and therefore they do not know about their opportunity to purchase the good. However, by choosing an advertising level  $\lambda \in [0, 1]$ , at a cost  $\frac{1}{2}\lambda^2$ , Firm D can reach a fraction  $\lambda$  of all consumers and thereby make them aware of their opportunity to buy the good at the price  $p \in [0, 1]$  (all consumers on the unit line have the same likelihood of being reached). All the other consumers, making up the fraction  $1 - \lambda$ , can by assumption not buy the good. The net utility of a consumer with valuation  $r$  is given by:

$$u = \begin{cases} r - p & \text{if buying} \\ 0 & \text{if not buying.} \end{cases}$$

All the units of the good that Firm D sells are supplied by an upstream firm, Firm U. The price that Firm D must pay Firm U to obtain  $q$  units of the good equals  $T = wq$ , where  $w$  is the (constant) per-unit wholesale price chosen by Firm U. In order to produce the quantity  $q$  of the good, Firm U must incur the production cost  $cq$ , where  $c \in [0, 1]$  is an exogenous parameter. Firm D does not have any costs on top of the costs of buying the goods from Firm U and the advertising costs. Each firm wants to maximize its profits. The profit functions of the two firms can be written as

$$\pi^D = \lambda(1 - p)(p - w) - \frac{1}{2}\lambda^2$$

and

$$\pi^U = \lambda(1 - p)(w - c),$$

respectively.

The sequence of events is as follows.

1. Firm U chooses  $w$ .
2. Firm D observes  $w$  and then chooses  $p$  and  $\lambda$ .
3. Each consumer is reached, or not reached, by the advertisements. Those who are reached decide whether to buy.

Answer the following questions:

(a) Solve for the subgame-perfect equilibrium values of  $p$ ,  $w$  and  $\lambda$ . You may assume that the second-order conditions are satisfied.

(b) Suppose the firms integrate and become one single firm. Calculate again the subgame-perfect equilibrium values of  $p$  and  $\lambda$ . You may assume that the second-order conditions are satisfied.

(c) Would you expect aggregate consumer surplus to be largest under integration or under non-integration? Spell out your reasons and the logic. Answer verbally only.

(d) Suppose now that, as under (a), the firms are not integrated. Moreover, the retail price  $p$  is now chosen *not* by Firm D at stage 2, but by Firm U at stage 1 (we can interpret this as resale price maintenance, RPM). Everything else in the model is unchanged. Would you expect RPM, modeled like this, to give rise to the same outcome (i.e., the same equilibrium values of  $p$  and  $\lambda$ ) as under integration? Spell out your reasons and the logic. Answer verbally only.

**End of Exam**